

# Chapter 1

## Exercise 1: Electrostatics.

### 1.1 PROBLEM: two coaxial conductors of square section

We have two coaxial conductors of square section, the interior one with a 1000 V electrostatic potential, respect to the exterior one. The gap between them is filled with air. The longitudinal length is larger than the transversal lengths, which are 50 mm for the interior conductor, and 150 mm for the interior face of the external conductor.

Boundary conditions and materials are shown in fig. 1.1.

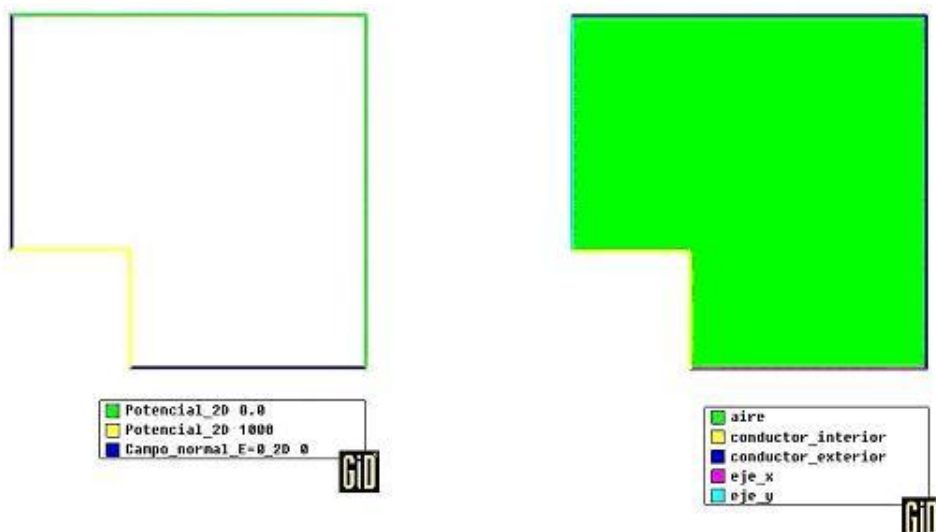
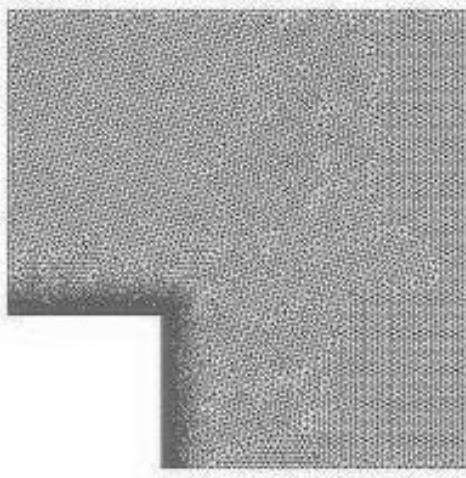


Figure 1.1: Materials and boundary conditions. Only one quarter of the full geometry is represented.

As we want to study the variation of the fields along the diagonal, we can include the diagonal as a part of our geometry.

In the figure 1.2 a possible mesh to solve the problem is shown. In order to draw XY graphs in every line, we have to activate the option **Meshing** → **Mesh Criteria** → **Mesh** → **Lines** and then select the seven geometry lines (two lines of the interior conductor + two lines of the exterior conductor + 2 symmetry lines + diagonal). See fig. 1.2.



**Figure 1.2: Mesh used.**

### 1. Theoretical background.

According to the first Maxwell's law:

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad (1.1)$$

where  $\rho$  is the electric charge density and  $\epsilon_0$  is just a constant.

We can also define the electrostatic field  $\vec{E}$  as the gradient of a electrostatic potential  $V$ , this is:

$$\vec{E} = -\nabla V \quad (1.2)$$

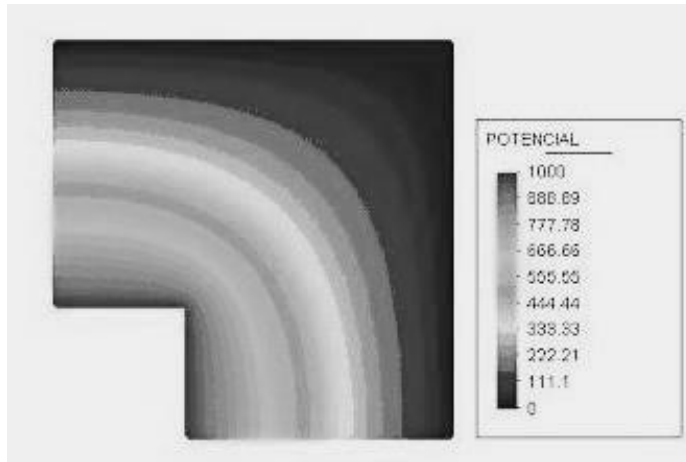
Substituting equation (1.2) in (1.1), we obtain the final differential equation known as the Poisson equation:

$$-\nabla^2 V = \frac{\rho}{\epsilon_0} \quad (1.3)$$

The equation (1.2) is the one we are going to solve by the Finite Element Method.

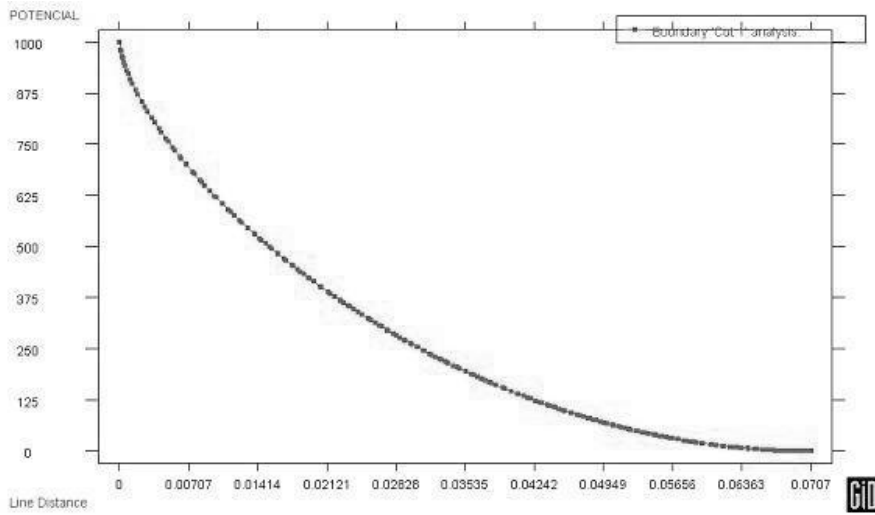
## 2. Postprocess.

The electrostatic potential in the air gap between both conductors is represented in figure 1.3.

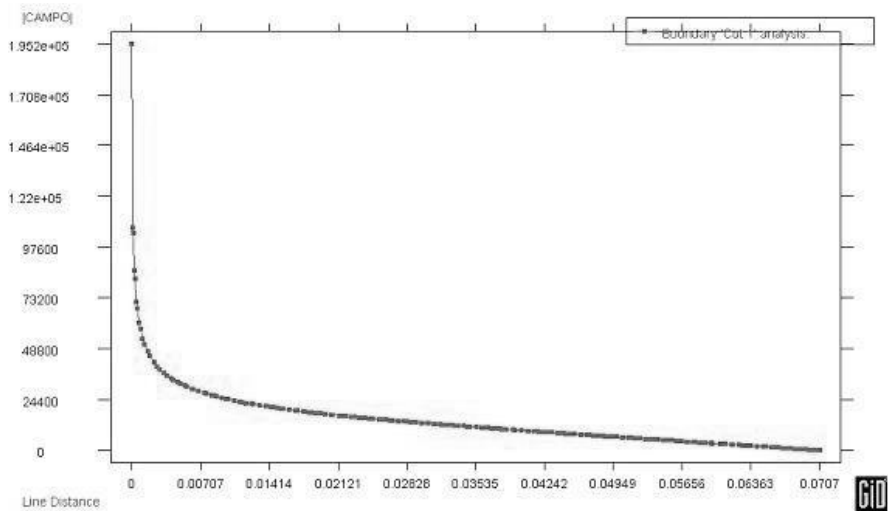


**Figure 1.3: Graph of the electric potential.**

The figure 1.4 show the potential variation along the diagonal and the figure 1.5 shows the variation of the electric field module  $|\vec{E}|$  in the diagonal. The maximum value of the electric field module can be seen in figure 1.5.



**Figure 1.4: Potential variation through the diagonal.**



**Figure 1.5: Electric field module through the diagonal.**