DETERMINING A CONIC

In the affine euclidean space we fix the coordinate system R={O,{e_1, e_2}}.

Determine the conic C with center Z(1/2,0) and going throught the points P(0,1), Q(1,1) .

Method 1. With 5 points of the conic. Going also through M(1,0).

```
> restart:with(linalg):
> A:=matrix([[a00,a01,a02],[a01,a11,a12],[a02,a12,a22]]);
```

$$A := \begin{bmatrix} a00 & a01 & a02 \\ a01 & a11 & a12 \\ a02 & a12 & a22 \end{bmatrix} \quad \textbf{(1.1.1)}$$

```
> P:=matrix(3,1,[1,0,1]);Q:=matrix(3,1,[1,1,1]):M:=matrix(3,1,
  [1,1,0]):
```

$$P := \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \textbf{(1.1.2)}$$

```
> ecu1:=evalm(transpose(P)&*A&*P)[1,1];
```

$$ecu1 := a00 + 2\,a02 + a22 \quad \textbf{(1.1.3)}$$

```
> ecu2:=evalm(transpose(Q)&*A&*Q)[1,1];
```

$$ecu2 := a00 + 2\,a01 + 2\,a02 + a11 + 2\,a12 + a22 \quad \textbf{(1.1.4)}$$

```
> ecu3:=evalm(transpose(M)&*A&*M)[1,1];
```

$$ecu3 := a00 + 2\,a01 + a11 \quad \textbf{(1.1.5)}$$

```
> Pp:=evalm(2*Z-P);Qp:=evalm(2*Z-Q);
```

$$Pp := \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

(1.1.6)

$$Qp := \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad (1.1.6)$$

```
> ecu4:=evalm(transpose(Pp)&*A&*Pp)[1,1];
```

$$ecu4 := a00 + 2\,a01 - 2\,a02 + a11 - 2\,a12 + a22 \quad (1.1.7)$$

```
> ecu5:=evalm(transpose(Qp)&*A&*Qp)[1,1];
```

$$ecu5 := a00 - 2\,a02 + a22 \quad (1.1.8)$$

```
> solve({ecu1,ecu2,ecu3,ecu4,ecu5,ecu6},{a00,a01,a02,a11,a12,
  a22});
```

$$\{a00 = 0, a01 = a01, a02 = 0, a11 = -2\,a01, a12 = 0, a22 = 0\} \quad (1.1.9)$$

Therefore the matrix of the conic is:

```
> AA:=matrix([[0,1,0],[1,-2,0],[0,0,0]]);
```

$$AA := \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (1.1.10)$$

```
> X:=matrix(3,1,[1,x,y]):
> CC:=simplify(evalm(transpose(X)&*AA&*X)[1,1]);
```

$$CC := 2\,x - 2\,x^2 \quad (1.1.11)$$

Which is a degenerate conic, two parallel lines x=0 and x=1 (it has a line of centers x=1/2).

Method 2. With conic bundles. Going through the point T(-1,0).

```
> PZ:=det(matrix([[1,x,y],[1,0,1],[1,1/2,0]]));
```

$$PZ := -\frac{1}{2} + \frac{1}{2}\,y + x \quad (1.2.1)$$

```
> QZ:=det(matrix([[1,x,y],[1,1,1],[1,1/2,0]]));
```

$$QZ := -\frac{1}{2} - \frac{1}{2}\,y + x \quad (1.2.2)$$

```
> PQ:=det(matrix([[1,x,y],[1,0,1],[1,1,1]]));
```

$$PQ := -1 + y \quad (1.2.3)$$

```
> Pp:=evalm(2*Z-P);Qp:=evalm(2*Z-Q);
```

$$Pp := \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$Qp := \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad (1.2.4)$$

```
> PpQp:=det(matrix([[1,x,y],[1,1,-1],[1,0,-1]]));
```

(1.2.5)

$$PpQp := -1 - y \tag{1.2.5}$$

```
> C1:=expand(PZ*QZ);
```

$$C1 := \frac{1}{4} - x - \frac{1}{4} y^2 + x^2 \tag{1.2.6}$$

```
> C2:=expand(PQ*PpQp);
```

$$C2 := 1 - y^2 \tag{1.2.7}$$

```
> H:=a*C1+b*C2;
```

$$H := a \left(\frac{1}{4} - x - \frac{1}{4} y^2 + x^2 \right) + b \left(1 - y^2\right) \tag{1.2.8}$$

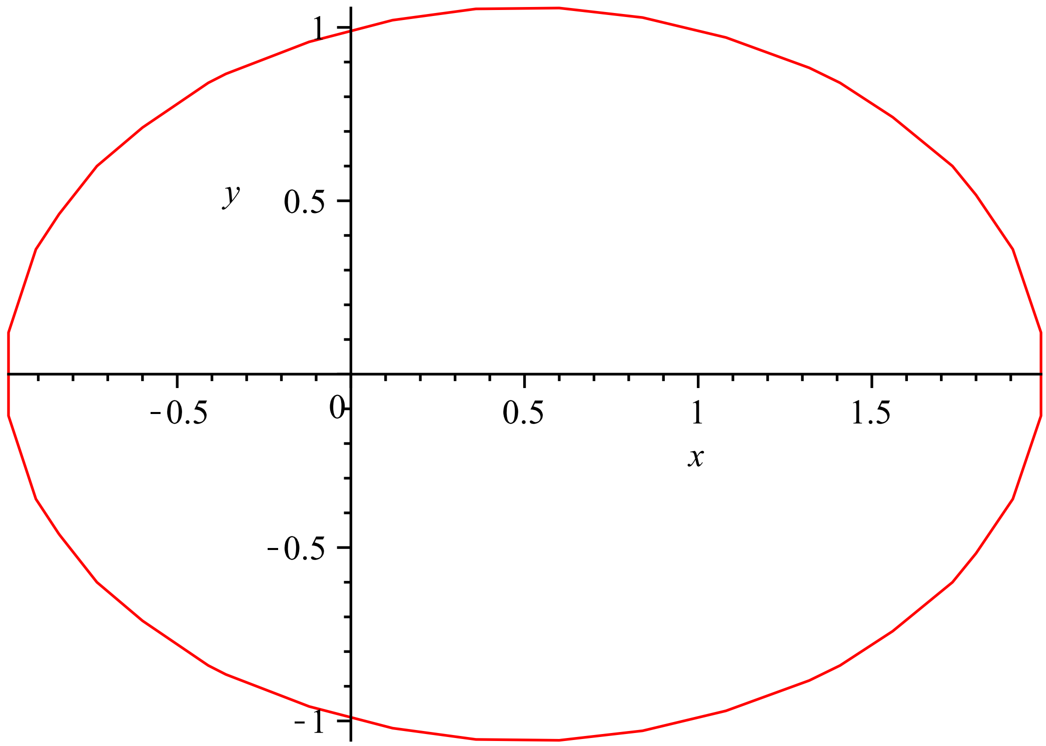
```
> subs({x=-1,y=0},H)=0;
```

$$\frac{9}{4} a + b = 0 \tag{1.2.9}$$

```
> C:=-4/9*C1+C2;
```

$$C := \frac{8}{9} + \frac{4}{9} x - \frac{8}{9} y^2 - \frac{4}{9} x^2 \tag{1.2.10}$$

```
> with(plots):
> implicitplot(C,x=-3..3,y=-3..3);
```



1
0.5
y
-0.5
-1
-0.5
0
0.5
1
1.5
x