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**UNIVERSIDAD POLITÉCNICA DE MADRID**

**Escuela Universitaria de  
Ingeniería Técnica Aeronáutica**

# **HELICOPTERS**

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**ROTOR AERODYNAMICS**

**Modified Momentum Theory  
Vertical Descent Flight**

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## Initial Thoughts:

- In vertical descending flight:
  - The flight velocity is in the upwards direction.
  - The induced velocity on the disc plane acts in the same direction as in climbing flight, which is, downwards.
  - The total velocity on the disc plane can have a positive or negative value.
  - Applying calculations based on the modified momentum theory, the results are not acceptable.



## ROTOR AERODYNAMICS



### VERTICAL DESCENDING FLIGHT



#### MODIFIED MOMENTUM THEORY.



**Diagrams of the different regimes of flight**



**Modified Momentum Theory. Induced Power.**



**Vertical autorotation.**



## FLIGHT REGIMES

This theory presents 6 different regimes of vertical flight.

These regimes keep the hypothesis' presented in chapter 2 (Momentum Theory in vertical climbing flight), according to the following ranges:

$$|V_V| \leq v_i \quad ; \quad v_i \leq |V_V| \leq 2v_i \quad ; \quad |V_V| \geq 2v_i$$



## FLIGHT REGIMES

### Vertical Climb

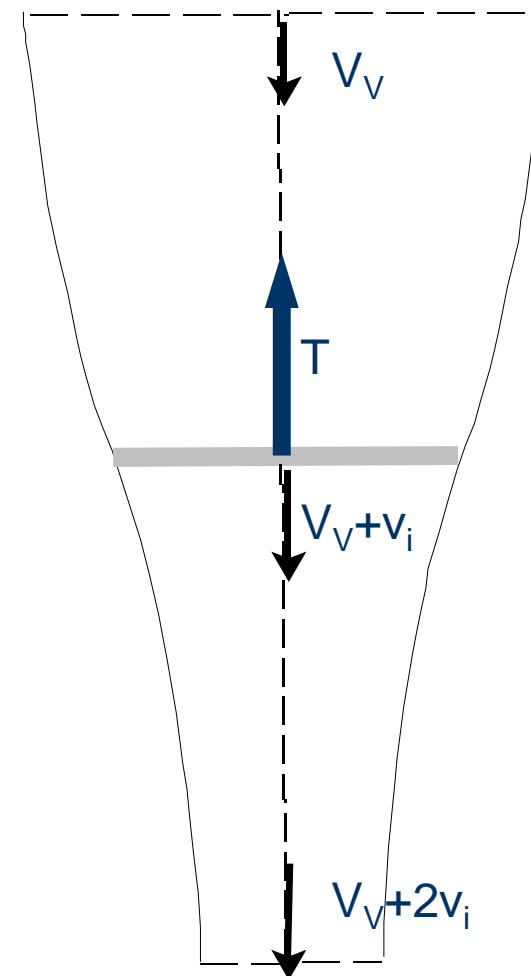
$$V_v > 0$$

$$v_i > 0$$

$$V_v + v_i > 0$$

$$V_v + 2 v_i > 0$$

$$P_i = T ( V_v + v_i ) > 0$$





## FLIGHT REGIMES

### Hover flight

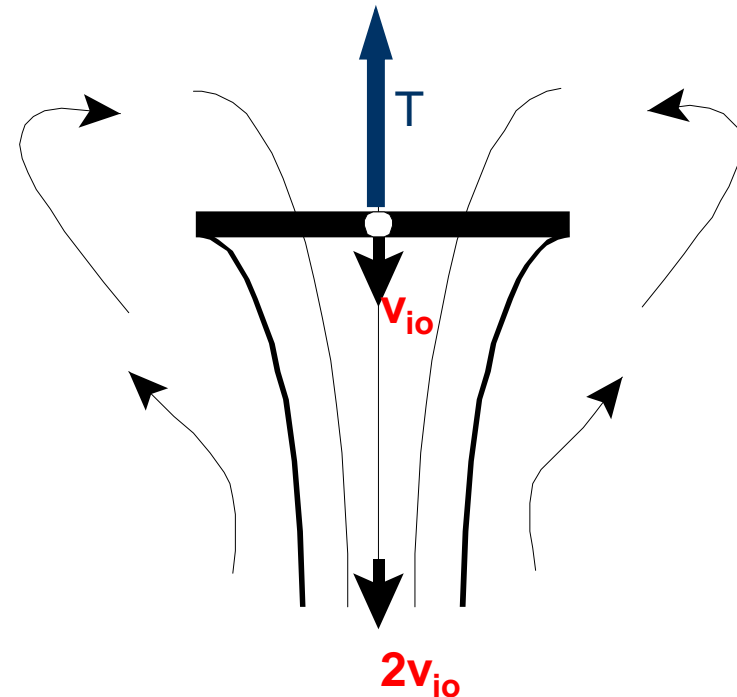
$$V_v = 0$$

$$v_i = v_{io} > 0$$

$$V_v + v_i = v_{io} > 0$$

$$V_v + 2 v_i = 2 v_{io} > 0$$

$$P_i = P_{io} = T v_{io}$$







## FLIGHT REGIMES

### Vortex Rings

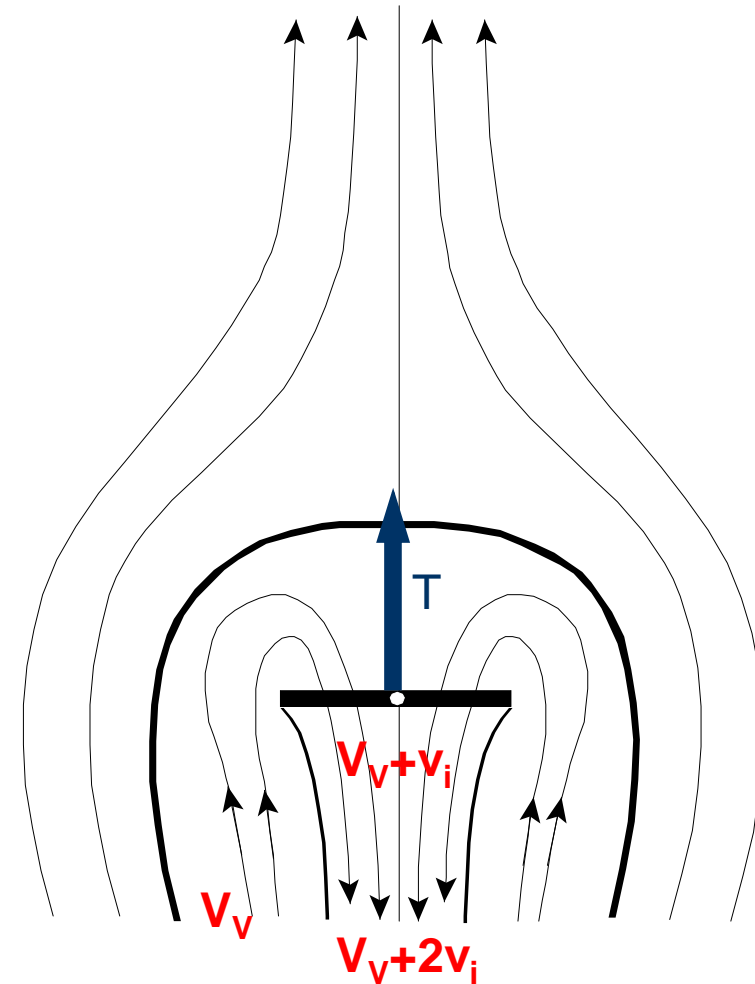
$$V_v < 0 \quad (|V_v| < v_i)$$

$$v_i > 0$$

$$V_v + v_i > 0$$

$$V_v + 2 v_i > 0$$

$$P_i = T (V_v + v_i) > 0$$





## FLIGHT REGIMES

### Autorotation

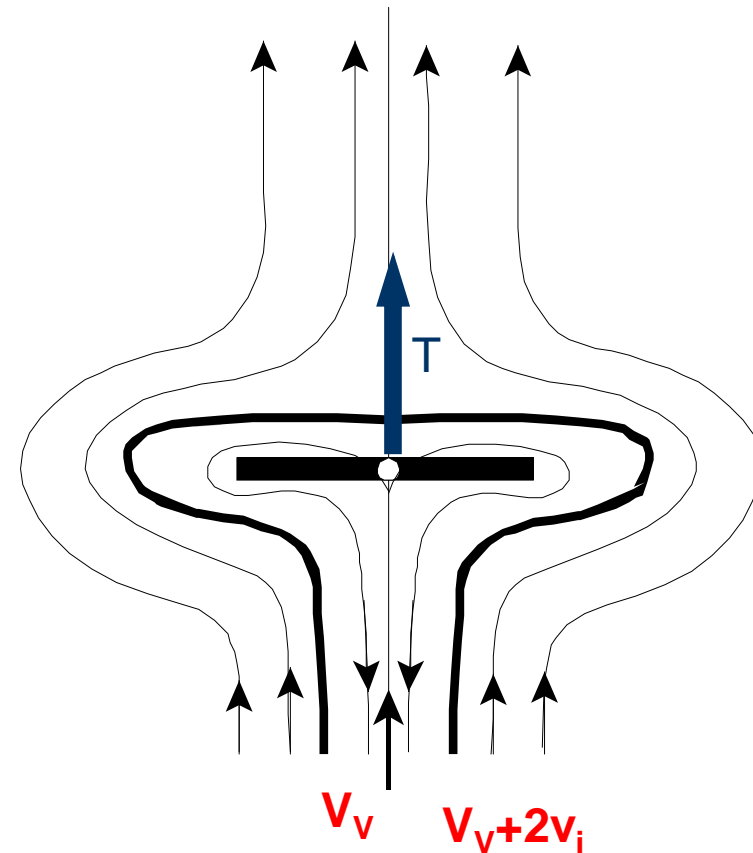
$$V_v < 0 \quad (|V_v| = v_i)$$

$$v_i > 0$$

$$V_v + v_i = 0$$

$$V_v + 2 v_i = v_i > 0$$

$$P_i = T (V_v + v_i) = 0$$





## FLIGHT REGIMES

### Turbulent Wake

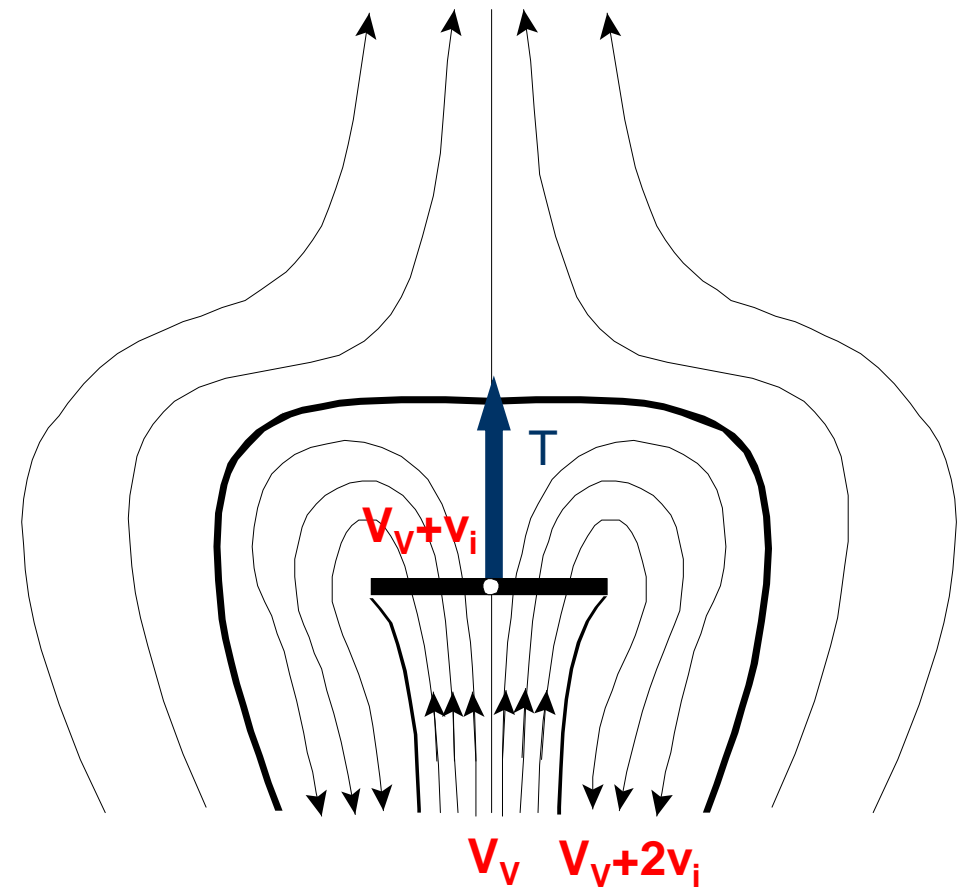
$$V_v < 0 \quad (v_i < |V_v| < 2 v_i)$$

$$v_i > 0$$

$$V_v + v_i < 0$$

$$V_v + 2 v_i > 0$$

$$P_i = T (V_v + v_i) < 0$$





## FLIGHT REGIMES

### Windmill brake

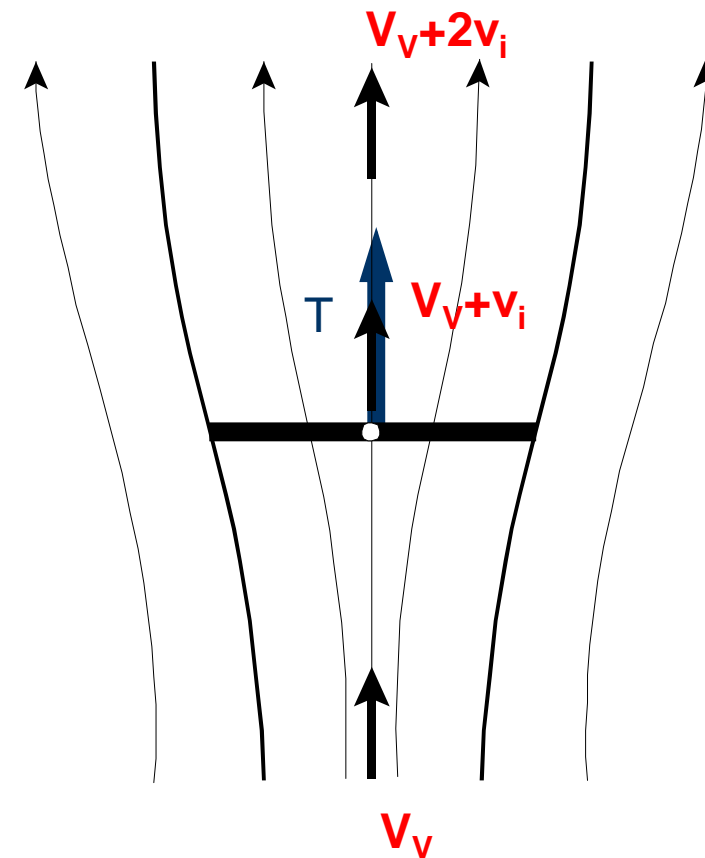
$$V_v < 0 \quad (|V_v| > 2 v_i)$$

$$v_i > 0$$

$$V_v + v_i < 0$$

$$V_v + 2 v_i < 0$$

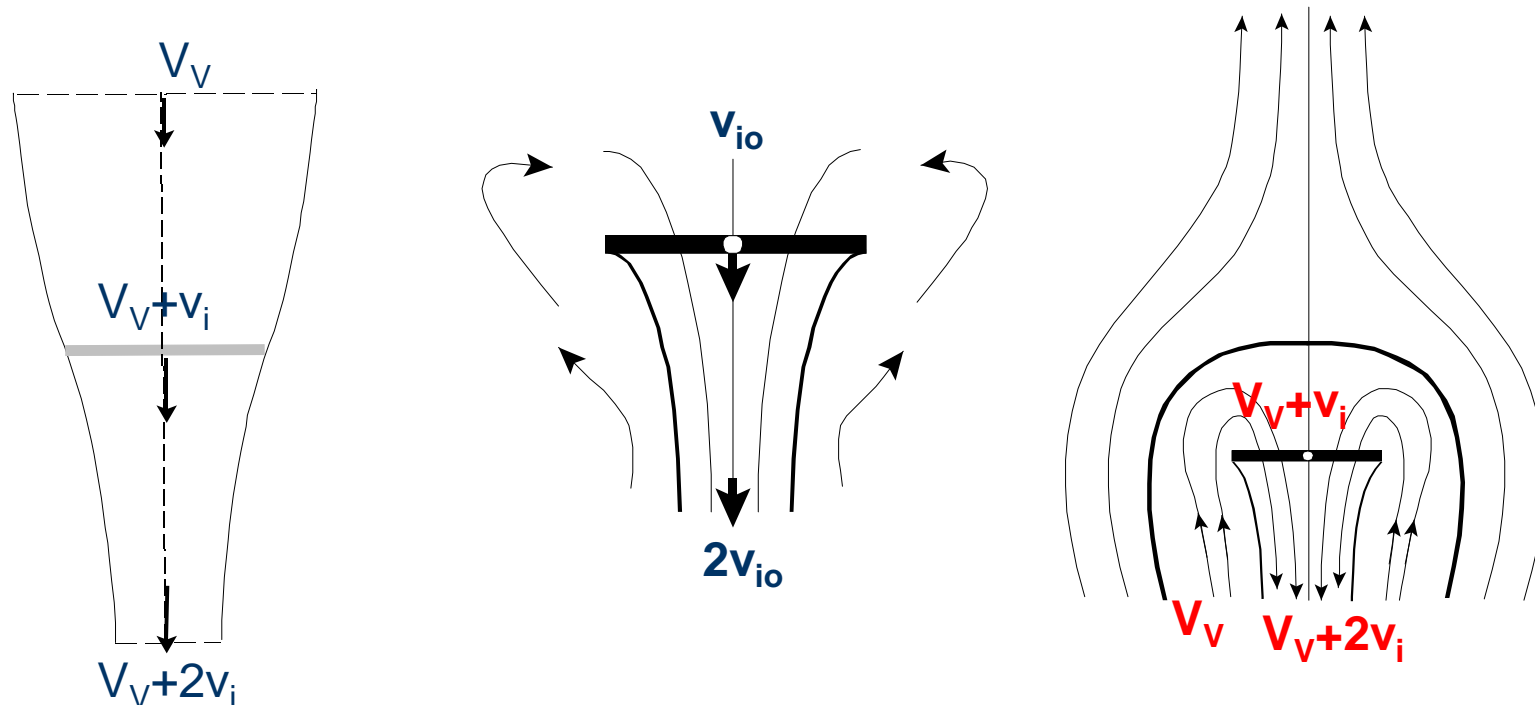
$$P_i = T (V_v + v_i) < 0$$





New concepts:

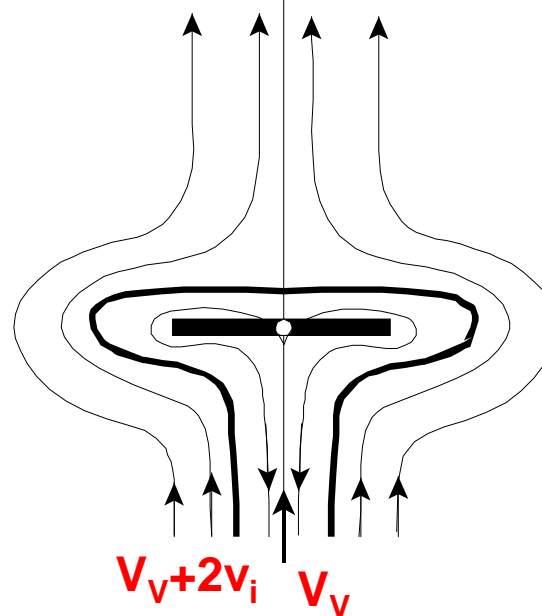
- In the first 3 regimes it is necessary to provide power to rotate the rotor.





## New concepts:

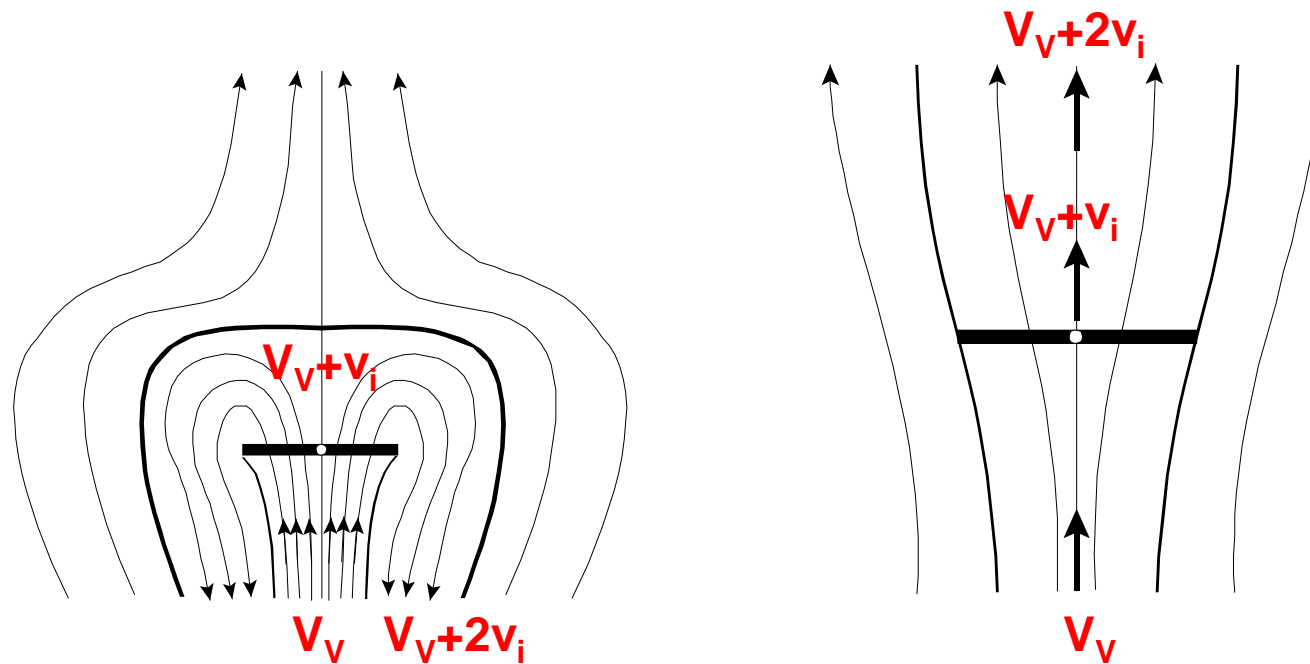
- In the fourth regime depicted, the rotor turns without absorbing power from the power supply or producing power.





New concepts:

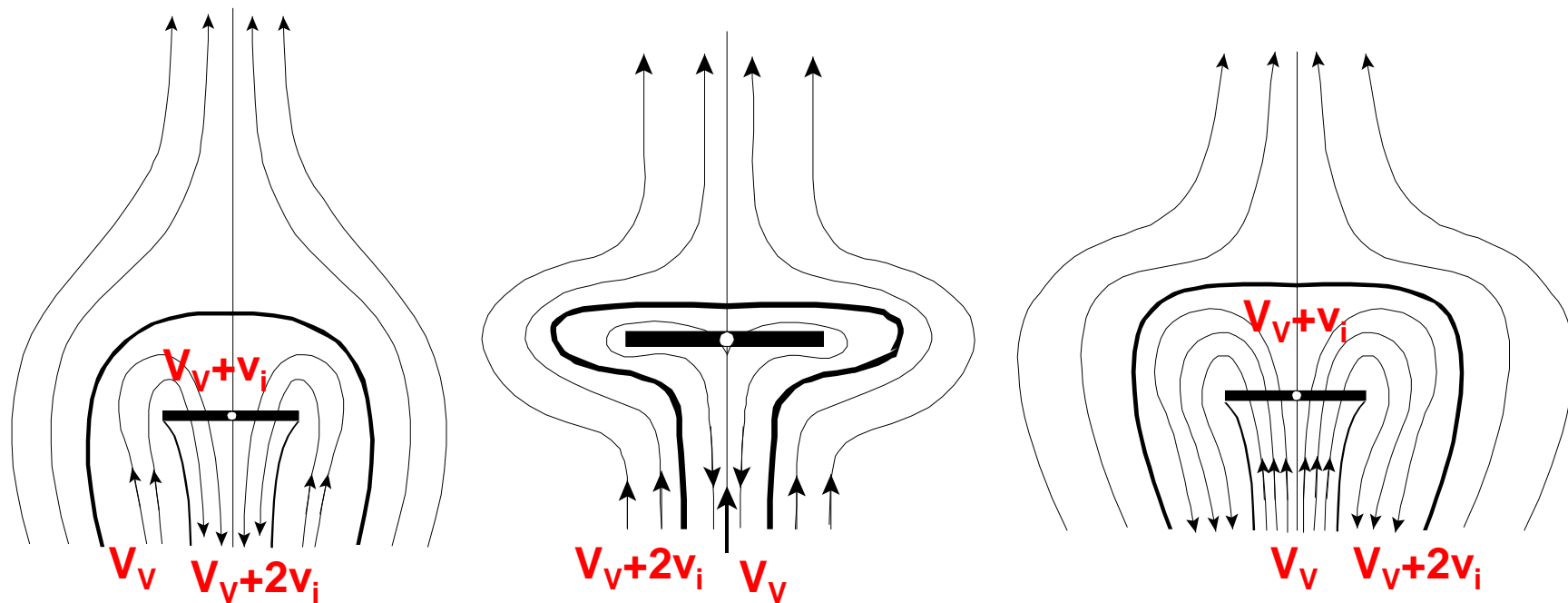
- In the last two regimes the rotor turns without the need for a power supply.



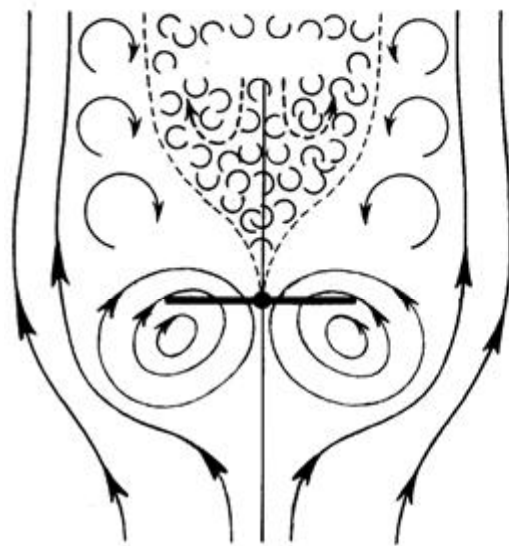


New concepts:

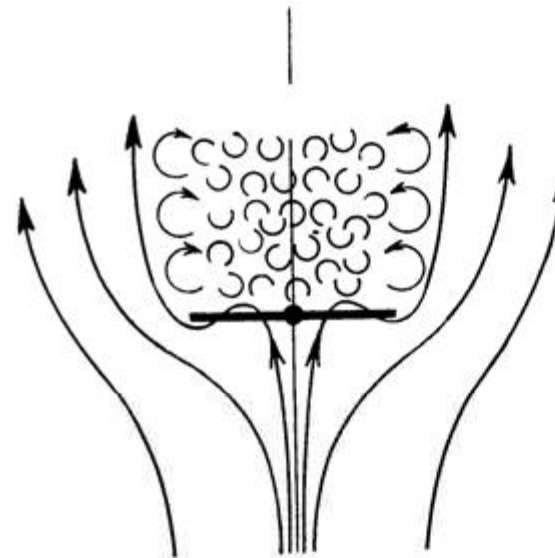
- In other regimes the configuration of the streamlines are clearly absurd.



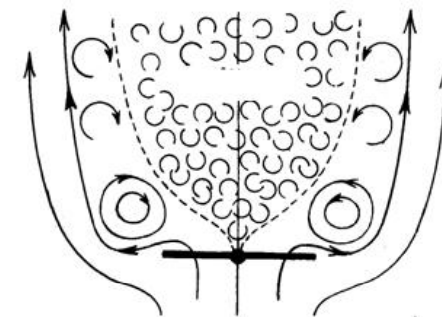




Vortex rings



Autorotation



Windmill brake

New concepts:

- The Momentum Theory is not applicable in these conditions.



## INDUCED POWER CALCULATION

- The aim of this section is to apply the equation of momentum to the corresponding regimes of  $V_v \geq 0$  and the windmill brake, and describe some empirical expressions.
- These expressions have to be adapted to the previous regimes, matching with what occurs in the regimes of the vortex rings, autorotation and windmill brake.
- The following expressions are used:

$$\frac{V_v}{V_{io}} = \overline{V_v}; \quad \frac{v_i}{V_{io}} = \overline{v_i}; \quad \frac{V_v + v_i}{V_{io}} = \overline{U_p}$$



## INDUCED POWER CALCULATION

### Vertical Climb, Hover.

$$\frac{V_v + v_i}{v_{io}} \bullet \frac{v_i}{v_{io}} = 1 \left\{ \begin{array}{l} \bar{V}_V = \frac{1}{\bar{v}_i} - \bar{v}_i \\ \bar{V}_V = \bar{U}_P - \frac{1}{\bar{U}_P} \end{array} \right\} \quad \bar{V}_v = \bar{U}_p - \frac{1.2}{\bar{U}_p}$$

$$\bar{P}_i = \frac{1}{2} \bar{V}_v + \sqrt{1.2 + \frac{1}{4} \bar{V}_v^2}$$



## INDUCED POWER CALCULATION

### Regime of vortex rings

$$0 < \overline{U}_p < 1,1 \quad y \quad -1,7 < \overline{V}_v < 0$$

$$0 < \overline{U}_p < 0.8 \quad \_$$

$$\overline{V}_v (\overline{V}_v + 1.18) = 0.812 + 0.072 \overline{U}_p - 1.75 \overline{U}_p^2$$

$$0.8 < \overline{U}_p < 1.1 \quad \_$$

$$\overline{V}_v = 3.726 - 0.693 \overline{U}_p - \frac{3.26}{\overline{U}_p}$$



## INDUCED POWER CALCULATION

### Regime of turbulent wake

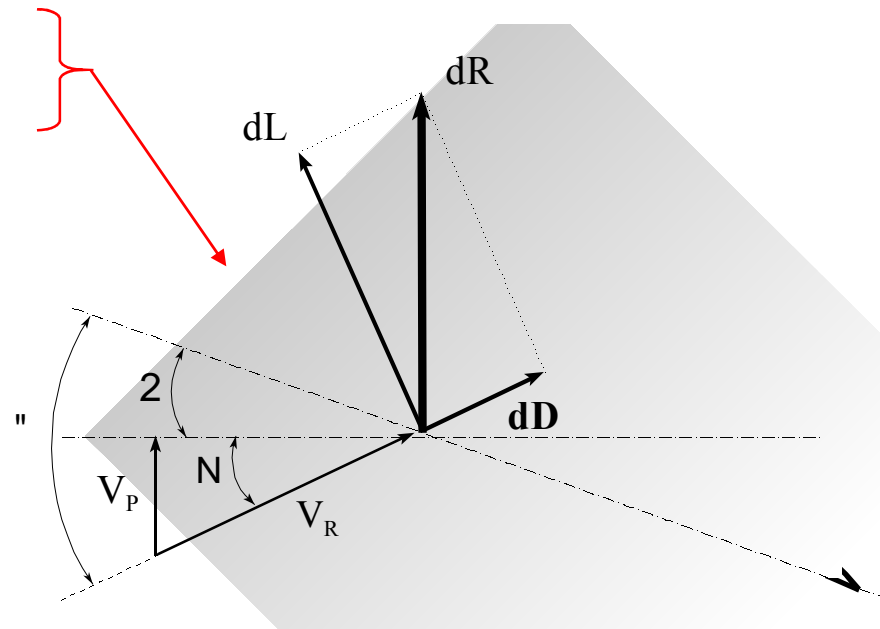
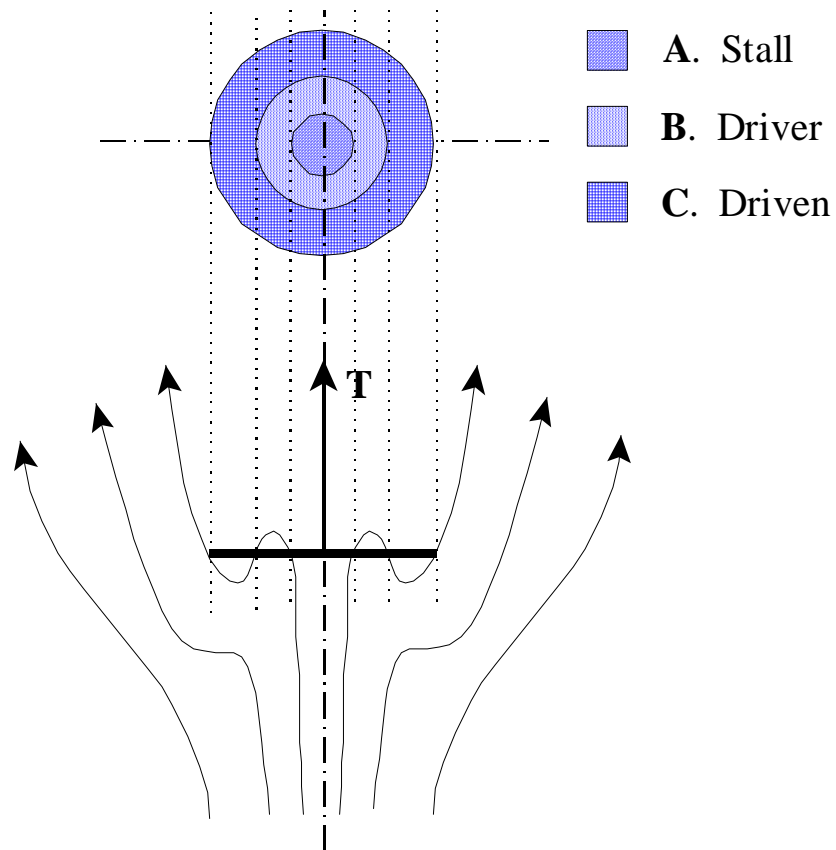
$$-1 < \overline{U}_P < 0 \quad y \quad -2 < \overline{V}_V < -1,7 \quad \overline{V}_V = -1.7 + 0.3\overline{U}_P$$

### Regime of windmill brake

$$\frac{V_v + v_i}{v_{io}} \bullet \frac{v_i}{v_{io}} = -1 \quad \left\{ \begin{array}{l} -\overline{V}_V = \frac{1}{\overline{v}_i} + \overline{v}_i \\ \overline{U}_P = -\frac{1}{\overline{v}_i} \end{array} \right\} \quad \overline{V}_V = \overline{U}_P + \frac{1}{\overline{U}_P}$$

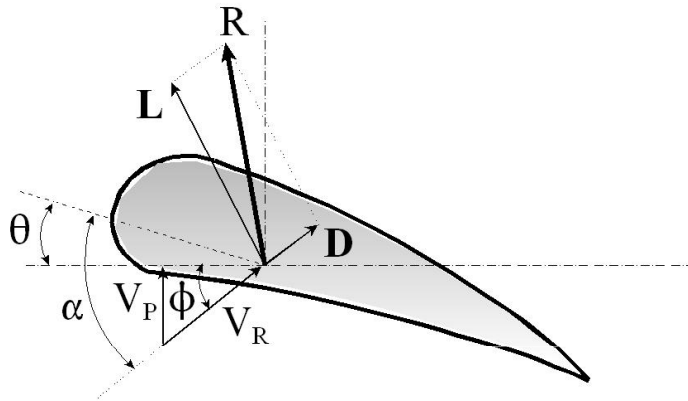


# AUTOROTATION

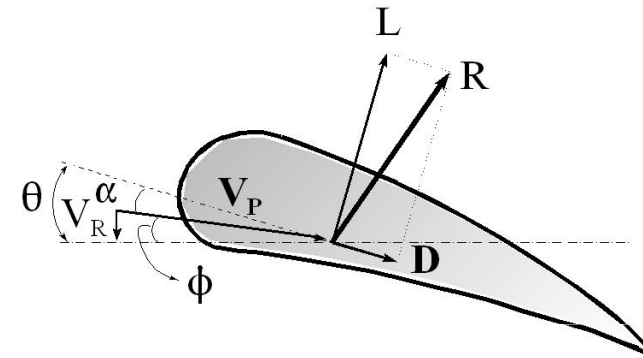




## AUTOROTATION



DRIVER DIAGRAM



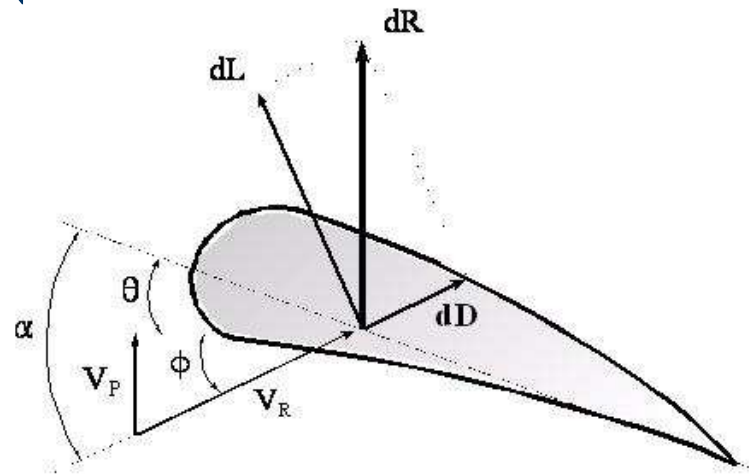
DRIVEN DIAGRAM

DRIVER ZONE TORQUE = DRIVEN ZONE TORQUE



# AUTOROTATION

## Stability



$$dQ_a = dQ_i = r \sin \Phi dL$$
$$dQ_d = dQ_o = r \cos \Phi dD$$

$$dQ_a = dQ_d \quad y \quad \tan|\phi| = \frac{dD}{dL} = \frac{C_L}{C_D}$$