DIAGONALIZATION

Obtain the eigenvalues, their multiplicities and dimension of the eigenspaces of the matrix A.

```
> restart;with(linalg):
> A:=matrix(3,3,[1,2,10,2,1,10,-1,-1,-6]);
```

$$A := \begin{bmatrix} 1 & 2 & 10 \\ 2 & 1 & 10 \\ -1 & -1 & -6 \end{bmatrix} \tag{1.1}$$

a) Compute the eigenvalues of f.

```
> I3:=diag(1,1,1);
```

$$I3 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1.1.1}$$

```
> M:=evalm(A-lambda*I3);
```

$$M := \begin{bmatrix} 1-\lambda & 2 & 10 \\ 2 & 1-\lambda & 10 \\ -1 & -1 & -6-\lambda \end{bmatrix} \tag{1.1.2}$$

```
> characteristic_polynomial:=det(A-lambda*I3);
```

$$characteristic\_polynomial := -2 - 5\,\lambda - 4\,\lambda^2 - \lambda^3 \tag{1.1.3}$$

```
> solve(characteristic_polynomial,lambda);
```

$$-2,\ -1,\ -1 \tag{1.1.4}$$

```
> factor(characteristic_polynomial);
```

$$-(\lambda+2)\,(\lambda+1)^2 \tag{1.1.5}$$

```
> lambda1:=-2;lambda2:=-1;
```

$$\lambda 1 := -2$$

$$\lambda 2 := -1 \tag{1.1.6}$$

b) Compute the dimension and a basis of the eigenspaces of f.

```
> M1:=evalm(A-lambda1*I3);
```

$$M1 := \begin{bmatrix} 3 & 2 & 10 \\ 2 & 3 & 10 \\ -1 & -1 & -4 \end{bmatrix} \quad (1.2.1)$$

```
> rank(M1);
```

$$2 \quad (1.2.2)$$

```
> zero:=matrix(3,1,[0,0,0]);
```

$$zero := \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (1.2.3)$$

```
> linsolve(M1,zero);
```

$$\begin{bmatrix} -2\,\_t_{1_1} \\ -2\,\_t_{1_1} \\ \_t_{1_1} \end{bmatrix} \quad (1.2.4)$$

A basis of the eigenspace associated to λ1 is:
B1={(-2,-2,1)}

```
> M2:=evalm(A-lambda2*I3);
```

$$M2 := \begin{bmatrix} 2 & 2 & 10 \\ 2 & 2 & 10 \\ -1 & -1 & -5 \end{bmatrix} \quad (1.2.5)$$

```
> rank(M2);
```

$$1 \quad (1.2.6)$$

```
> zero:=matrix(3,1,[0,0,0]);
```

$$zero := \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (1.2.7)$$

```
> linsolve(M2,zero);
```

$$\begin{bmatrix} -\_t_{1_1} - 5\,\_t_{1_2} \\ \_t_{1_1} \\ \_t_{1_2} \end{bmatrix} \quad (1.2.8)$$

A basis of the eigenspace associated to λ1 is:
B2={(-1,1,0), (-5,0,1)}

c)Check the answers with LINALG:

```
> charpoly(A,lambda);
```

$$2+5\lambda+4\lambda^2+\lambda^3 \quad (1.3.1)$$

```
> eigenvalues(A);
```

$$-2,\ -1,\ -1 \quad (1.3.2)$$

```
> eigenvectors(A);
```

$$\left[-1, 2, \left\{\left[\begin{array}{ccc}-5 & 0 & 1\end{array}\right], \left[\begin{array}{ccc}-1 & 1 & 0\end{array}\right]\right\}\right], \left[-2, 1, \left\{\left[\begin{array}{ccc}-2 & -2 & 1\end{array}\right]\right\}\right] \quad (1.3.3)$$

d) Find the matrix associated to f in the basis B'=B1∪B2 of eigenvectors of f.

The matrix of change of basis from B' to B is:

```
> MBpB:=matrix(3,3,[-2,-1,-5,-2,1,0,1,0,1]);
```

$$MBpB := \left[\begin{array}{ccc}-2 & -1 & -5\\ -2 & 1 & 0\\ 1 & 0 & 1\end{array}\right] \quad (1.4.1)$$

The matrix associated to f in the basis B' is:

```
> MfBp:=evalm(inverse(MBpB)&*A&*MBpB);
```

$$MfBp := \left[\begin{array}{ccc}-2 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{array}\right] \quad (1.4.2)$$

Therefore f is diagonalizable.

Let us consider the linear map f(x,y,z)=(x+3y+3z, 5x+3z, 6x+6y+3z).

```
> restart;with(linalg):
```

a) Obtain the matrix A associated to f in the standard basis of R^3.

b) Compute the eigenvalues of f.

c) Compute the dimension and a basis of the eigenspaces of f.

d) Check the answers with LINALG:

- e) **Find the matrix associated to f in the basis B'=B1∪B2∪B3 of eigenvectors of f.**

- **Repete the previous exercise with the endomorphism g(x,y,z)= (7x+10y+4z, -3x-4y-3z, -x-2y).**