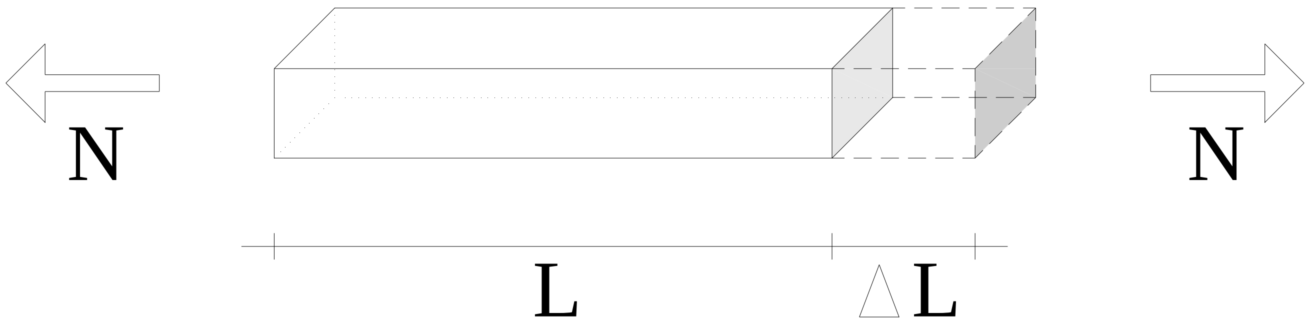
Statically determinate trusses

- static analysis
  stability and resistance evaluation
  (ultimate limit states)

- kinematic analysis
  axial deformations
  deflections / movements
  (serviceability limit states)

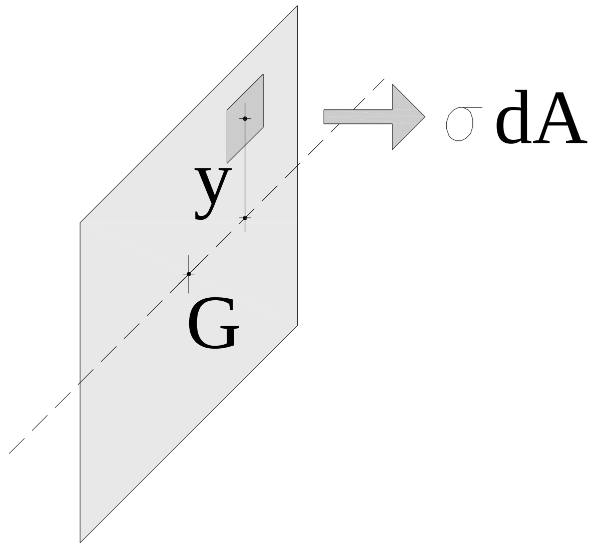
axial loading stress, strain and deformation



EQUILIBRIUM :

$$\int_A \sigma \cdot dA = N$$

$$\int_A \sigma \cdot y \cdot dA = 0$$



PLANE CROSS-SECTION HYPOTHESIS (NAVIER) :

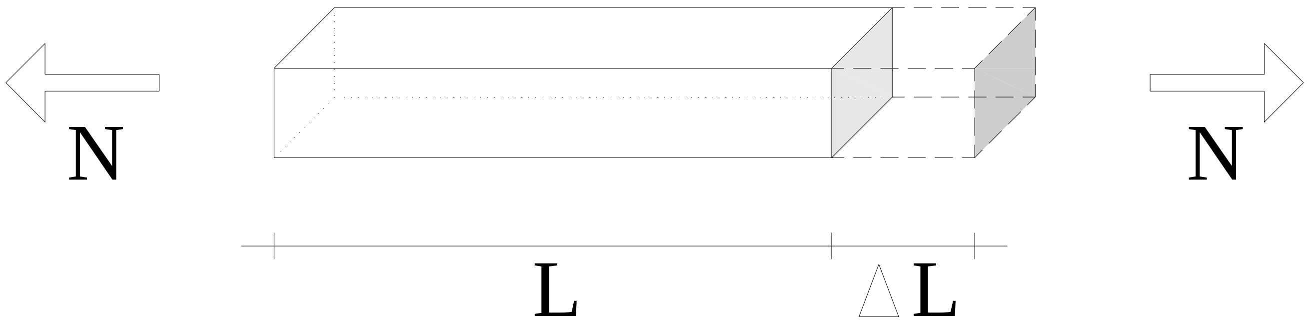
$$\epsilon = \epsilon_G + C \cdot y$$

ELASTIC-LINEAR MATERIAL (YOUNG) :

$$\sigma = E \cdot \epsilon$$

$$\boxed{\sigma = \frac{N}{A}} \qquad \boxed{\epsilon = \frac{N}{EA}}$$

elastic deformation of a constant cross-section bar loaded in tension



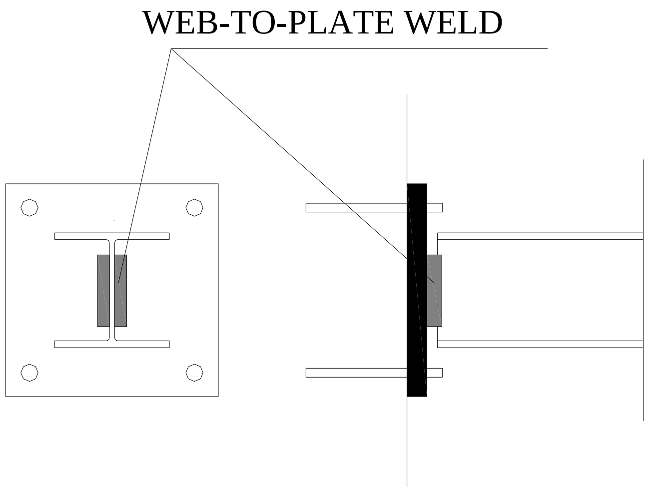
$$\sigma = \frac{N}{A}$$

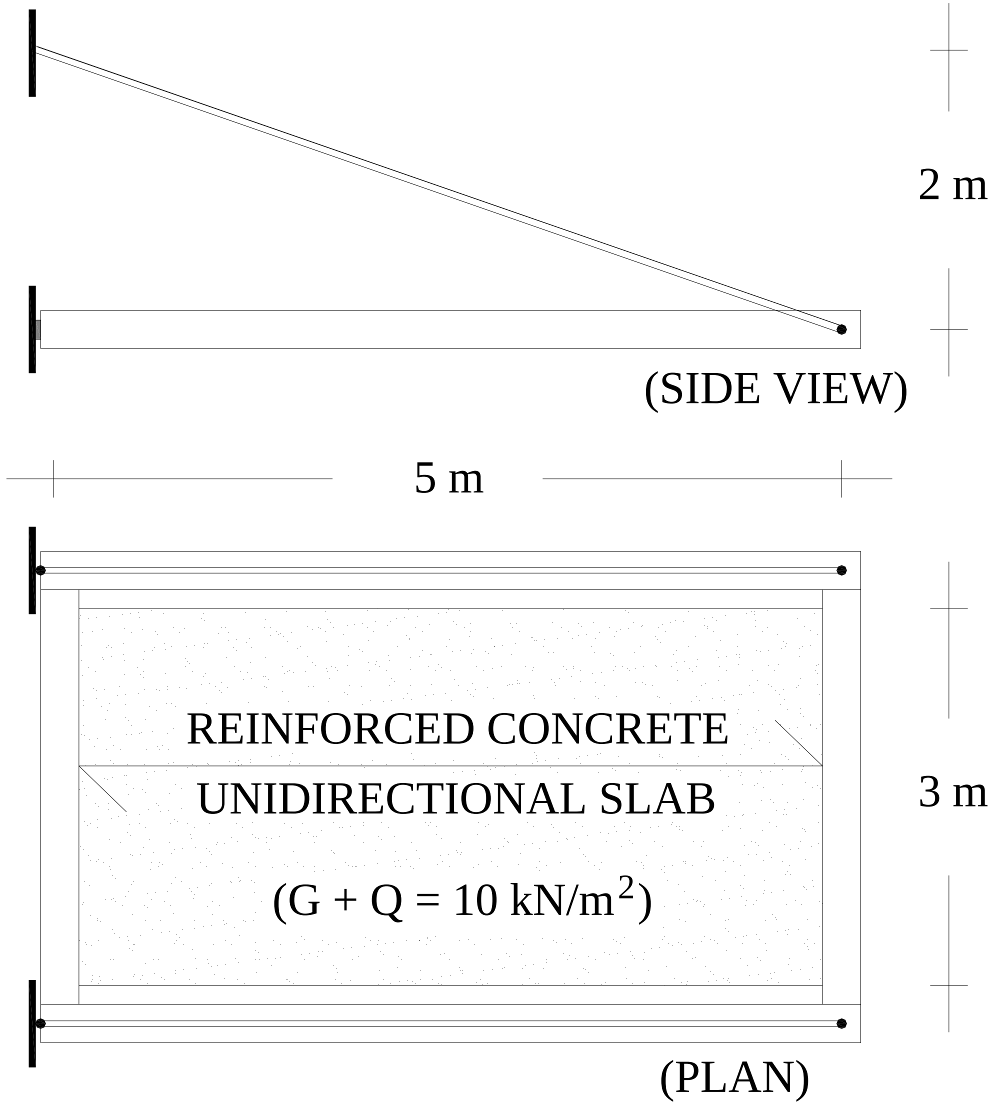
$$\epsilon = \frac{N}{EA}$$

$$\frac{\Delta L}{L} = \epsilon = \frac{N}{EA}$$

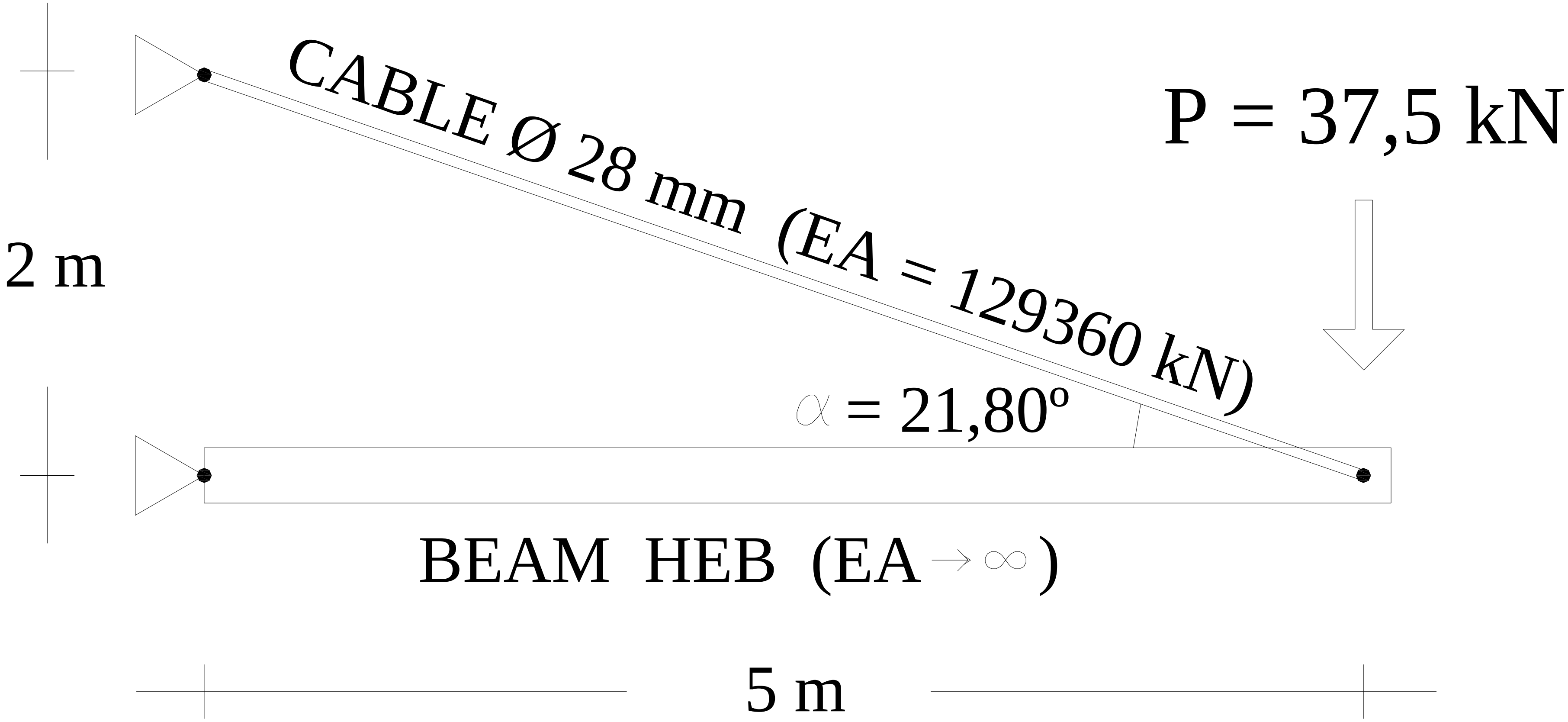
$$\boxed{\Delta L = \frac{N\,L}{E\,A}}$$

an elementary "real" example





elementary example : model



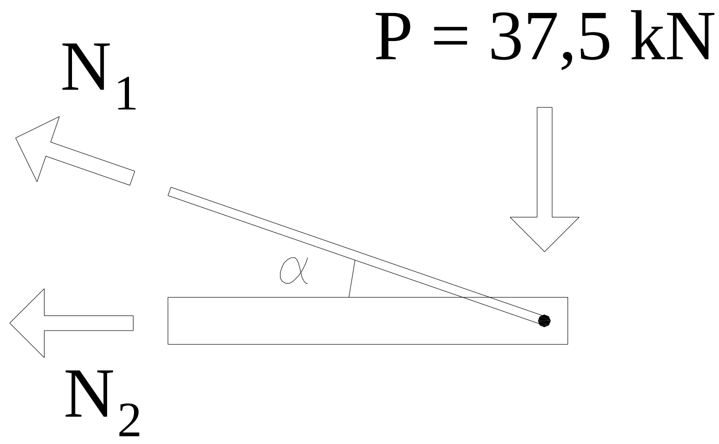
(internal) joint equilibrium

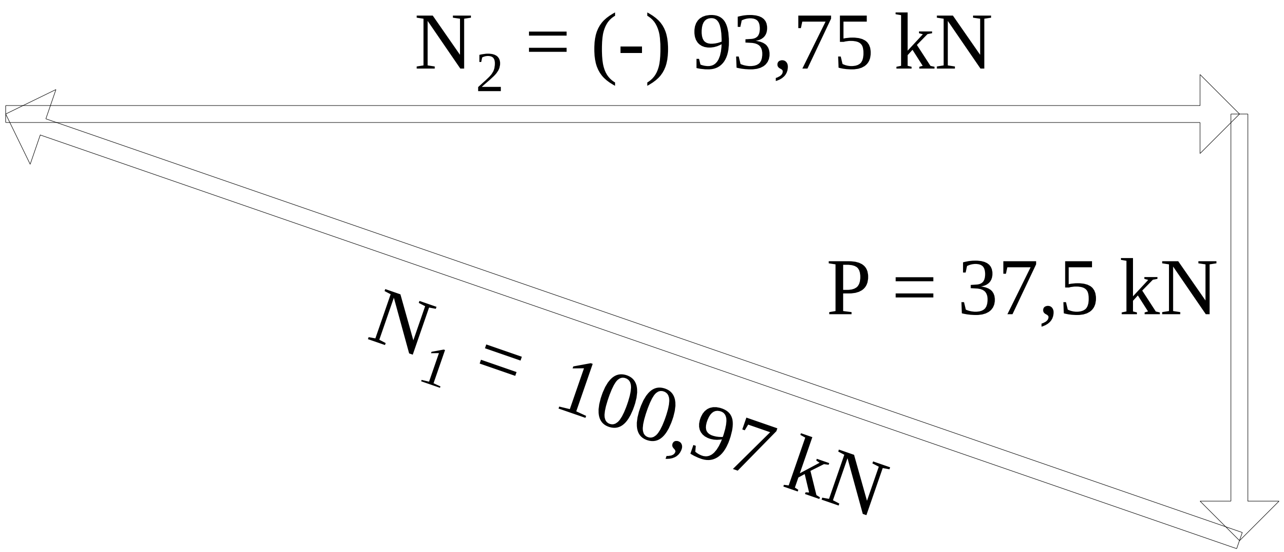
$$\Sigma F_x = 0 \ : \ -N_1 \cos\alpha - N_2 = 0$$

$$\Sigma F_y = 0 \ : \ -37,5 + N_1 \operatorname{sen}\alpha = 0$$

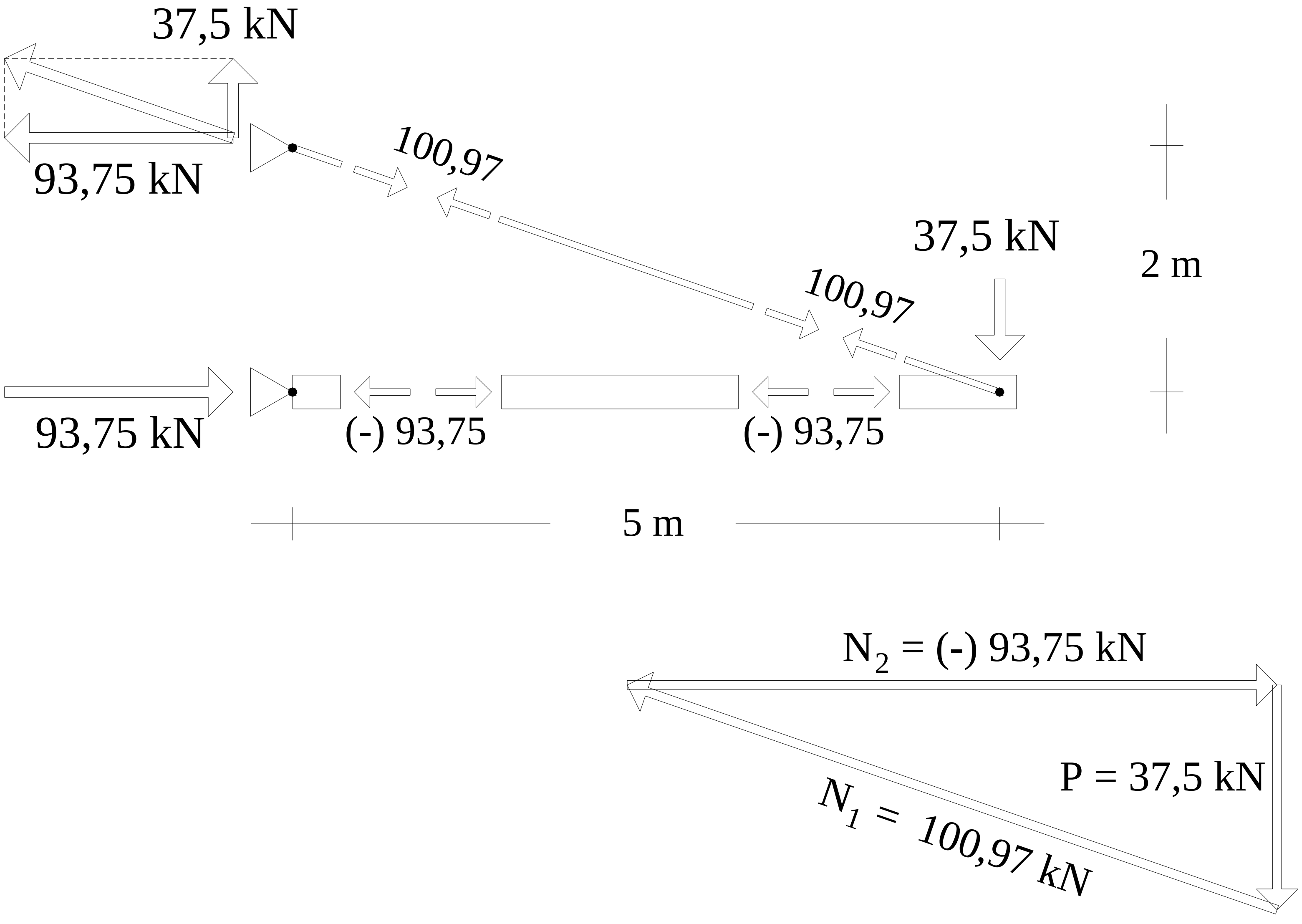
$$\boxed{N_1 = 100,972\,\text{kN}}$$

$$\boxed{N_2 = \ -93,75\ \text{kN}}$$

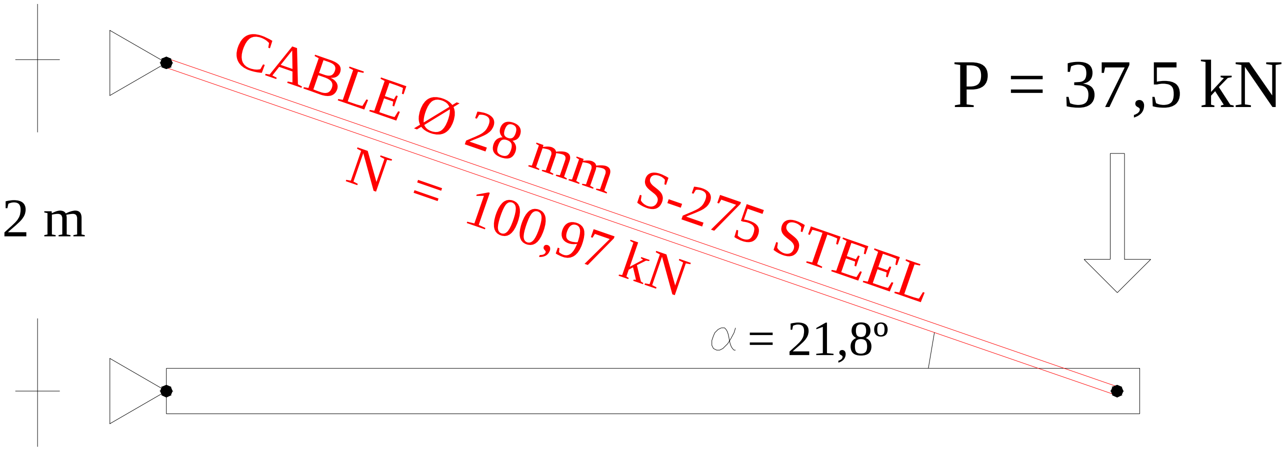




¡¡¡ equilibrium !!!



u.l.s. cable axial tension check



DATA :

$A_{\Phi 28} = 6,16\,\text{cm}^2$

$f_{y_{S\,275}} = 275\,\text{N/mm}^2$

$\gamma_F = 1,5$

$\gamma_M = 1,05$

DESIGN TENSILE FORCE, $N_{E,d}$:

$$N_{E,d} = N \cdot \gamma_F = 100,97\,\text{kN} \cdot 1,5 = 151,46\text{kN}$$

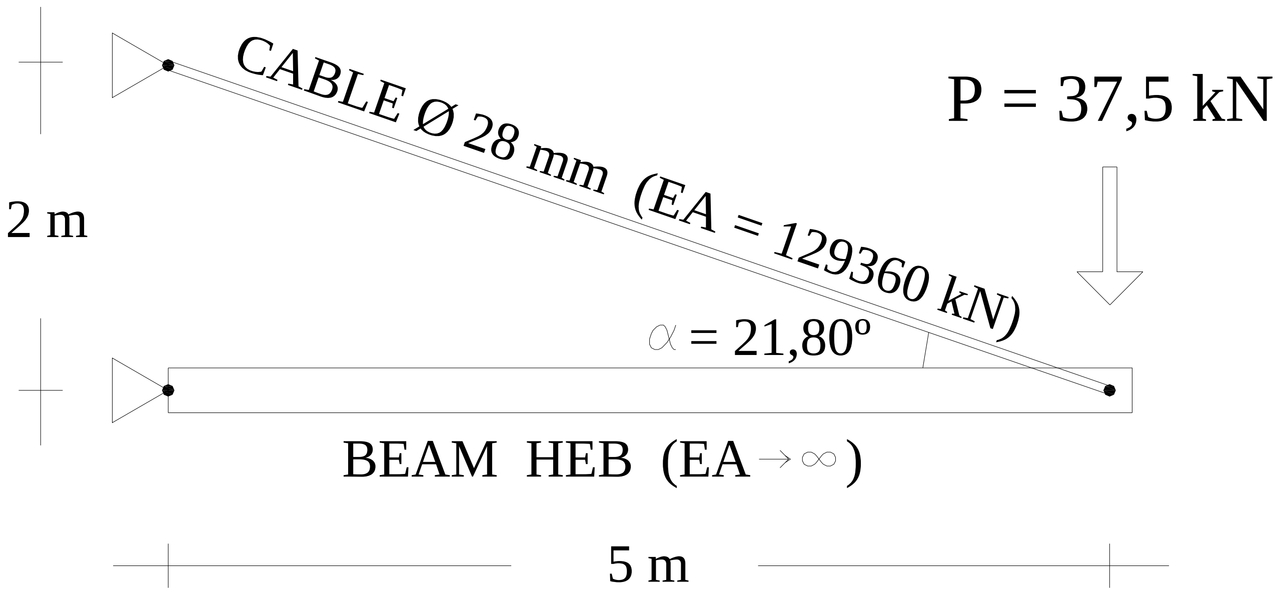
DESIGN TENSION RESISTANCE, $N_{R,d}$:

$$N_{R,d} = A \cdot f_{yd} = A \cdot \frac{f_y}{\gamma_M} = 6,16\text{cm}^2 \cdot \frac{275\,\text{N/mm}^2}{1,05} = 161,33\,\text{kN}$$

AXIAL TENSION CHECK:

$$\boxed{\frac{N_{E,d}}{N_{R,d}} = \frac{151,46\,\text{kN}}{161,33\,\text{kN}} < 1}$$

axial deformations



DATA :

$E = 210000\,\text{N/mm}^2$

$A_1 = A_{\Phi_{25}} = 6,16\,\text{cm}^2$

$E\,A_1 = 129360\,\text{kN}$

$A_2 >> A_1$

$E\,A_2 \longrightarrow \infty$

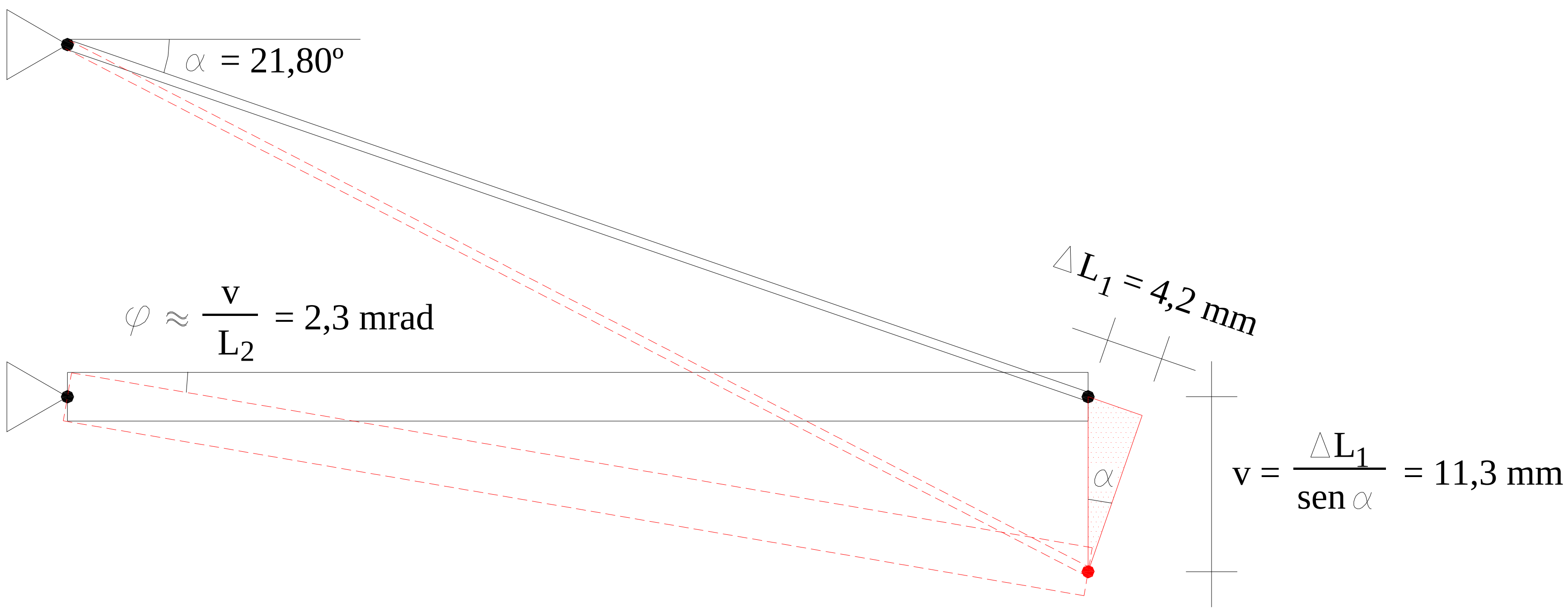
CABLE AXIAL DEFORMATION, ΔL_1 :

$$\Delta L_1 = \frac{N_1 \cdot L_1}{E \cdot A_1} = \frac{100,97\,\text{kN} \cdot 5,385\,\text{m}}{210000\,\text{N/mm}^2 \cdot 6,16\,\text{cm}^2} = 0,004203\,\text{m} = 4,203\,\text{mm}$$

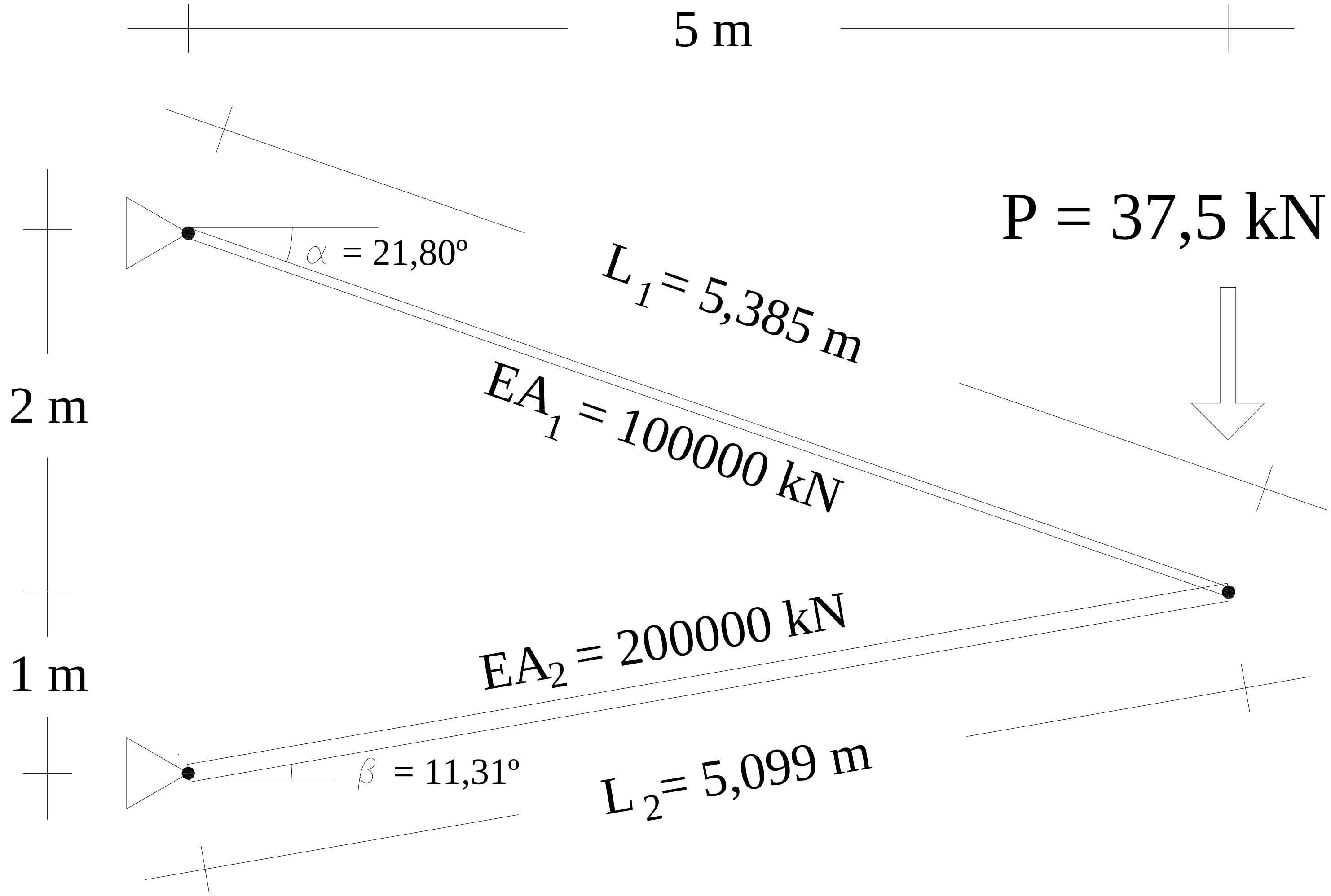
BEAM AXIAL DEFORMATION, ΔL_2 :

$$\Delta L_2 = \frac{N_2 \cdot L_2}{E \cdot A_2} = \frac{-93,75\,\text{kN} \cdot 5\,\text{m}}{EA_2 \to \infty} \simeq 0$$

axial deformations and movements



another (not so) elementary example



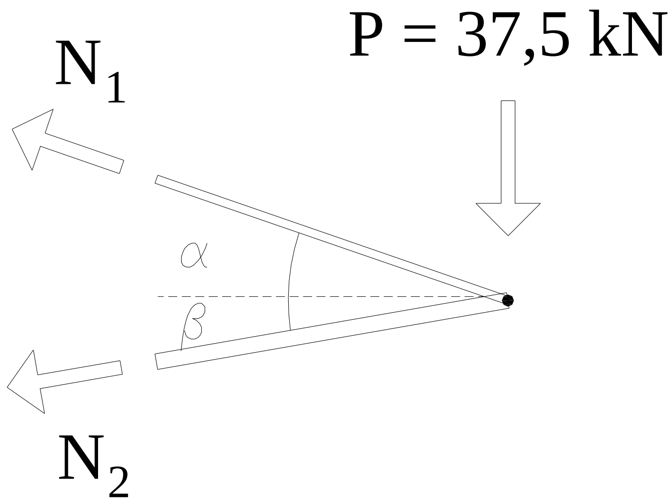
(internal) joint equilibrium

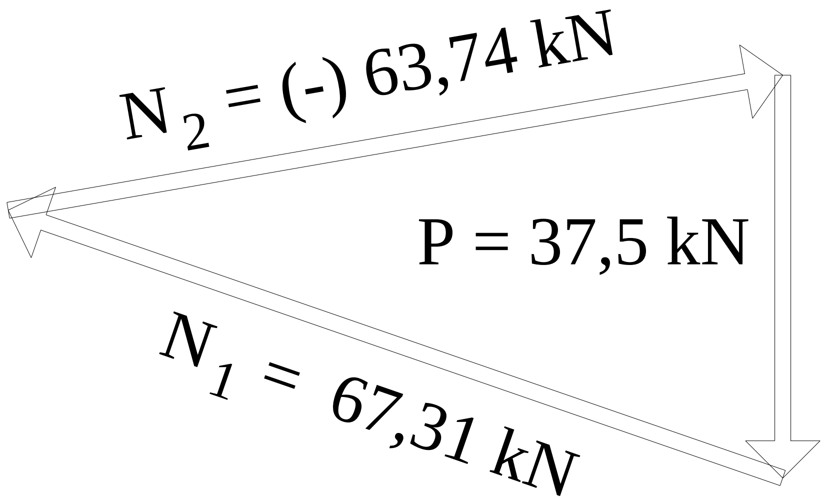
$$\Sigma F_x = 0 \;:\; -N_1\cos\alpha - N_2\cos\beta = 0$$

$$\Sigma F_y = 0 \;:\; -37,5 + N_1\,\text{sen}\alpha - N_2\,\text{sen}\beta = 0$$

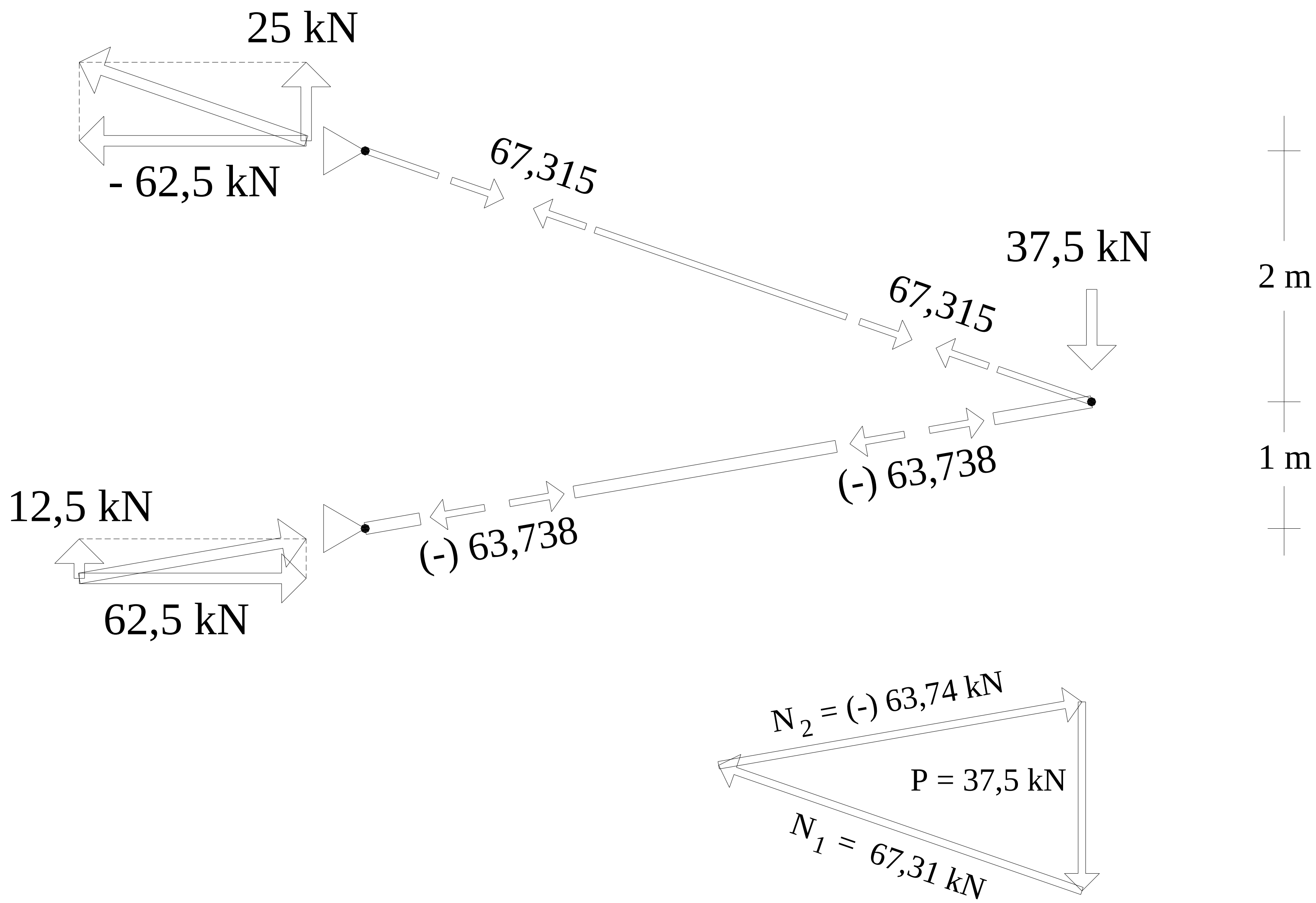
$$\boxed{N_1 = \;\; 67,315 \text{ kN}}$$

$$\boxed{N_2 = -63,738 \text{ kN}}$$

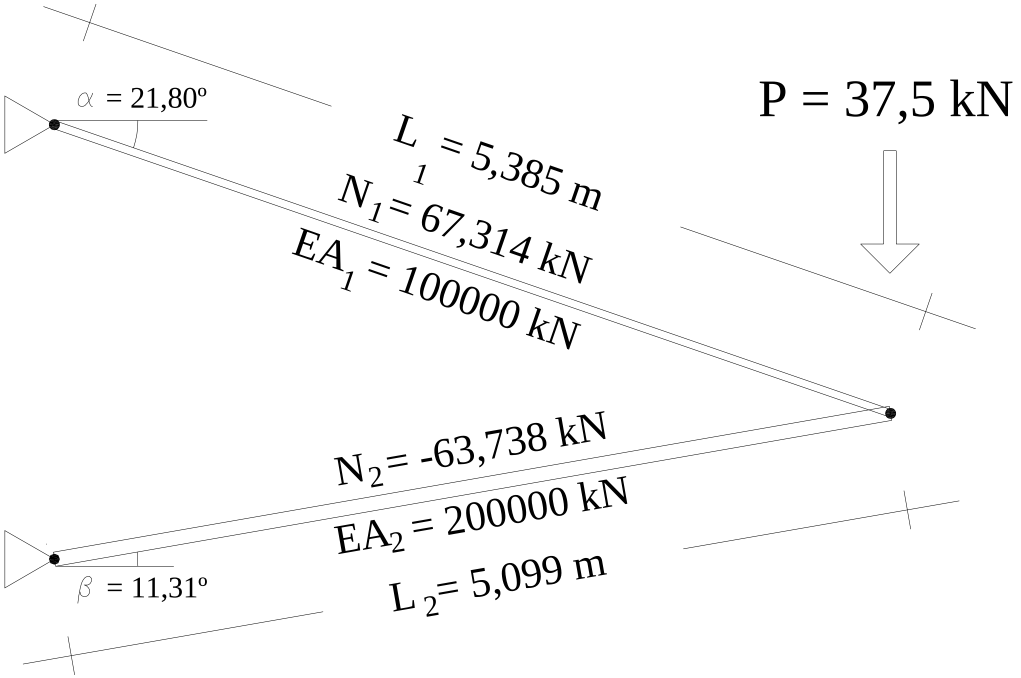




¡¡¡ equilibrium !!!



axial deformations



$$\Delta L = \frac{N\,L}{E\,A}$$

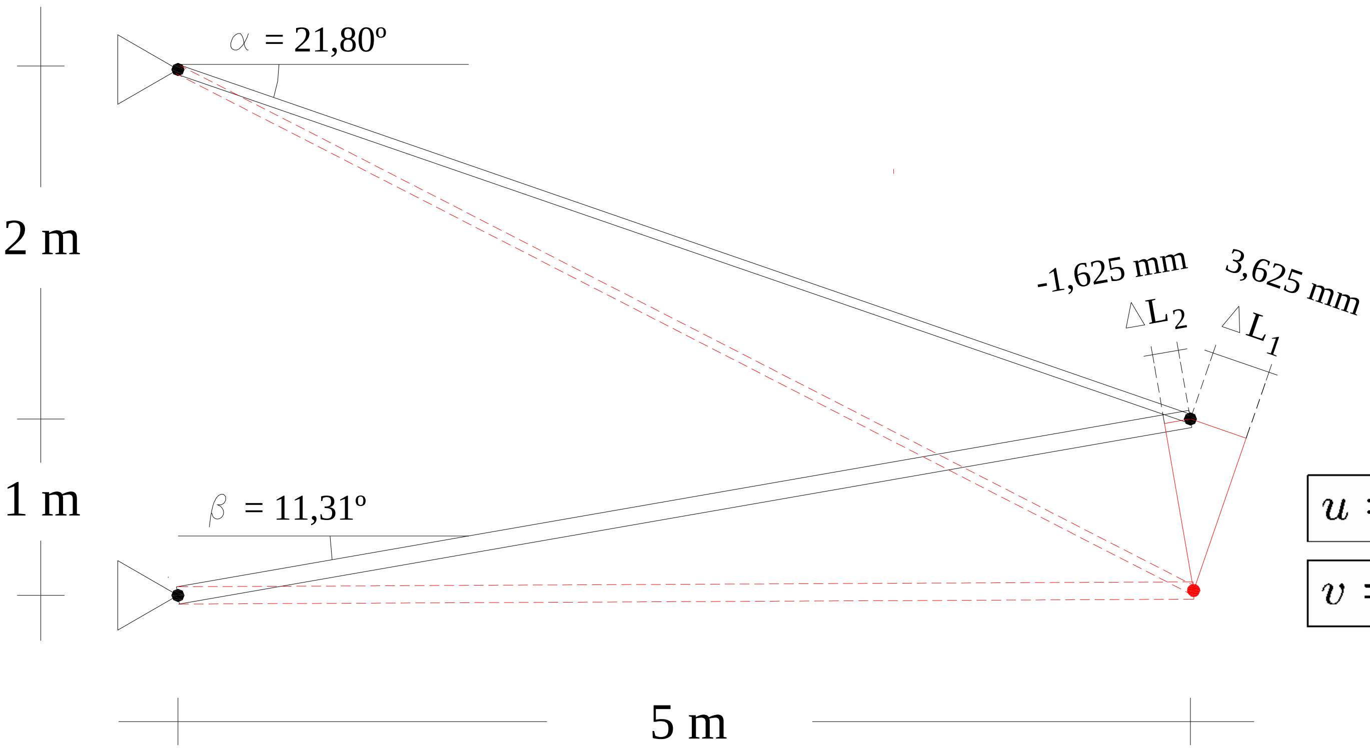
ALARGAMIENTO BARRA 1, ΔL_1 :

$$\Delta L_1 = \frac{N_1\,L_1}{E\,A_1} = \frac{67,314\,\text{kN} \cdot 5,385\,\text{m}}{100000\,\text{kN}} = 0,003625\,\text{m} = 3,625\,\text{mm}$$

ACORTAMIENTO BARRA 2, ΔL_2 :

$$\Delta L_2 = \frac{N_2\,L_2}{E\,A_2} = \frac{-63,738\,\text{kN} \cdot 5,099\,\text{m}}{200000\,\text{kN}} = -0,001625\,\text{m} = -1,625\,\text{mm}$$

axial deformations and movements



$u = \quad 0,196\,\text{mm}$

$v = -9,269\,\text{mm}$