LAB # 5

(SOLVING SYSTEMS OF ALGEBRAIC EQUATIONS:
RESULTANTS AND GRÖEBNER BASIS)

1. Find the solution, using resultants, of the systems of polynomial equations in two variables:

   1.1 $\{x(y^2-x)^2-y^5=0, y^4+y^3-x^2=0\}$.

   1.2 $\{x^4+y^4-y^2=0, x^4+y^4-2y^3-2x^2y-xy^2+y^2=0\}$

   1.3. $\{p_1(x,y)=0, p_2(x,y)=0\}$ where
   $p_1(x,y)=2+9y+16y^2+y^5+4x^5+12x^4+24xy^3-36x^2y^2-5x^4y$
   $p_2(x,y)=1+4y-4x^5+9x^4y-3y^2x^3-5x^2y^3+3xy^4+10x^4-7yx^3-16x^2y^2+13xy^3-3x^2+3xy+6y^2$

2. Find the solution, using resultants, of the system of polynomial equations in two variables:

$$\{p_1(x,y)=0, p_2(x,y)=0, p_3(x,y)=0\}$$

   where $p_1(x,y)=-16+6y+y^2-17x+9xy-x^2=0$,
   $p_2(x,y)=-136+124y-12y^2-8y^3+143x-164xy+40xy^2+2x^2+24x^2y-25x^3=0$,
   $p_3(x,y)=70-73y+21y^2-y^3+x+xy-6x^2+3x^3=0$.

3. Implement in Maple an algorithm using resultants to determine the implicit equation of a planar curve defined by a parametrization. Apply such algorithm to compute the polynomial defining implicitly the curve defined by the rational parametrization

$$\mathcal{P}(t)=\left(\frac{t^5+1}{t^2+3}, \frac{t^3+t+1}{t^2+1}\right).$$

4. Apply Gröbner basis to decide the existence of solution for each one of the following systems of algebraic equations. In the affirmative case, find the solution set.

   4.1. $\{x^2+y^2+z^2-1=0, x^2+z^2-y=0, x-z=0\}$.

   4.2. $\{x^2+y^4+xy=0, x^3+zy=0, xyz=0\}$.

   4.3. $\{x^6+y^4+z^3=0, x^2+z^2y=0, xyz-w^3=0\}$.

5. Decide if the polynomials $f=x^3+3yx^2+xy^2-x+z+z^2x+z^2+zy+xz+xy$ and $g=x^4-xz$ belong to the ideal $I=(x^2+yx, xy-1, xz+z+x)$.

6. Implement in Maple an algorithm using Grö ebner basis to determine the implicit expression of a geometric object defined by n parametric equations in $n-1$ variables. Apply such algorithm to compute the polynomial defining implicitly the object defined by the rational parametrization

$$\mathcal{P}(t)=\left(\frac{t_1t_2}{t_1^2+2t_2^2+1}, \frac{t_2^2+4}{t_1^2+2t_2^2+1}, \frac{3t_1+5}{t_1^2+2t_2^2+1}\right).$$