Ejercicio 2, 4º parcial 13/06/2000

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- Ecuaciones de la dinámica de Lagrange generales (no lineales);
- Pequeñas oscilaciones: modos de vibración y frecuencias propias.

```
> restart:
```

```
> with(linalg):
```

```
Warning, new definition for norm

Warning, new definition for trace
```

```
> T:=(1/2)*2*m*diff(x(t),t)^2
+(1/2)*m*(diff(x(t),t)^2+(R/sqrt(3))^2*diff(theta(t),t)^2+2*(R/sqrt(3))*diff(x(t),t)*diff(theta
(t),t)*cos(theta(t)))
+(1/2)*((m/12)*R^2)*diff(theta(t),t)^2;
```

$$T := m\left(\frac{\partial}{\partial t}x(t)\right)^2 + \frac{1}{2}m\left(\left(\frac{\partial}{\partial t}x(t)\right)^2 + \frac{1}{3}R^2\left(\frac{\partial}{\partial t}\theta(t)\right)^2 + \frac{2}{3}R\sqrt{3}\left(\frac{\partial}{\partial t}x(t)\right)\left(\frac{\partial}{\partial t}\theta(t)\right)\cos(\theta(t))\right) + \frac{1}{24}m R^2\left(\frac{\partial}{\partial t}\theta\right.$$

```
> V:=2*(1/2)*k*x(t)^2-m*g*(R/sqrt(3))*cos(theta(t));
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$$V := k\,x(t)^2 - \frac{1}{3}m\,g\,R\sqrt{3}\cos(\theta(t))$$

```
> L:=T-V;
```

$$L := m\left(\frac{\partial}{\partial t}x(t)\right)^2 + \frac{1}{2}m\left(\left(\frac{\partial}{\partial t}x(t)\right)^2 + \frac{1}{3}R^2\left(\frac{\partial}{\partial t}\theta(t)\right)^2 + \frac{2}{3}R\sqrt{3}\left(\frac{\partial}{\partial t}x(t)\right)\left(\frac{\partial}{\partial t}\theta(t)\right)\cos(\theta(t))\right) + \frac{1}{24}m R^2\left(\frac{\partial}{\partial t}\theta\right.$$

$$- k\,x(t)^2 + \frac{1}{3}m\,g\,R\sqrt{3}\cos(\theta(t))$$

```
> L||1:=subs({diff(x(t),t)=x||1,diff(theta(t),t)=theta||1,x(t)=x,theta(t)=theta},L);
```

$$L1 := m\,x1^2 + \frac{1}{2}m\left(x1^2 + \frac{1}{3}R^2\,\theta1^2 + \frac{2}{3}R\sqrt{3}\,x1\,\theta1\cos(\theta)\right) + \frac{1}{24}m R^2\,\theta1^2 - k\,x^2 + \frac{1}{3}m\,g\,R\sqrt{3}\cos(\theta)$$

```
> t1:=diff(L||1,x||1);
```

$$t1 := 2\,m\,x1 + \frac{1}{2}m\left(2\,x1 + \frac{2}{3}R\sqrt{3}\,\theta 1\cos(\theta)\right)$$

> **t2:=subs({x||1=diff(x(t),t),theta||1=diff(theta(t),t),x=x(t),theta=theta(t)},t1);**

$$t2 := 2\,m\left(\frac{\partial}{\partial t}\mathrm{x}(t)\right) + \frac{1}{2}m\left(2\left(\frac{\partial}{\partial t}\mathrm{x}(t)\right) + \frac{2}{3}R\sqrt{3}\left(\frac{\partial}{\partial t}\theta(t)\right)\cos(\theta(t))\right)$$

> **t3:=diff(t2,t)-diff(L||1,x);**

$$t3 := 2\,m\left(\frac{\partial^2}{\partial t^2}\mathrm{x}(t)\right) + \frac{1}{2}m\left(2\left(\frac{\partial^2}{\partial t^2}\mathrm{x}(t)\right) + \frac{2}{3}R\sqrt{3}\left(\frac{\partial^2}{\partial t^2}\theta(t)\right)\cos(\theta(t)) - \frac{2}{3}R\sqrt{3}\left(\frac{\partial}{\partial t}\theta(t)\right)^2\sin(\theta(t))\right) + 2\,k\,x$$

> **ecu||1:=subs({diff(x(t),t,t)=x||2,diff(theta(t),t,t)=theta||2,diff(x(t),t)=x||1,diff(theta(t),t)
=theta||1,x(t)=x,theta(t)=theta},t3);**

$$ecu1 := 2\,m\,x2 + \frac{1}{2}m\left(2\,x2 + \frac{2}{3}R\sqrt{3}\,\theta 2\cos(\theta) - \frac{2}{3}R\sqrt{3}\,\theta 1^2\sin(\theta)\right) + 2\,k\,x$$

> **ecu||1:=simplify(ecu||1);**

$$ecu1 := 3\,m\,x2 + \frac{1}{3}m\,R\sqrt{3}\,\theta 2\cos(\theta) - \frac{1}{3}m\,R\sqrt{3}\,\theta 1^2\sin(\theta) + 2\,k\,x$$

> **t4:=diff(L||1,theta||1);**

$$t4 := \frac{1}{2}m\left(\frac{2}{3}R^2\,\theta 1 + \frac{2}{3}R\sqrt{3}\,x1\cos(\theta)\right) + \frac{1}{12}m\,R^2\,\theta 1$$

> **t5:=subs({x||1=diff(x(t),t),theta||1=diff(theta(t),t),x=x(t),theta=theta(t)},t4);**

$$t5 := \frac{1}{2}m\left(\frac{2}{3}R^2\left(\frac{\partial}{\partial t}\theta(t)\right) + \frac{2}{3}R\sqrt{3}\left(\frac{\partial}{\partial t}\mathrm{x}(t)\right)\cos(\theta(t))\right) + \frac{1}{12}m\,R^2\left(\frac{\partial}{\partial t}\theta(t)\right)$$

> **t6:=diff(t5,t)-diff(L||1,theta);**

$$t6 := \frac{1}{2}m\left(\frac{2}{3}R^2\left(\frac{\partial^2}{\partial t^2}\theta(t)\right) + \frac{2}{3}R\sqrt{3}\left(\frac{\partial^2}{\partial t^2}\mathrm{x}(t)\right)\cos(\theta(t)) - \frac{2}{3}R\sqrt{3}\left(\frac{\partial}{\partial t}\mathrm{x}(t)\right)\sin(\theta(t))\left(\frac{\partial}{\partial t}\theta(t)\right)\right) + \frac{1}{12}m\,R^2\left(\frac{\partial^2}{\partial t}\right.$$

$$+ \frac{1}{3}m\,R\sqrt{3}\,x1\,\theta 1\sin(\theta) + \frac{1}{3}m\,g\,R\sqrt{3}\sin(\theta)$$

> **ecu||2:=subs({diff(x(t),t,t)=x||2,diff(theta(t),t,t)=theta||2,diff(x(t),t)=x||1,diff(theta(t),t)
=theta||1,x(t)=x,theta(t)=theta},t6);**

$$ecu2 := \frac{1}{2} m \left(\frac{2}{3} R^2 \theta 2 + \frac{2}{3} R \sqrt{3}\, x2 \cos(\theta) - \frac{2}{3} R \sqrt{3}\, x1\, \theta 1 \sin(\theta) \right) + \frac{1}{12} m R^2 \theta 2 + \frac{1}{3} m R \sqrt{3}\, x1\, \theta 1 \sin(\theta) + \frac{1}{3} m g R$$

> **ecu||2:=simplify(ecu||2);**

$$ecu2 := \frac{5}{12} m R^2 \theta 2 + \frac{1}{3} m R \sqrt{3}\, x2 \cos(\theta) + \frac{1}{3} m g R \sqrt{3} \sin(\theta)$$

> **### WARNING: persistent store makes one-argument readlib obsolete**
readlib(mtaylor):

> **ecu1l:=mtaylor(ecu||1,[x||1,theta||1,x,theta],2);**

$$ecu1l := 3\, m\, x2 + \frac{1}{3} m R \sqrt{3}\, \theta 2 + 2\, k\, x$$

> **ecu2l:=mtaylor(ecu||2,[x||1,theta||1,x,theta],2);**

$$ecu2l := \frac{5}{12} m R^2 \theta 2 + \frac{1}{3} m R \sqrt{3}\, x2 + \frac{1}{3} m g R \sqrt{3}\, \theta$$

> **M:=matrix(**
[[coeff(ecu1l,x||2),coeff(ecu1l,theta||2)],
[coeff(ecu2l,x||2),coeff(ecu2l,theta||2)]]
);

$$M := \begin{bmatrix} 3\, m & \frac{1}{3} m R \sqrt{3} \\ \frac{1}{3} m R \sqrt{3} & \frac{5}{12} m R^2 \end{bmatrix}$$

> **K:=matrix(**
[[coeff(ecu1l,x),coeff(ecu1l,theta)],
[coeff(ecu2l,x),coeff(ecu2l,theta)]]
);

$$K := \begin{bmatrix} 2\, k & 0 \\ 0 & \frac{1}{3} m g R \sqrt{3} \end{bmatrix}$$

> **K||1:=map2(subs,{k=m*g,R=1},K);**
M||1:=map2(subs,{k=m*g,R=1},M);

$$K1 := \begin{bmatrix} 2\, m\, g & 0 \\ 0 & \frac{1}{3} m g \sqrt{3} \end{bmatrix}$$

$$M1 := \begin{bmatrix} 3\,m & \frac{1}{3} m \sqrt{3} \\ \frac{1}{3} m \sqrt{3} & \frac{5}{12} m \end{bmatrix}$$

> **ecu_car:=det(K||1-lambda*M||1);**

$$ecu\_car := \frac{2}{3} m^2 g^2 \sqrt{3} - \frac{5}{6} m^2 g \lambda - \lambda m^2 g \sqrt{3} + \frac{11}{12} \lambda^2 m^2$$

> **s_lambda:=solve(ecu_car,lambda);**

$$s\_lambda := \left(\frac{5}{11} + \frac{6}{11}\sqrt{3} + \frac{1}{11}\sqrt{133 - 28\sqrt{3}}\right) g, \left(\frac{5}{11} + \frac{6}{11}\sqrt{3} - \frac{1}{11}\sqrt{133 - 28\sqrt{3}}\right) g$$

> **evalf(s_lambda);**

$$2.234984836\, g, .5636160460\, g$$

> **s_omega:=map(sqrt,[s_lambda]);**

$$s\_omega := \left[\sqrt{\left(\frac{5}{11} + \frac{6}{11}\sqrt{3} + \frac{1}{11}\sqrt{133 - 28\sqrt{3}}\right) g}, \sqrt{\left(\frac{5}{11} + \frac{6}{11}\sqrt{3} - \frac{1}{11}\sqrt{133 - 28\sqrt{3}}\right) g}\right]$$

> **map(evalf,s_omega);**

$$[1.494986567\sqrt{g}, .7507436620\sqrt{g}]$$

> **evalm(M||1&*inverse(K||1));**

$$\begin{bmatrix} \frac{3}{2}\frac{1}{g} & \frac{1}{g} \\ \frac{1}{6}\frac{\sqrt{3}}{g} & \frac{5}{12}\frac{\sqrt{3}}{g} \end{bmatrix}$$

> **eigenvals(K||1 &* inverse(M||1));**

$$\left(\frac{5}{11} + \frac{6}{11}\sqrt{3} + \frac{1}{11}\sqrt{133 - 28\sqrt{3}}\right) g, \left(\frac{5}{11} + \frac{6}{11}\sqrt{3} - \frac{1}{11}\sqrt{133 - 28\sqrt{3}}\right) g$$

> **modos:=eigenvectors(K||1 &* inverse(M||1));**

$$modos := \left[\left[\left(\frac{5}{11} + \frac{6}{11}\sqrt{3} + \frac{1}{11}\sqrt{133 - 28\sqrt{3}}\right) g, 1, \left\{\left[\frac{1}{4}\,\frac{-11\left(\frac{5}{11} + \frac{6}{11}\sqrt{3} + \frac{1}{11}\sqrt{133 - 28\sqrt{3}}\right) g + 12\, g\sqrt{3}}{g}, 1\right]\right\}\right]\right.$$

$$\left[\left[\left(\frac{5}{11}+\frac{6}{11}\sqrt{3}-\frac{1}{11}\sqrt{133-28\sqrt{3}}\right)g, 1, \left\{\left[\frac{1}{4}\frac{-11\left(\frac{5}{11}+\frac{6}{11}\sqrt{3}-\frac{1}{11}\sqrt{133-28\sqrt{3}}\right)g+12\,g\sqrt{3}}{g}, 1\right]\right\}\right]\right]$$

\> **evalf(modos);**

$$[2.234984836\,g, 1., \{[-.9500558750, 1.]\}], [.5636160460\,g, 1., \{[3.646208298, 1.]\}]$$

\> **lambda[1]:=modos[1,1];omega[1]:=sqrt(lambda[1]);**

$$\lambda_1 := \left(\frac{5}{11}+\frac{6}{11}\sqrt{3}+\frac{1}{11}\sqrt{133-28\sqrt{3}}\right)g$$

$$\omega_1 := \sqrt{\left(\frac{5}{11}+\frac{6}{11}\sqrt{3}+\frac{1}{11}\sqrt{133-28\sqrt{3}}\right)g}$$

\> **evalf(lambda[1]);evalf(omega[1]);**

$$2.234984836\,g$$

$$1.494986567\sqrt{g}$$

\> **lambda[2]:=modos[2,1];omega[2]:=sqrt(lambda[2]);**

$$\lambda_2 := \left(\frac{5}{11}+\frac{6}{11}\sqrt{3}-\frac{1}{11}\sqrt{133-28\sqrt{3}}\right)g$$

$$\omega_2 := \sqrt{\left(\frac{5}{11}+\frac{6}{11}\sqrt{3}-\frac{1}{11}\sqrt{133-28\sqrt{3}}\right)g}$$

\> **evalf(lambda[2]);evalf(omega[2]);**

$$.5636160460\,g$$

$$.7507436620\sqrt{g}$$

\> **modo[1]:=simplify(modos[1,3][1]);**

$$modo_1 := \left[-\frac{5}{4}+\frac{3}{2}\sqrt{3}-\frac{1}{4}\sqrt{133-28\sqrt{3}}, 1\right]$$

\> **map(evalf,modo[1]);**

$$[-.950055874, 1.]$$

> **modo[2]:=simplify(modos[2,3][1]);**

$$modo_2 := \left[-\frac{5}{4}+\frac{3}{2}\sqrt{3}+\frac{1}{4}\sqrt{133-28\sqrt{3}}, 1\right]$$

> **map(evalf,modo[2]);**

$$[3.646208298, 1.]$$

>