Virtual Work Principle (VWP)

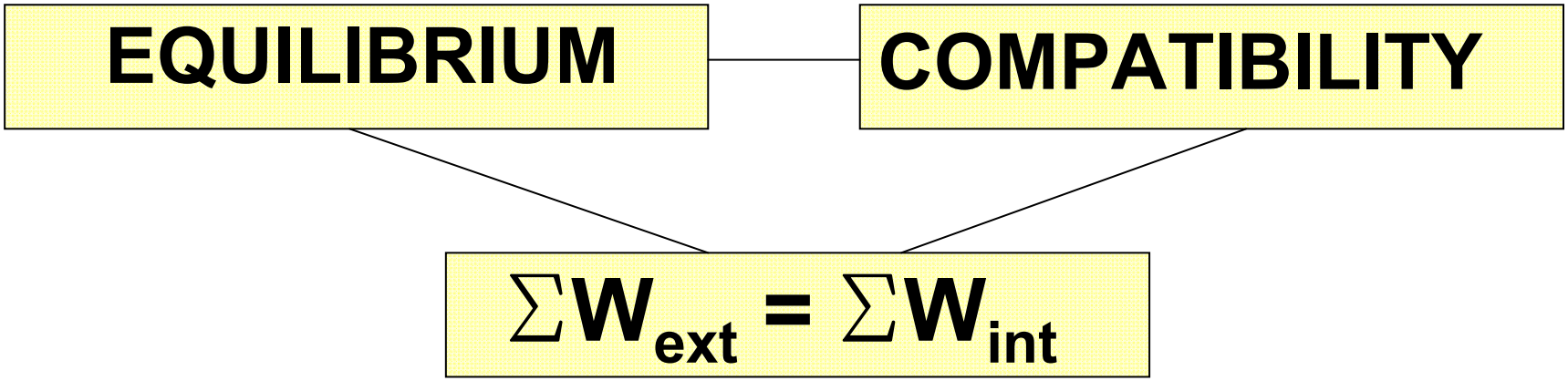
ARQUÍMEDES ……..(Simple machines, c. III b.c.)
GALILEO ……(Della Scienza Meccanica, 1594)
JOHANN BERNOUILLI ……(Letter to Varignon, 1717)
EULER .....(Methodus inveniendi lineas curvas …, 1744)

LAGRANGE ……(Méchanique Analythique, 1788)

"The necessary and sufficient condition for the equilibrium of a system is that total work done by all forces is zero for any virtual displacement consistent with system constraints."

CLAPEYRON .(Théorie mathematique de l'élasticité, 1852)
MAXWELL . (On the calculation of equilibrium and stiffnes of frames, 1864)
BETTI ……(Teoria della ellastictá, 1872)
CASTIGLIANO (Nuova teoria intorno all'equilibrio dei sistemi ellastici,1875)

V. W. P.
“triangular” relationship



V. W. P.

independence of static and kinematic systems

static system (virtual or real)	**kinematic system** (virtual or real)

V.W.P. POSSIBLE FORMULATIONS

((REAL FORCES AND DISPLACEMENTS))

REAL FORCES AND VIRTUAL DISPLACEMENTS
REAL DISPLACEMENTS AND VIRTUAL FORCES

((VIRTUAL FORCES AND DISPLACEMENTS))

V. W. P.

independence of material behaviour model

(elastic or not …)

independence of deformation origin

(mechanical, temperature changes, …)

V.W.P. in structural analysis

ISOSTATIC SYSTEMS (STATIC ANALYSIS)

- DETERMINATION OF REACTIONS
- DETERMINATION OF INTERNAL FORCES

ISOSTATIC SYSTEMS (KINEMATIC ANALYSIS)

- DETERMINATION OF MOVEMENTS / DEFLECTIONS

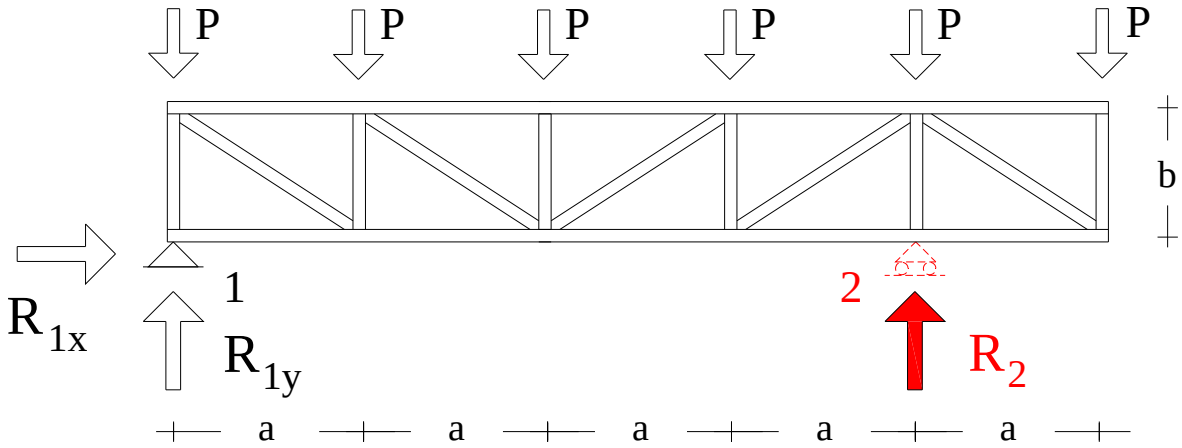
HIPERESTATIC SYSTEMS (STATIC/KINEMATIC ANALYSIS)

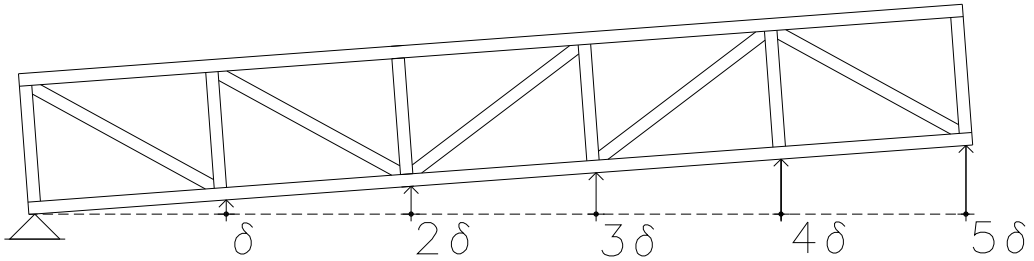
- BASIS OF BOTH ATERNATIVE ANALYSIS METHODS
  - Forthe Method (“virtual” forces)
  - Displacement Method (“virtual” displacements)

APPROXIMATED ANALYSIS BY DISCRETIZATION METHODS

- “WEAK” FORMULATION OF EQUILIBRIUM CONDITIONS

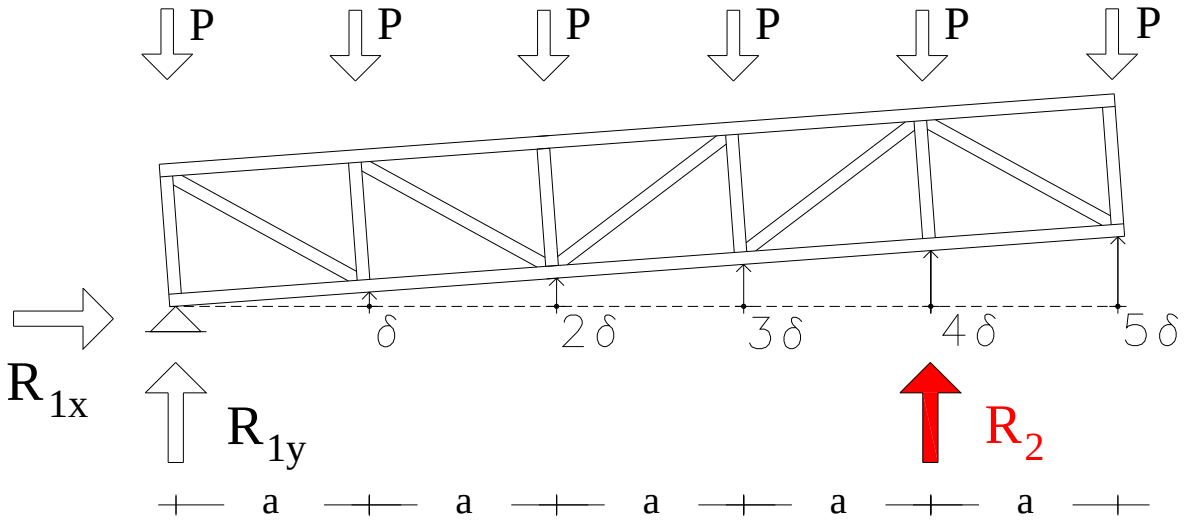
V.W.P. determination of reactions





CONSTRAINT CORRESPONDING TO REACTION TO CALCULATE IS ELIMINATED …
A VIRTUAL RIGID-BODY MOVEMENT COMPATIBLE WITH OTHER CONSTRAINTS IS CONSIDERED

V.W.P. determination of reactions



$$\boxed{\text{RIGID BODY} \Rightarrow \sum W_{int} = 0 \Rightarrow \sum W_{ext} = 0}$$

$$\sum W_{ext} = -P \cdot \delta - P \cdot 2\delta - P \cdot 3\delta - P \cdot 4\delta - P \cdot 5\delta + R_2 \cdot 4\delta = 0$$

$$R_2 = \frac{15P}{4} = 3,75P$$