Introduction to Symbolic Computation for Engineers

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4.-SYMBOLIC ALGORITHMS IN LINEAR ALGEBRA: BAREISS METHOD.

4.1. Setting out the problem.

4.2. Direct Methods of resolution of linear systems. Bareiss method

Based on the book of Winkler F. *Polynomial Algorithms in Computer Algebra.* **Springer Verlag (1996).**

4.1- Setting out the problem

We want to solve a system of linear equations with integer coefficients.

We assume that the matrix of the system is square and the system has unique solution.

(The ideas here exposed can be generalized to the general case).

Therefore, we want to solve the system of equations:

$$A \cdot X = b, \quad \textbf{donde} \quad A \in GL_n(\mathbb{Z}), \quad b^T \in \mathbb{Z}^n$$

Theoretically, the problem can be solve using Cramer's method. But this method involve determinants, and the computing time is of order $\mathcal{O}(n!)$.

On the other hand, the gaussian elimination is of order $\mathcal{O}(n^3)$. In consequence

the gaussian elimination provides a very good method to solve our problem.

Example: **(blackboard).**

- Gaussian Elimination ▶

But the difficulty of the gaussian elimination is the order of the matrices involved in the method grows considerably.

- Example Gaussian Elimination ▶

In the following, we present a direct method that consists in applying gaussian elimination avoiding divisions and reducing the size of the integers, without computations.

$$\downarrow$$

Bareiss Method

4.2- Direct Methods of resolution of linear systems.

This method is from Bareiss and is namely Fraction Free Gaussian Elimination.

Difficulties of the Gaussian Elimination:

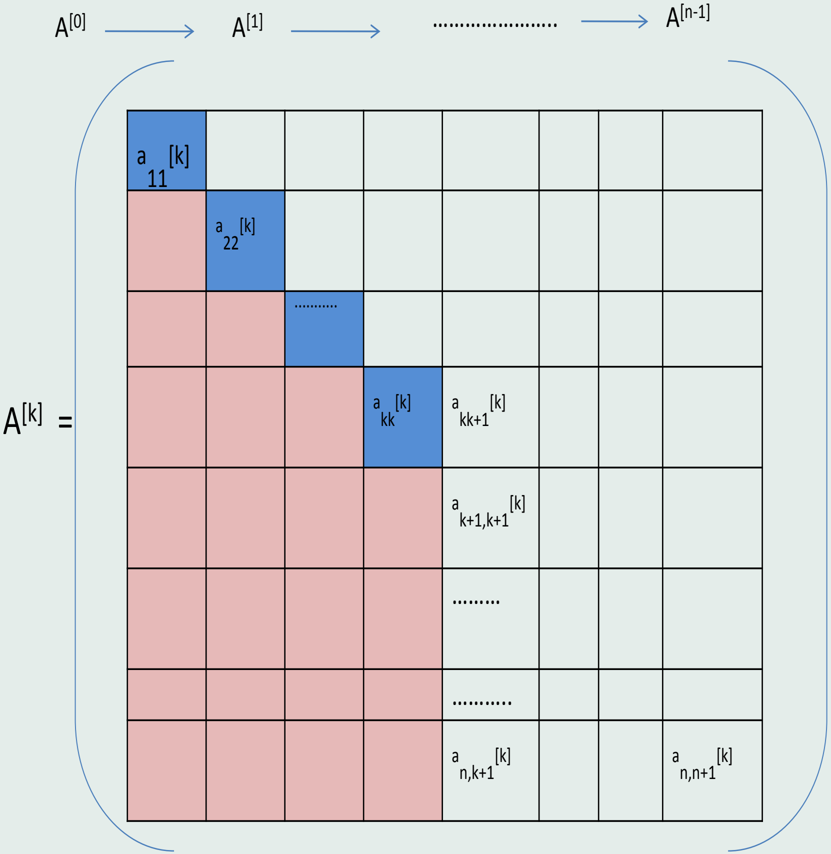
We introduce rational numbers. We overcome it '*clearing denominators*"in each step of the triangulation. But, this leads to a second difficulty, the size of the integers grows drastically.

To solve it, we could divide all the elements of each row by the gcd of its elements. But it is not advisable, since now we have to compute many gcd and it makes the complexity gets worse.

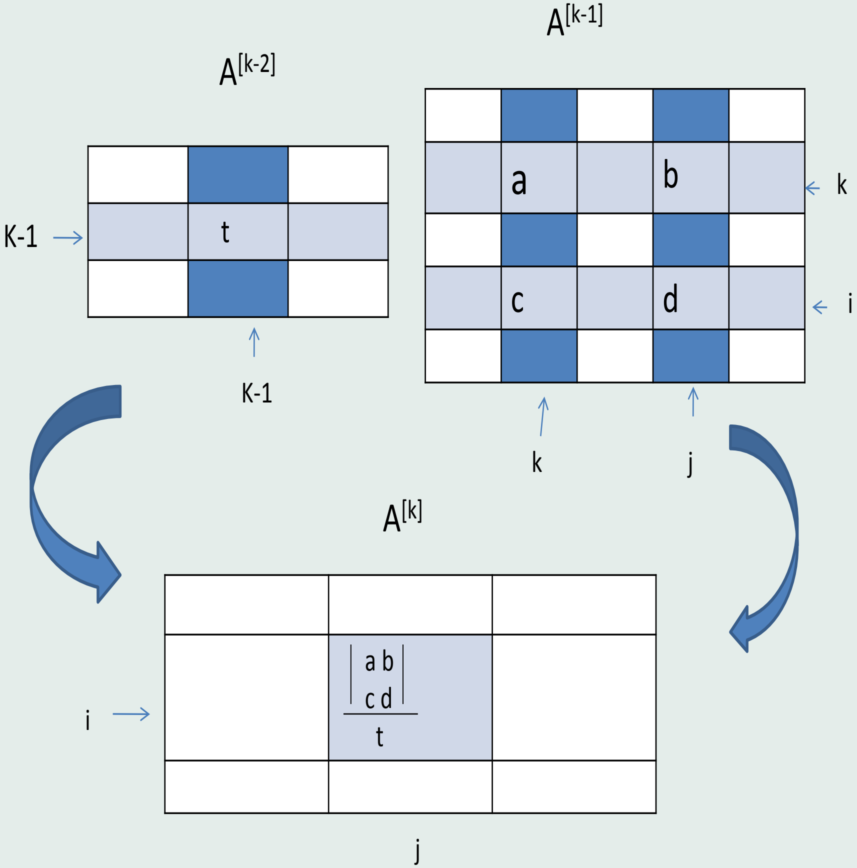
Bareiss solution:

It basically consists on determine, without extra computations, a common factor to the elements or each row big enough to control the size of the integers.

Description of the Method:



A[0]
A[1]
....
A[n-1]
A[k] =
a 11 [k]
a 22 [k]
....
a kk [k]
a kk+1 [k]
a k+1,k+1 [k]
....
....
a n,k+1 [k]
a n,n+1 [k]



A[k-2]
K-1
t
K-1
A[k-1]
a
b
k
c
d
i
k
j
A[k]
i
a b
c d
t
j

Remark.

- $a_{00}^{[-1]} = 1$.
- This method is specially appropriate when the coefficients of the matrix are polynomials.
- If n is the order of the matrix and ℓ is an upper bound for the size of the coefficients of the matrix, the method is of order $\mathcal{O}(n^5\ell^2)$.
- Maple command: ffgausselim