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UNIVERSIDAD POLITÉCNICA DE MADRID
Escuela Universitaria de
Ingeniería Técnica Aeronáutica

HELICOPTERS

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ROTOR AERODYNAMICS
Momentum Theory
Vertial Climb

Inital thoughts:

- Vertical climb flight is the easiest flight condition.
  - The velocities in the rotor plane are symmetrical about the rotation axis.
  - The aerodynamic forces on the blades are constant regardless of their angular position.
  - The plane formed by the blade rotor tips is perpendicular to the drive shaft.

Inital thoughts:

- Vertical climbing flight is the easiest flight condition.
- There are different theories for studying rotor aerodynamics.
  - The momentum theory.
  - The blade element theory.
  - The vortex theory.

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ROTOR AERODYNAMICS

VERTICAL CLIMBING FLIGHT

MOMENTUM THEORY

- ✓ **Thrust and Power Calculations.**
- ✓ **Hover flight.**
- ✓ **Velocity and Power ratios.**
- ✓ **Thurst and Power coefficients.**
- ✓ **Dimensionless expressions.**

INITIAL ASSUMPTIONS

- High values of Re number flow.
- Replace the original rotor with blades that rotate for a totally porous disc of the same radius (**R**) as the rotor replaced.
- We assume the affected flow through the disc is defined by the streamtube.
- The fluid flow in the streamtube is considered to be unidimensional, steady and incompressible.
- The effects of the rotation of the slipstream and loses in the blade tips, are negleted.

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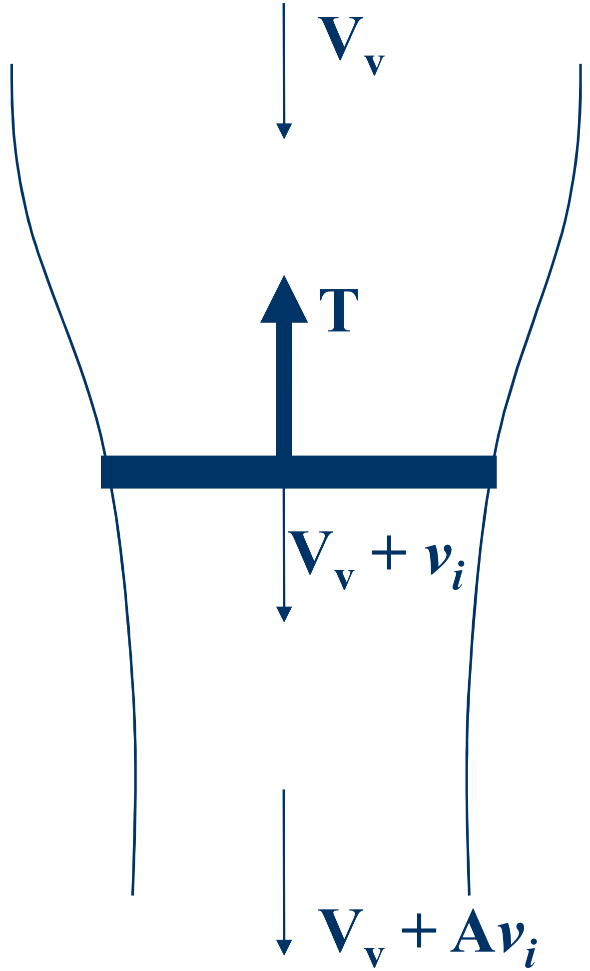
MATHEMATICAL MODEL



- The velocity of the upstream rotor fluid is the vertical velocity of the rotor. (V_v).
- The fluid velocity in this section of the disc is the climbing velocity of the rotor plus the induced velocity by the lifting disc. (Vv+*vi*).
- The velocity of the downstream rotor fluid is the vertical velocity of the rotor plus the induced velocity in the disc plane affected by a factor of A. (Vv+A*vi*).

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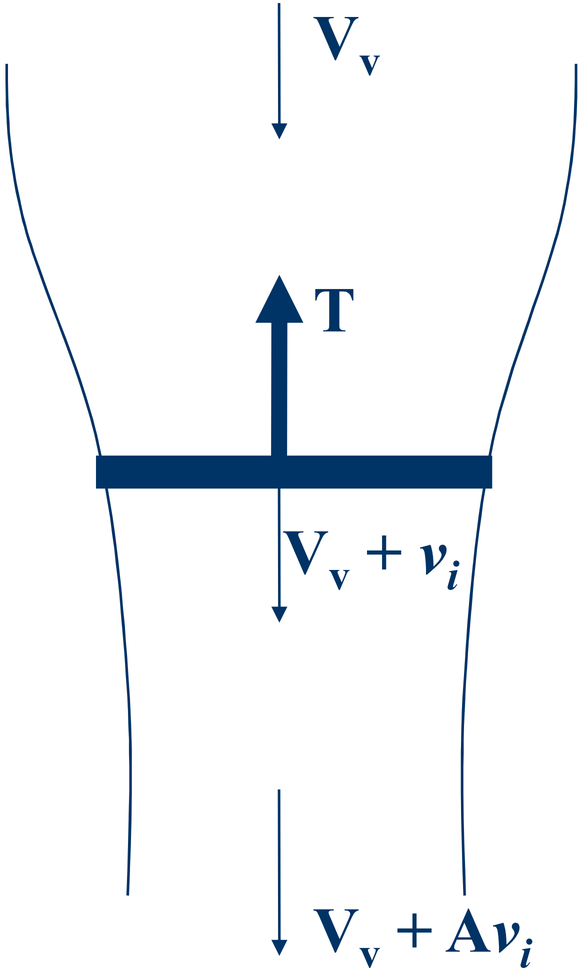
THRUST AND POWER CALCULATIONS



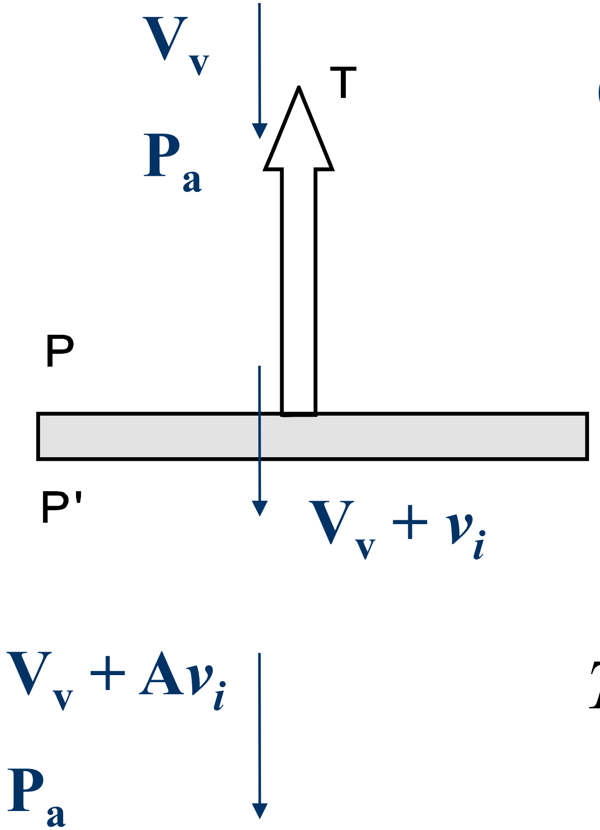
$$\vec{F}_{ex} - \int_A P\,\vec{n}\,dA = G(\vec{V}_s - \vec{V}_e)$$

$$G = \rho V A = \rho \pi R^2 (V_v + v_i)$$

$$T = \rho(\pi R^2)(V_v + v_i) A v_i$$

A?

THRUST AND POWER CALCULATIONS



Calculation of Parameter "A"

$$T = (P' - P)\pi R^2$$

$$P_a + \tfrac{1}{2}\rho V_v^2 = P + \tfrac{1}{2}\rho (V_v + v_i)^2$$

$$P' + \tfrac{1}{2}\rho (V_v + v_i)^2 = Pa + \tfrac{1}{2}\rho (V_v + A v_i)^2$$

$$T = \tfrac{1}{2}\rho (\pi R^2)(2V_v + Av_i)\, Av_i$$

A = 2

THRUST AND POWER CALCULATIONS

THRUST

$$T = 2\rho(\pi R^2)v_i(V_v + v_i)$$

POWER

$$P_i = T(V_v + v_i)$$

$$P - P_a = -\rho v_i(V_v + \tfrac{1}{2}v_i)$$

$$P' - P_a = \rho v_i(V_v + \tfrac{3}{2}v_i)$$

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HOVER FLIGHT (THRUST AND POWER)

Flight condition ⟶ $V_V=0$

$$T = 2\rho(\pi R^2)v_{io}^2$$

$$P_{io} = 2\rho(\pi R^2)v_{io}^3$$

$$v_{io} = \sqrt{\frac{T}{2\rho(\pi R^2)}} = \sqrt{\frac{W}{2\rho(\pi R^2)}}$$

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VELOCITY RATIO

$$\frac{W}{2\rho\left(\pi R^2\right)} = v_{io}^2 = v_i\left(V_v + v_i\right) \longrightarrow \left(\frac{V_V + v_i}{v_{io}}\right)\left(\frac{v_i}{v_{io}}\right) = 1$$

$$\left(\frac{v_i}{v_{io}}\right)^2 + \left(\frac{v_i}{v_{io}}\right)\left(\frac{V_v}{v_{io}}\right) - 1 = 0$$

$$\begin{cases} \dfrac{v_i}{v_{io}} = \dfrac{1}{2}\left[\sqrt{\left(\dfrac{V_v}{v_{io}}\right)^2 + 4} - \left(\dfrac{V_v}{v_{io}}\right)\right] \\ \dfrac{V_v + v_i}{v_{io}} = \dfrac{1}{2}\left[\sqrt{\left(\dfrac{V_v}{v_{io}}\right)^2 + 4} + \left(\dfrac{V_v}{v_{io}}\right)\right] \end{cases}$$

POWER RATIO

$$\frac{P_i}{P_{io}} = \frac{T(V_V + v_i)}{Tv_{io}} = \frac{V_v + v_i}{v_{io}} = \frac{1}{2}\left[\sqrt{\left(\frac{V_v}{v_{io}}\right)^2 + 4} + \left(\frac{V_v}{v_{io}}\right)\right]$$

$$\frac{P_i}{P_{io}} = \frac{V_V + v_i}{v_{io}} = \frac{1}{\left(\frac{v_i}{v_{io}}\right)}$$

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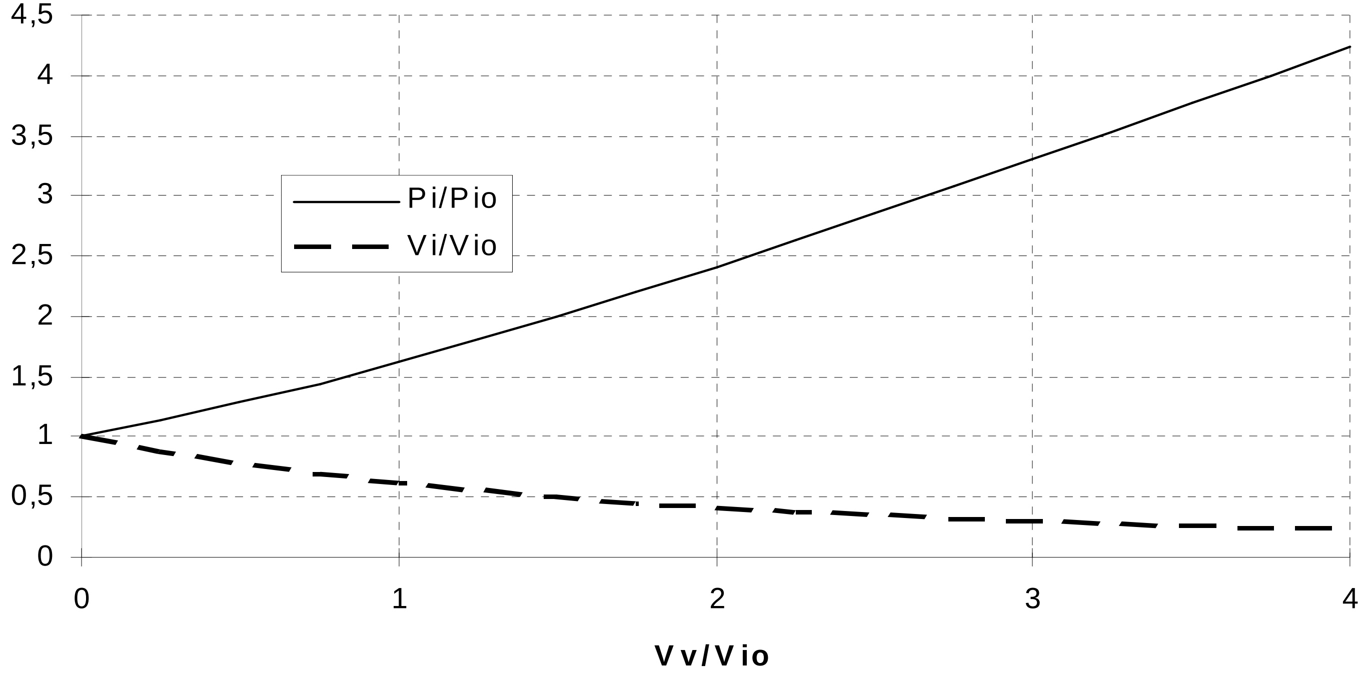
Power Ratio vs Vertical Velocity Ratio

Induced Velocity Ratio vs Vertical Velocity Ratio.



DIMENSIONLESS COEFFICIENTS

Thrust coefficient (Dimensionless)

$$C_N = \frac{F}{\rho S V^2} \qquad C_T = \frac{T}{\rho\left(\pi R^2\right)\left(\Omega R\right)^2}$$

Power coefficient (Dimensionless)

$$C_W = \frac{W}{\rho S V^3} \qquad C_{P_i} = \frac{P_i}{\rho(\pi R^2)(\Omega R)^3}$$

$$C_T = \frac{2\rho\left(\pi R^2\right)v_{io}^2}{\rho\left(\pi RSUP2\right)\left(\Omega R\right)^2} = 2\left(\frac{v_{io}}{\Omega R}\right)^2 \qquad C_{P_i} = 2\frac{v_i}{\Omega R}\left(\frac{V_v + v_i}{\Omega R}\right)^2$$

DIMENSIONLESS EXPRESSIONS

$$\frac{v_i}{v_{io}} = \frac{v_i}{\Omega R} \bullet \frac{\Omega R}{v_{io}} = \frac{1}{2}\left[\sqrt{\left(\frac{V_v}{\Omega R}\frac{\Omega R}{v_{io}}\right)^2 + 4} - \left(\frac{V_v}{\Omega R}\frac{\Omega R}{v_{io}}\right)\right] \rightarrow \frac{v_i}{\Omega R} = \frac{1}{2}\left[\sqrt{2C_T + \left(\frac{V_v}{\Omega R}\right)^2} - \left(\frac{V_v}{\Omega R}\right)\right]$$

$$\frac{P_i}{P_{io}} = \frac{C_{P_i}}{C_{P_{io}}} = \frac{V_v + v_{io}}{v_{io}} \longrightarrow \frac{C_{P_i}}{C_{P_{io}}} = \frac{1}{2}\frac{1}{\sqrt{\frac{C_T}{2}}}\left[\sqrt{2C_T + \left(\frac{V_v}{\Omega R}\right)^2} + \left(\frac{V_v}{\Omega R}\right)\right]$$

$$\frac{C_{P_i}}{C_T} = \frac{C_{P_i}}{C_{P_{io}}}\frac{C_{P_{io}}}{C_T} \longrightarrow \frac{C_{P_i}}{C_T} = \frac{1}{2}\left[\sqrt{2C_T + \left(\frac{V_v}{\Omega R}\right)^2} + \left(\frac{V_v}{\Omega R}\right)\right]$$