POLITÉCNICA

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Universidad Politécnica de Madrid
Escuela Universitaria de
Ingeniería Técnica Aeronáutica

HELICOPTERS

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UNIVERSIDAD POLITÉCNICA DE MADRID
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POLITÉCNICA

ROTOR AERODYNAMICS
Blade Element Theory
Vertical Climb Flight

Inital thoughts:

- The vertical climb flight is the easiest condition of flight.
  - The velocities on the plane of the rotor are symmetrical about the axis of rotation.
  - The aerodynamic forces on the blades are constant regardless of their angular position.
  - The plane formed by the points of the rotor is perpendicular to the drive shaft.

Inital thoughts:

- The vertical climb flight is the easiest condition of flight.
- There are different theories for studying rotor aerodynamics.
  - The momentum theory.
  - The blade element theory.
  - The vortex theory.

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ROTOR AERODYNAMIC

VERTICAL CLIMB FLIGHT

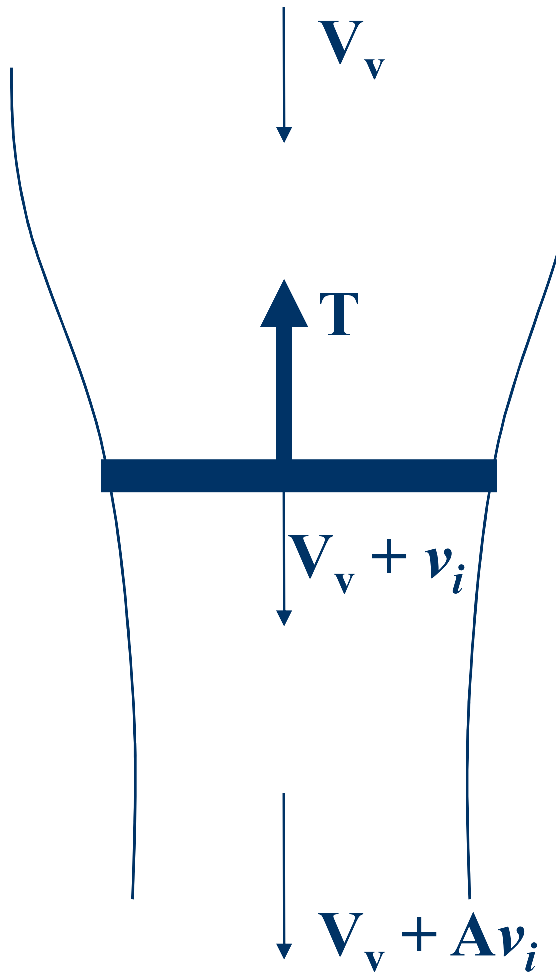
MOMENTUM THEORY

- ✓ **Thrust and Power Calculations.**
- ✓ **Hover Flight.**
- ✓ **Velocity and Power Ratios**
- ✓ **Thurst and Power coefficients.**
- ✓ **Dimensionless expressions.**

INITIAL ASSUMPTIONS

- High values of Re number flow.
- Replace the rotor with a totally porous disc of the same radius (**R**) as the rotor replaced.
- We assume the affected current through the disc is defined by the streamtube.
- The fluid flow in the streamtube is considered to be unidimensional, steady and incompressible.
- The effects of the rotation of the slipstream and loses in the blade tips, are negleted.

MATHEMATICAL MODEL



- The velocity of the upstream fluid of the rotor is the vertical velocity of the rotor. (Vv).
- The fluid velocity in this section of the disc is the vertical velocity of the rotor plus the induced velocity by the lifting disc. (V_v+v_i).
- The velocity of the downstream fluid of the rotor is the vertical velocity of the rotor plus the induced velocity in the disc plane affected by a factor of A. (Vv+A*vi*).

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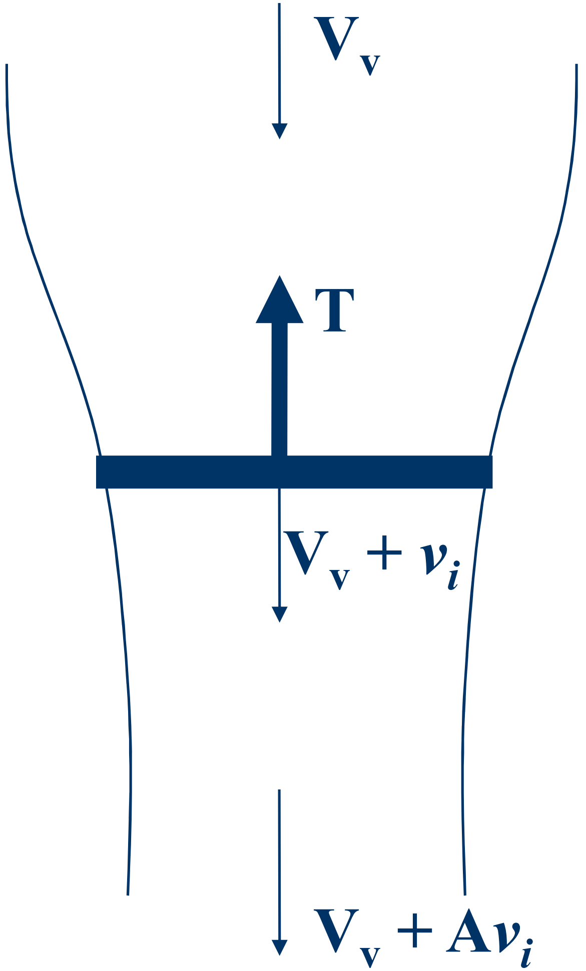
THRUST AND POWER CALCULATIONS



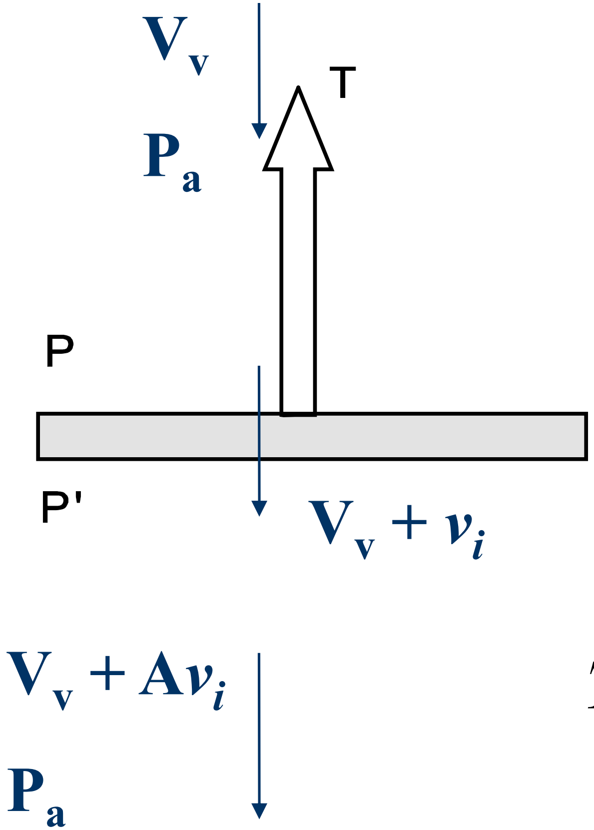
$$\vec{F}_{ex} - \int_A P\,\vec{n}\,dA = G(\vec{V}_s - \vec{V}_e)$$

$$G = \rho V A = \rho\pi R^2 (V_v + v_i)$$

$$T = \rho(\pi R^2)(V_v + v_i) A v_i$$

A?

THRUST AND POWER CALCULATIONS



Calculation of Parameter “A”

$$T = (P' - P)\pi R^2$$

$$P_a + \tfrac{1}{2}\rho V_v^2 = P + \tfrac{1}{2}\rho (V_v + v_i)^2$$

$$P' + \tfrac{1}{2}\rho (V_v + v_i)^2 = Pa + \tfrac{1}{2}\rho (V_v + A v_i)^2$$

$$T = \tfrac{1}{2}\rho(\pi R^2)(2V_v + A v_i)\, A v_i$$

$$A = 2$$

THRUST AND POWER CALCULATION

THRUST

$$T = 2\rho(\pi R^2)v_i(V_v + v_i)$$

POWER

$$P_i = T(V_v + v_i)$$

$$P - P_a = -\rho v_i (V_v + \tfrac{1}{2} v_i)$$

$$P' - P_a = \rho v_i (V_v + \tfrac{3}{2} v_i)$$

HOVER FLIGHT (THRUST AND POWER)

Flight condition → $V_V=0$

$$T = 2\rho(\pi R^2)v_{io}^2$$

$$P_{io} = 2\rho(\pi R^2)v_{io}^3$$

$$v_{io} = \sqrt{\frac{T}{2\rho(\pi R^2)}} = \sqrt{\frac{W}{2\rho(\pi R^2)}}$$

VELOCITY RATIOS

$$\frac{W}{2\rho\left(\pi R^2\right)} = v_{io}^2 = v_i\left(V_v + v_i\right) \longrightarrow \left(\frac{V_V + v_i}{v_{io}}\right)\left(\frac{v_i}{v_{io}}\right) = 1$$

$$\left(\frac{v_i}{v_{io}}\right)^2 + \left(\frac{v_i}{v_{io}}\right)\left(\frac{V_v}{v_{io}}\right) - 1 = 0$$

$$\begin{cases} \dfrac{v_i}{v_{io}} = \dfrac{1}{2}\left[\sqrt{\left(\dfrac{V_v}{v_{io}}\right)^2 + 4} - \left(\dfrac{V_v}{v_{io}}\right)\right] \\ \dfrac{V_v + v_i}{v_{io}} = \dfrac{1}{2}\left[\sqrt{\left(\dfrac{V_v}{v_{io}}\right)^2 + 4} + \left(\dfrac{V_v}{v_{io}}\right)\right] \end{cases}$$

POWER RATIOS

$$\frac{P_i}{P_{io}} = \frac{T(V_V + v_i)}{T v_{io}} = \frac{V_v + v_i}{v_{io}} = \frac{1}{2}\left[\sqrt{\left(\frac{V_v}{v_{io}}\right)^2 + 4} + \left(\frac{V_v}{v_{io}}\right)\right]$$

$$\frac{P_i}{P_{io}} = \frac{V_V + v_i}{v_{io}} = \frac{1}{\left(\frac{v_i}{v_{io}}\right)}$$

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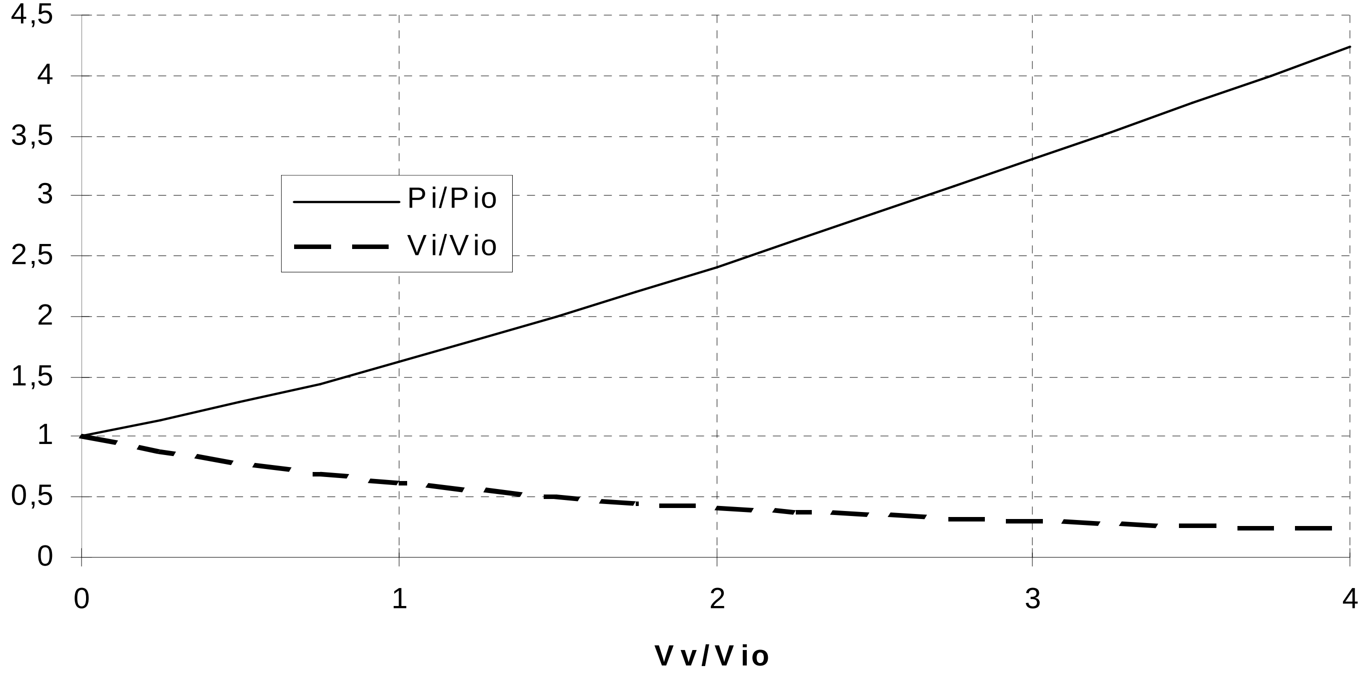
Power Ratio vs Vertical Velocity Ratio

Induced Velocity Ratio vs Vertical Velocity Ratio



DIMENSIONLESS COEFFICIENTS

Thrust Coefficient (Dimensionless) $C_N = \frac{F}{\rho S V^2}$ $C_T = \frac{T}{\rho\left(\pi R^2\right)\left(\Omega R\right)^2}$

Power Coefficient (Dimensionless) $C_W = \frac{W}{\rho S V^3}$ $C_{P_i} = \frac{P_i}{\rho(\pi R^2)(\Omega R)^3}$

$$C_T = \frac{2\rho\left(\pi R^2\right)v_{io}^2}{\rho\left(\pi RSUP2\right)\left(\Omega R\right)^2} = 2\left(\frac{v_{io}}{\Omega R}\right)^2$$

$$C_{P_i} = 2\frac{v_i}{\Omega R}\left(\frac{V_v + v_i}{\Omega R}\right)^2$$

DIMENSIONLESS EXPRESSIONS

$$\frac{v_i}{v_{io}} = \frac{v_i}{\Omega R} \bullet \frac{\Omega R}{v_{io}} = \frac{1}{2}\left[\sqrt{\left(\frac{V_v}{\Omega R}\frac{\Omega R}{v_{io}}\right)^2 + 4} - \left(\frac{V_v}{\Omega R}\frac{\Omega R}{v_{io}}\right)\right] \longrightarrow \frac{v_i}{\Omega R} = \frac{1}{2}\left[\sqrt{2C_T + \left(\frac{V_v}{\Omega R}\right)^2} - \left(\frac{V_v}{\Omega R}\right)\right]$$

$$\frac{P_i}{P_{io}} = \frac{C_{P_i}}{C_{P_{io}}} = \frac{V_v + v_{io}}{v_{io}} \longrightarrow \frac{C_{P_i}}{C_{P_{io}}} = \frac{1}{2}\frac{1}{\sqrt{\frac{C_T}{2}}}\left[\sqrt{2C_T + \left(\frac{V_v}{\Omega R}\right)^2} + \left(\frac{V_v}{\Omega R}\right)\right]$$

$$\frac{C_{P_i}}{C_T} = \frac{C_{P_i}}{C_{P_{io}}}\frac{C_{P_{io}}}{C_T} \longrightarrow \frac{C_{P_i}}{C_T} = \frac{1}{2}\left[\sqrt{2C_T + \left(\frac{V_v}{\Omega R}\right)^2} + \left(\frac{V_v}{\Omega R}\right)\right]$$

Initial thoughts:

- The Momentum Theory has excessive limitations, including:
  - It doesn't take into account the parasite drag.
  - It doesn't consider the geometry of the rotor.
  - It doesn't take into account the three dimensional effects.

Initial thoughts:

- The Momentum Theory has excessive limitations.
- The lifting disc is a rotor with b blades rotating with an angular velocity Ω, with the following considerations:
    - The blades are high aspect ratio wings, and
    - The application of aerodynamics knowledge is possible.

ROTOR AERODYNAMICS

VERTICAL CLIMBING FLIGHT

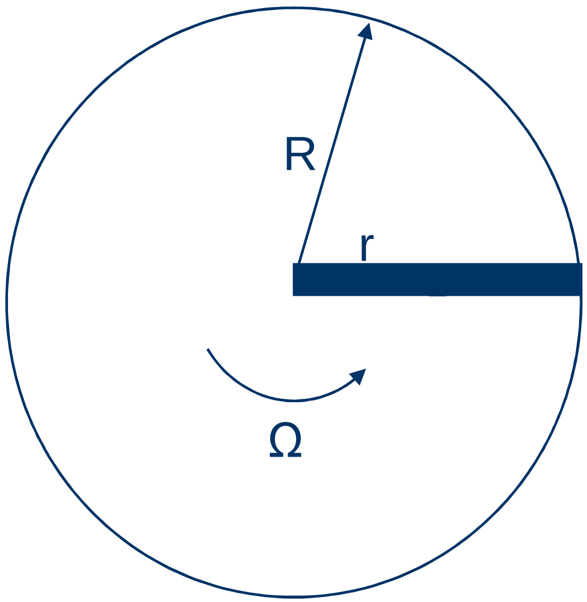
BLADE ELEMENT THEORY

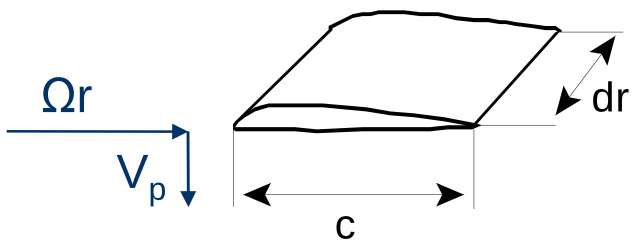
- ✓ **Hypothesis, Mathematical Model and Aerodynamic Forces.**
- ✓ **Thrust and Torque Calculations.**
- ✓ **Dimensionless expressions.**
- ✓ **Induced velocity distribution.**
- ✓ **Constant induced velocity rotors.**

ASSUMPTIONS

- High values of Re number flow.
- The fluid flow is considered steady and incompressible.
- The aerodynamic forces will be obtained from the lift curve and the airfoil polars, which are two-dimensional curves.
- The three-dimensional effects are calculated using semiempirical methods.
- It does not take into account the effects of wake rotation.

MATHEMATICAL MODEL





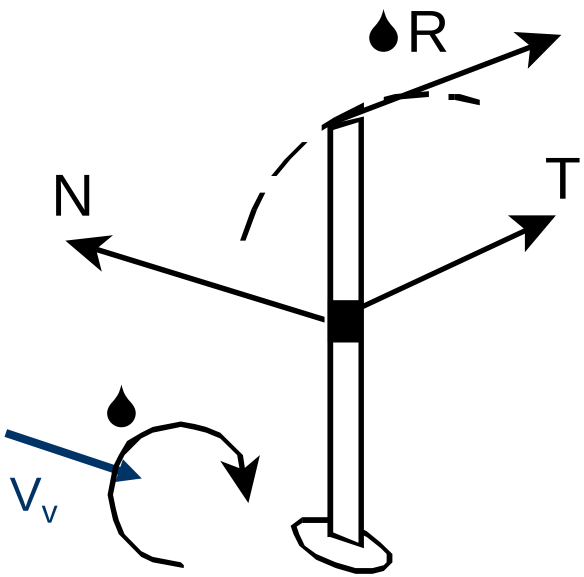
- We consider the rotor with a radius "R" formed by "b" blades that rotate with a constant angular velocity Ω, in ascending constant vertical flight.
- The blade element has a chord "c", a span "dr" and is situated a distance "r" from the rotor centre.
- The undisturbed incident velocity is a sum of the tangential velocity and the normal velocity.

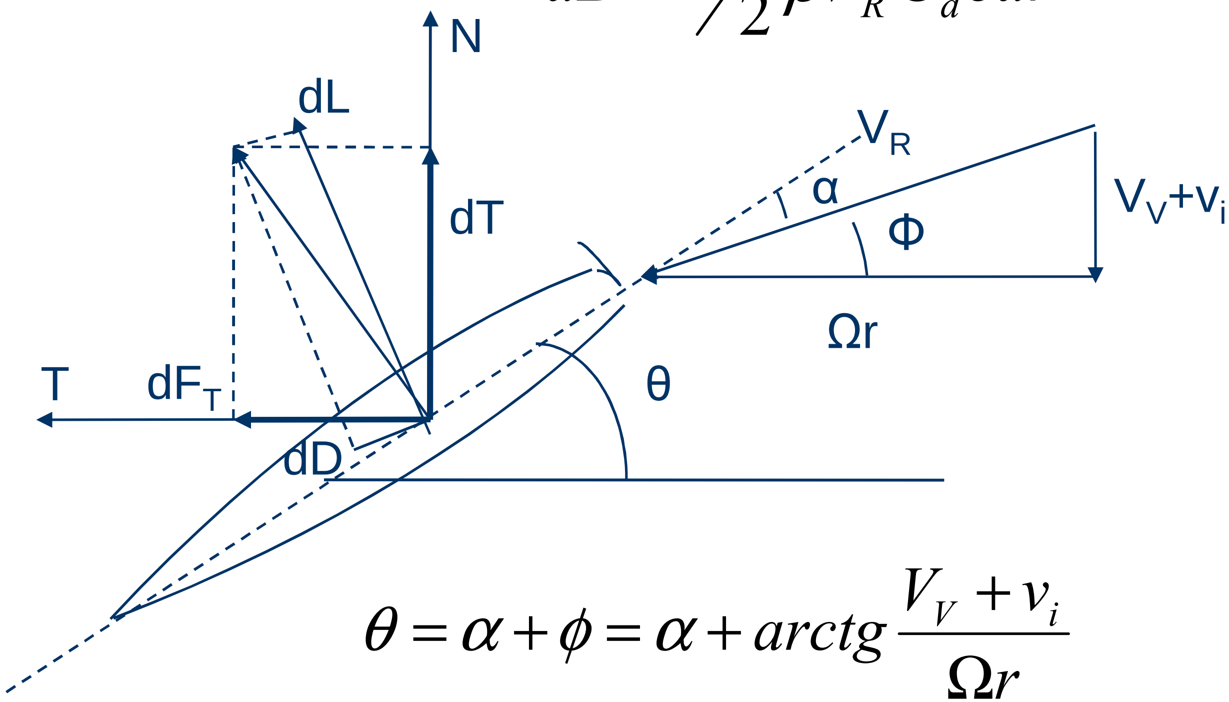
AERODYNAMIC FORCES

Euler axes.

$$dL = \tfrac{1}{2}\rho V_R^2 C_l c\,dr$$

$$dD = \tfrac{1}{2}\rho V_R^2 C_d c\,dr$$





$$\theta = \alpha + \phi = \alpha + arctg\,\frac{V_V + v_i}{\Omega r}$$

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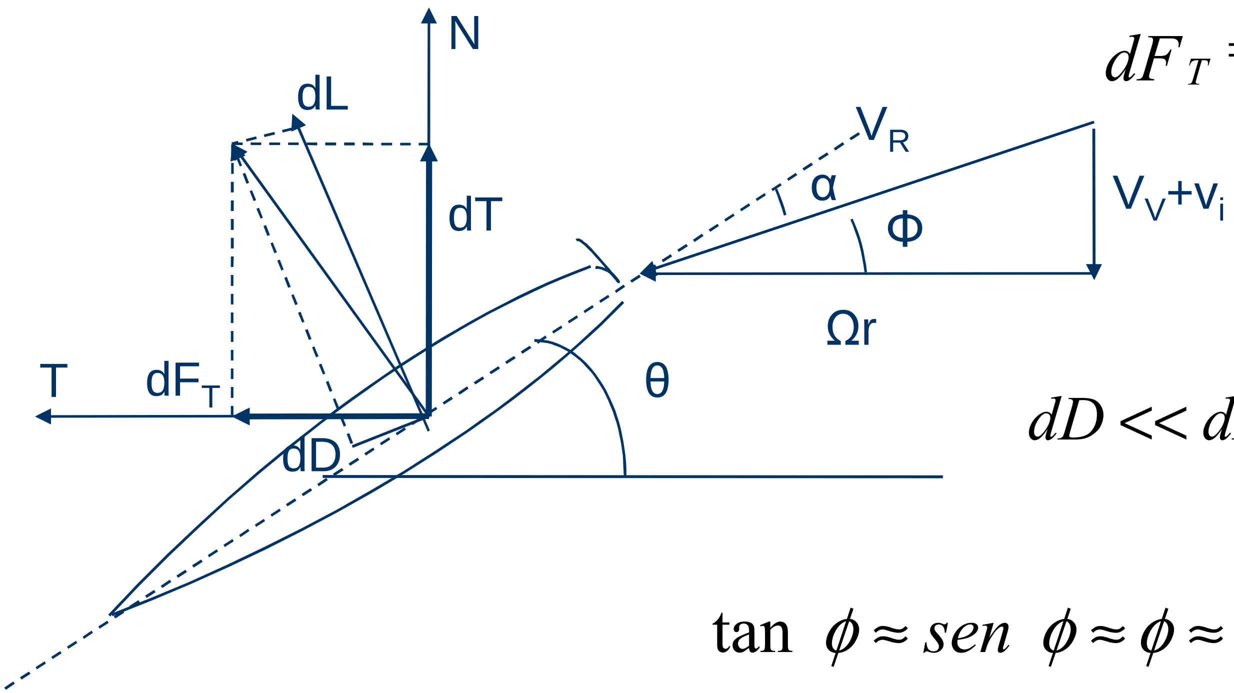
THRUST AND TORQUE CALCULATIONS

Equations of to start with

$$dT = \cos\phi \bullet dL - sen\phi \bullet dD$$

$$dF_T = sen\phi \bullet dL + \cos\phi \bullet dD$$



$$dD << dL \quad y \quad \frac{V_V + v_i}{\Omega r} << 1$$

$$\tan\ \phi \approx sen\ \phi \approx \phi \approx \frac{V_v + v_i}{\Omega r} \quad y \quad \cos\ \phi \approx 1$$

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THRUST AND TORQUE CALCULATIONS

Simplified Equations

$$\left.\begin{array}{l} dD << dL \\ \phi \approx sen\phi \approx tag\phi = \dfrac{V_V + v_i}{\Omega r} \end{array}\right\}$$

$$dT \approx dL$$

$$dF_T \approx sen\phi \, dL + dD$$

$$dT = \tfrac{1}{2}\rho(\Omega r)^2 c\, C_l \; dr$$

$$dF_{T_i} = \tfrac{1}{2}\rho(\Omega r)^2 \phi c\, C_l \; dr = \phi dT$$

$$dF_{T_o} = \tfrac{1}{2}\rho(\Omega r)^2 c\, C_d \; dr = \frac{C_d}{C_l} \; dT$$

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THRUST AND TORQUE CALCULATIONS

Differential equations

Differential of thrust

$$dT = \tfrac{1}{2}\rho(\Omega r)^2 c\, C_l\, dr$$

Differential of induced torque

$$dQ_i = \phi r\, dT = \tfrac{1}{2}\rho(\Omega r)^2 \phi c\, C_l\, r dr$$

Differential of parasite torque

$$dQ_o = \tfrac{C_d}{C_l}\, r dT = \tfrac{1}{2}\rho(\Omega r)^2 c\, C_d\, r dr$$

THRUST AND TORQUE CALCULATIONS

Dimensionless equations $$C_T = \frac{T}{\rho(\pi R^2)(\Omega R)^2} \quad \text{y} \quad C_Q = \frac{Q}{\rho(\pi R^2)R(\Omega R)^2}$$

Local solidity $$\sigma = \frac{bc}{\pi R}$$

Dimensionless radius $$x = \frac{r}{R}$$

$$dC_T = \frac{bdT}{\rho(\pi R^2)(\Omega R)^2} = \frac{\sigma}{2}x^2 C_l dx$$

$$dC_{Q_i} = \frac{bQ_i}{\rho(\pi R^2)R(\Omega R)^2} = \frac{\sigma}{2}\phi x^3 C_l dx = \phi x dC_T$$

$$dC_{Q_0} = \frac{bQ_0}{\rho(\pi R^2)R(\Omega R)^2} = \frac{\sigma}{2}x^3 C_d dx$$

THRUST AND TORQUE CALCULATIONS

Set of simultaneous equations

$$C_T = \int_0^1 \frac{\sigma}{2} x^2 C_l dx$$

$$C_{Q_i} = \int_0^1 \frac{\sigma}{2} \phi x^3 C_l dx$$

$$C_{Q_0} = \int_0^1 \frac{\sigma}{2} x^3 C_d dx$$

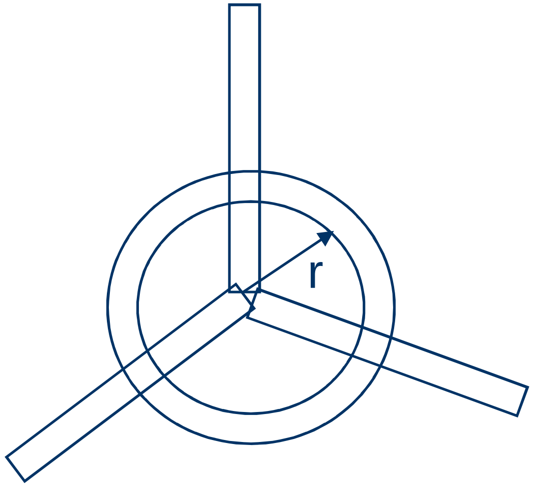
$¿\theta(x)?$

$¿v_i(x)?$

$¿\sigma(x)?$

DISTRIBUTION OF INDUCED VELOCITIES

Combination of TCM and TEP



$$dT = \frac{b}{2}\rho(\Omega r)^2 ca\alpha dr$$

$$dT = 4\rho v_i (V_V + v_i)\pi r dr$$

$$\frac{v_i}{\Omega R}\frac{V_v + v_i}{\Omega R} = \frac{\sigma a}{8}\left(\theta x - \frac{V_v + v_i}{\Omega R}\right)$$

DISTRIBUTION OF INDUCED VELOCITIES

$$\frac{v_i}{\Omega R}\frac{V_v+v_i}{\Omega R}=\frac{\sigma a}{8}\left(\theta x-\frac{V_v+v_i}{\Omega R}\right)$$

$$\frac{v_i}{\Omega R}=\frac{1}{2}\left[\sqrt{\left(\frac{V_v}{\Omega R}-\frac{a\sigma}{8}\right)^2+\frac{a\sigma}{2}\theta x}-\left(\frac{V_v}{\Omega R}+\frac{a\sigma}{8}\right)\right]$$

Hover flight

$$\frac{v_{io}}{\Omega R}=\frac{a\sigma}{16}\left(\sqrt{1+\frac{32}{a\sigma}\theta x}-1\right)$$

CONSTANT INDUCED VELOCITY ROTORS

Ideally twisted rotor

$$\frac{v_i}{\Omega R}\frac{V_v+v_i}{\Omega R}=\frac{\sigma a}{8}\left(\theta x - \frac{V_v+v_i}{\Omega R}\right)$$

Constant local solidity, σ=cte

Hyperbolic twisted rotor, xθ=cte

$$\frac{v_i}{\Omega R}\frac{V_V+v_i}{\Omega R}=\frac{\sigma a}{8}\left(\theta_t-\frac{V_V+v_i}{\Omega R}\right)$$

CONSTANT INDUCED VELOCITY ROTORS

Optimum Rotor

$$\frac{v_i}{\Omega R}\frac{V_v + v_i}{\Omega R} = \frac{\sigma a}{8}\left(\theta x - \frac{V_v + v_i}{\Omega R}\right)$$

Hyperbolic local solidity, σx=cte

Constant angle of attack, α=cte

$$\frac{v_i}{\Omega R}\frac{V_v + v_i}{\Omega R} = \frac{a \quad \sigma_t}{8} \quad \alpha_o$$

AERODYNAMIC ROTOR

VERTICAL CLIMBING FLIGHT

BLADE ELEMENT THEORY

- ✓ **Constant induced velocity rotors.**
- ✓ **Rotors with an Ideal twist distribution.**
- ✓ **Optimum rotor.**
- ✓ **Figure of Merit.**
- ✓ **Blade tip losses.**
- ✓ **Ground effect.**

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CALCULATION OF THRUST AND REQUIRED TORQUE

$$\sigma = \frac{bc}{\pi R} \qquad x = \frac{r}{R}$$

$$dC_T = \frac{bdT}{\rho(\pi R^2)(\Omega R)^2} = \frac{\sigma}{2} x^2 C_l dx$$

$$dC_{Q_i} = \frac{bQ_i}{\rho(\pi R^2)R(\Omega R)^2} = \frac{\sigma}{2} \phi x^3 C_l dx = \phi x dC_T$$

$$dC_{Q_0} = \frac{bQ_0}{\rho(\pi R^2)R(\Omega R)^2} = \frac{\sigma}{2} x^3 C_d dx$$

$$\frac{v_i}{\Omega R}\frac{V_v + v_i}{\Omega R} = \frac{\sigma a}{8}\left(\theta x - \frac{V_v + v_i}{\Omega R}\right)$$

CONSTANT INDUCED VELOCITY ROTORS

Ideally twisted rotor

$$\frac{v_i}{\Omega R}\frac{V_V+v_i}{\Omega R}=\frac{\sigma a}{8}\left(\theta_t-\frac{V_V+v_i}{\Omega R}\right)$$

Constant local solidity, σ=cte

Hyperbolic twisted rotor, xθ=cte

Optimum rotor

$$\frac{v_i}{\Omega R}\frac{V_v+v_i}{\Omega R}=\frac{a\ \sigma_t}{8}\alpha_o$$

Hyperbolic local solidity, σx=cte

Constant angle of attack, α=cte

IDEALLY TWISTED ROTOR

Thrust Coefficient

$$\theta = \theta_t \frac{1}{x} \quad y \quad \phi = \phi_t \frac{1}{x}$$

$$\alpha = \theta - \phi = (\theta_t - \phi_t)\frac{1}{x} = \alpha_t \frac{1}{x}$$

$$dC_T = \frac{a\sigma}{2}\alpha x^2 dx$$

$$C_T = \frac{a\sigma}{2}\int_0^1 \alpha x^2 dx = \frac{a\sigma}{2}\alpha_t \int_0^1 x dx$$

$$C_T = \frac{a\sigma}{4}\alpha_t$$

IDEALLY TWISTED ROTOR

Induced Torque Coefficent

$$\theta = \theta_t \frac{1}{x} \quad y \quad \phi = \phi_t \frac{1}{x}$$

$$\alpha = \theta - \phi = (\theta_t - \phi_t)\frac{1}{x} = \alpha_t \frac{1}{x}$$

$$dC_{Q_i} = \frac{a\sigma}{2}\phi\alpha x^3 dx$$

$$C_{Q_i} = \int_0^1 \frac{a\sigma}{2}\phi\alpha x^3 dx = \frac{a\sigma}{2}\phi_t\alpha_t\int_0^1 x dx$$

$$C_{Q_i} = \frac{a\sigma}{4}\phi_t\alpha_t = \phi_t C_T$$

IDEALLY TWISTED ROTOR

Parasite Torque Coefficient

$$\theta = \theta_t \frac{1}{x} \quad y \quad \phi = \phi_t \frac{1}{x}$$

$$\alpha = \theta - \phi = (\theta_t - \phi_t)\frac{1}{x} = \alpha_t \frac{1}{x}$$

$$dC_{Q_0} = \frac{\sigma}{2} C_d x^3 dx$$

$$C_{Q_0} = \frac{\sigma}{2} \int_o^1 C_d x^3 dx$$

It is necessary to know the mathematical relationship between the C_d and the position of the blade element.

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IDEALLY TWISTED ROTOR

Parasite Torque Coefficient

$$C_{Q_0} = \frac{\sigma}{2}\int_o^1 C_d x^3 dx$$

For blade airfoils which have a parabolic polar curve, the results are:

$$C_d = \delta_o + \delta_1\alpha + \delta_2\alpha^2 \qquad para\ r.t.i \qquad C_d = \delta_o + \delta_1\alpha_t\frac{1}{x} + \delta_2\alpha_t^2\frac{1}{x^2}$$

$$C_{Q_0} = \frac{\sigma}{8}\left(\delta_0 + \frac{4}{3}\delta_1\alpha_t + 2\delta_2\alpha_t^2\right)$$

OPTIMUM ROTORS

Thrust Coefficient

$$\sigma = \frac{\sigma_t}{x}$$

$$\alpha = \alpha_0$$

$$\theta = \alpha_0 + \frac{(V_V + v_i)}{\Omega R}\frac{1}{x}$$

$$dC_T = \frac{a\sigma}{2}\alpha x^2 dx$$

$$C_T = \int_0^1 \frac{a\sigma}{2}\alpha x^2 dx = \frac{a\sigma_t}{2}\alpha_0 \int_0^1 x dx$$

$$C_T = \frac{a\sigma_t}{4}\alpha_0 = \frac{\sigma_t}{4}C_l$$

OPTIMUM ROTORS

Induced Torque Coefficient

$$\sigma = \frac{\sigma_t}{x}$$

$$\alpha = \alpha_0$$

$$\theta = \alpha_0 + \frac{(V_V + v_i)}{\Omega R}\frac{1}{x}$$

$$dC_{Q_i} = \frac{a\sigma}{2}\phi\alpha x^3 dx$$

$$C_{Q_i} = \int_0^1 \frac{a\sigma}{2}\phi\alpha x^3 dx = \frac{a\sigma_t}{2}\phi_t\alpha_0\int_0^1 x dx$$

$$C_{Q_i} = \frac{a\sigma_t}{4}\phi_t\alpha_0 = \phi_t C_T$$

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OPTIMUM ROTORS

Parasite Torque Coefficient

$$\sigma = \frac{\sigma_t}{x}$$

$$\alpha = \alpha_0$$

$$\theta = \alpha_0 + \frac{(V_V + v_i)}{\Omega R}\frac{1}{x}$$

$$dC_{Q_0} = \frac{\sigma}{2} C_d x^3 dx$$

$$C_{Q_0} = \frac{\sigma_t}{2} C_d \int_o^1 x^2 dx$$

$$C_{Q_0} = \frac{\sigma_t}{6} C_d$$

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Figure of Merit

Is defined as the ratio between the minimum possible power required to hover an ideal rotor and the actual power required to hover. Is denoted by the letter "M".

$$M = \frac{Tv_{io}}{P} = \frac{C_T \dfrac{V_{io}}{\Omega R}}{C_{Q_{io}} + C_{Q_{0o}}}$$

$$M = \frac{1}{1 + C_{Q_{0o}} / C_{Q_{io}}}$$

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LOSSES IN THE BLADE TIPS

General Thoughts

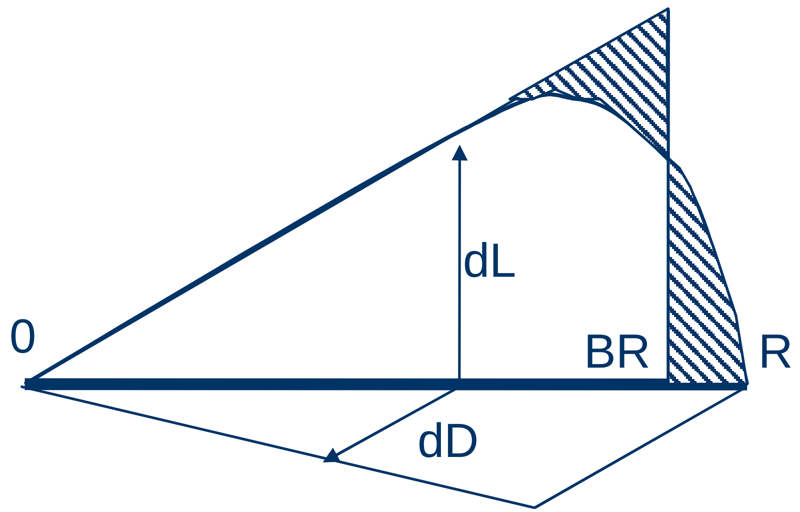
- Until now the “three-dimensional effects” have not been taken into account.
- There is a circulation around the marginal tips of the blades caused by the pressure difference between the upper and lower surfaces of the airfoil.
- **“the lift force of the blade element on the marginal tips is null”**

LOSSES IN THE BLADE TIP

Tip Loss Correction

Prandtl and Wald. They created a dimensionless coefficient “B”.

From a blade section located a distance “BR” from the centre, all the airfoils do not generate lift but have drag. With this tip losses correction, the overall value of the thrust is valid, even though the thrust distribution along the blade span is not correct.



LOSSES IN THE BLADE TIPS

Tip Loss Correction, expressions

$$B = 1 - \frac{\sqrt{2C_T}}{b}$$

$$\begin{cases} x_e > 0{,}5 \;\rightarrow B = 1 - \sqrt{2C_T}\left(\dfrac{1}{b} - 0{,}6x_e\right) \\ x_e > 0{,}5 \;\rightarrow B = 1 - \sqrt{2C_T}\left(\dfrac{1}{b} - 0{,}3\right) \end{cases}$$

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LOSSES IN THE BLADE TIPS

Ideally Twisted Rotor

$$C_T = \frac{a\sigma}{2}\alpha_t \int_0^B x dx \longrightarrow C_T = B^2 \frac{a\sigma}{4}\alpha_t$$

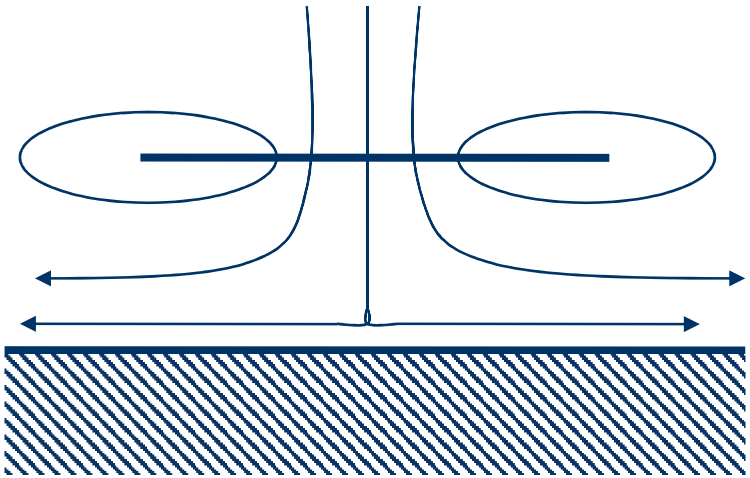
$$C_{Q_i} = \frac{a\sigma}{2}\phi_t \alpha_t \int_0^B x dx \longrightarrow C_{Q_i} = B^2 \frac{a\sigma}{4}\phi_t \alpha_t = \phi_t C_T$$

$$C_{Q_0} = \frac{\sigma}{2}\int_o^1 C_d x^3 dx \longrightarrow C_{Q_0} = \frac{\sigma}{8}\left(\delta_0 + \frac{4}{3}\delta_1 \alpha_t + 2\delta_2 \alpha_t^2\right)$$

GROUND EFFECT (IGE)

A helicopter in close proximity to the ground **(z~R):**

- Less power is needed for a given thrust.
  Or
- It lifts more weight while using the same power.



The configuration of the streamlines in close proximity to the ground changes and the induced velocity is less than out of ground effect.

GROUND EFFECT

Tip Loss Correction

$$\Lambda = \frac{1}{0.9926 + 0.03794\left(\frac{2R}{z}\right)^2} < 1$$

Correction factor is applied to the terms that contain the variable v_i

GROUND EFFECT

Tip Loss Correction, expression

IDEALLY TWISTED ROTOR (Torque Coefficient)

$$C_Q = C_{Q_i} + C_{Q_o} = \phi_t C_T + \frac{\sigma}{8}\left(\delta_o + \frac{4}{3}\delta_1\alpha_t + 2\delta_2\alpha_t^2\right)$$

$$C_{Q_{c.e.s.}} = \Lambda\left(C_{Q_i} + C_{Q_o}\right)_{s.e.s} + (1+\Lambda)\frac{\sigma}{8}\delta_o$$

GENERAL CALCULATION FOR ANY ROTOR

Some General Thoughts

Gessow (he called "quick estimation of power") proposed to carry out calculations on any rotor that:

- 1st step: Calculate the Equivalent ideally twisted rotor.
- 2nd step: Correct these calculations taking into account the actual taper ratio and the actual twist of the blade.

GENERAL CALCULATIONS FOR ROTORS

Expressions of the I.T.R. equivalent

I.T.R. equivalent $\sigma_e = 3\int_0^1 \sigma\, x^2\, dx$; $\theta = \frac{1}{x}\theta_t$

$$\phi_t = \frac{1}{2}\left[\sqrt{\frac{2C_T}{B^2} + \left(\frac{V_v}{\Omega R}\right)^2} + \left(\frac{V_V}{\Omega R}\right)\right]$$

$$C_{T_{RTIe}} = B^2 \frac{a\sigma_e}{4}\alpha_t$$

$$C_{Q_{TRTIe}} = C_{Q_i} + C_{Q_o} = \phi_t C_T + \frac{\sigma}{8}\left(\delta_o + \frac{4}{3}\delta_1\alpha_t + 2\delta_2\alpha_t^2\right)$$

GENERAL CALCULATIONS FOR ROTORS

Expressions of the actual Rotor

$$C_Q = (1 + F)\, C_{Q_{RTIe}}$$

F	λ = 1:1			λ = 3:1		
torsión	0	- 8	- 12	0	- 8	- 12
$C_T/\sigma=0.067$	+ 5.5	+ 3.0	+ 1.5	+ 3.5	0	0
$C_T/\sigma=0.100$	+ 7.5	+ 3.5	+ 1.5	+ 3.0	-0.5	- 0.5

GENERAL CALCULATIONS FOR ROTORS

Expressions of the actual Rotor

$$C_Q = (1 + F)\, C_{Q_{RTIe}}$$

$$c_l = \frac{6\, C_T}{\sigma_e} = \frac{6\, C_T}{B_2 \sigma}$$

$$G = 0.015\lambda + 0.0625\,(2.6 - \lambda)\, c_l$$

$$H = 0.0017708\,(1.3543 - \lambda) - 0.0046875\,(3 - \lambda)\, c_l$$

$$F = G - H\,\theta_t$$