ENUNCIADO EJEMPLO 13

Disco homogéneo de masa M y radio R que rueda sin deslizar sobre el plano horizontal y gira alrededor del eje vertical a una distancia d. Sobre su borde se mueve una partícula de masa m.

Paso 0. Reiniciación de las variables del sistema y llamada a los paquetes linalg, plots y plottools.

```
> restart;
> with(linalg):with(plots):with(plottools):
Warning, the protected names norm and trace have been redefined and unprotected

Warning, the name changecoords has been redefined

Warning, the name arrow has been redefined

> libname:="C:/",libname:
> with(mecapac3d):
```

Paso 1. Definimos las coordenadas generalizadas del sistema en una lista que se denominará cg.

```
> cg:=[alpha,theta]:
```

Paso 2. Definición mediante variables de los elementos que forman el sistema mecánico. Es decir, el aro, el disco y el muelle.

```
> rotdisc:=evalm(rota(d/R*alpha,1) &*rota(Pi/2,2)):
> d1:=[disco,[0,0,0],rotdisc,M,R]:
> m1:=[punto,0,R*cos(theta),R*sin(theta),m]:
```

Paso 3. Definición de los elementos gráficos que definiran nuestro sistema de ejes.

```
> ejeX:=[vector,[0,0,0],[10,0,0],red]:
> ejeY:=[vector,[0,0,0],[0,10,0],green]:
> ejeZ:=[vector,[0,0,0],[0,0,10],blue]:
> TO := [texto,[0,0,-1],"O"]:
> TX := [texto,[10,0,1],"X"]:
> TY := [texto,[0,10,1],"Y"]:
> TZ := [texto,[0,0,11],"Z"]:
```

Paso 4. Definición de la variable sistema que agrupa en una lista todos los elementos anteriores.

```
> cgsub:=[d*cos(alpha),d*sin(alpha),R]:
> rotsub:=rota(alpha,3):
> s1:=[subsistema2,cgsub,rotsub,[d1,m1]]:
> sistema:=[s1,ejeX,ejeY,ejeZ,TO,TX,TY,TZ]:
```

Paso 5. Obtención de la energía cinética del sistema mediante fT asignándola a la variable T.

```
> T:=fT(sistema);
```

$$T := \frac{135}{2}\sin(\alpha)^2\,\alpha1^2 + \frac{135}{2}\cos(\alpha)^2\,\alpha1^2 + 11.71875000\cos(1.200000000\,\alpha)^2\,\alpha1^2$$
$$+ 11.71875000\sin(1.200000000\,\alpha)^2\,\alpha1^2$$
$$+ \frac{5}{2}(-3\sin(\alpha)\,\alpha1 - 2.5\cos(\alpha)\,\alpha1\cos(\theta) + 2.5\sin(\alpha)\sin(\theta)\,\theta1)^2$$
$$+ \frac{5}{2}(3\cos(\alpha)\,\alpha1 - 2.5\sin(\alpha)\,\alpha1\cos(\theta) - 2.5\cos(\alpha)\sin(\theta)\,\theta1)^2 + 15.62500000\cos(\theta)^2\,\theta1^2$$

Paso 6. Obtención de la energía potencial del sistema mediante fV asignándola a la variable V.

```
> V:=fV(sistema);
```

$$V := 490.00 + 122.50\sin(\theta)$$

Paso 7. Obtención de la lagrangiana como diferencia de energías entre la energía cinética y la potencial.

```
> L:=T-V:
```

Paso 8. Obtención de las ecuaciones de lagrange para las dos coordenadas generalizadas mediante el operador Ec_lag

```
> ecua:=ec_lag();
```

$$ecua := \Bigg[$$

$$135\sin(\alpha(t))^2\left(\frac{d^2}{dt^2}\alpha(t)\right) + 135\cos(\alpha(t))^2\left(\frac{d^2}{dt^2}\alpha(t)\right)$$
$$+ 23.43750000\cos(1.200000000\,\alpha(t))^2\left(\frac{d^2}{dt^2}\alpha(t)\right) + 23.43750000\sin(1.200000000\,\alpha(t))^2\left(\frac{d^2}{dt^2}\alpha(t)\right)$$
$$+ 5\left(-3\cos(\alpha(t))\left(\frac{d}{dt}\alpha(t)\right)^2 - 3\sin(\alpha(t))\left(\frac{d^2}{dt^2}\alpha(t)\right) + 2.5\sin(\alpha(t))\left(\frac{d}{dt}\alpha(t)\right)^2\cos(\theta(t))\right.$$
$$- 2.5\cos(\alpha(t))\left(\frac{d^2}{dt^2}\alpha(t)\right)\cos(\theta(t)) + 5.0\cos(\alpha(t))\left(\frac{d}{dt}\alpha(t)\right)\sin(\theta(t))\left(\frac{d}{dt}\theta(t)\right)$$
$$\left.+ 2.5\sin(\alpha(t))\cos(\theta(t))\left(\frac{d}{dt}\theta(t)\right)^2 + 2.5\sin(\alpha(t))\sin(\theta(t))\left(\frac{d^2}{dt^2}\theta(t)\right)\right)(-3\sin(\alpha(t))$$

$$- 2.5 \cos(\alpha(t)) \cos(\theta(t)))$$

$$+ 5 \left(-3 \sin(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right)^2 + 3 \cos(\alpha(t)) \left(\frac{d^2}{dt^2} \alpha(t) \right) - 2.5 \cos(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right)^2 \cos(\theta(t)) \right.$$

$$\left. - 2.5 \sin(\alpha(t)) \left(\frac{d^2}{dt^2} \alpha(t) \right) \cos(\theta(t)) + 5.0 \sin(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) \sin(\theta(t)) \left(\frac{d}{dt} \theta(t) \right) \right.$$

$$\left. - 2.5 \cos(\alpha(t)) \cos(\theta(t)) \left(\frac{d}{dt} \theta(t) \right)^2 - 2.5 \cos(\alpha(t)) \sin(\theta(t)) \left(\frac{d^2}{dt^2} \theta(t) \right) \right) (3 \cos(\alpha(t))$$

$$- 2.5 \sin(\alpha(t)) \cos(\theta(t))),$$

$$12.50000000 \left(-3 \cos(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right)^2 - 3 \sin(\alpha(t)) \left(\frac{d^2}{dt^2} \alpha(t) \right) + 2.5 \sin(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right)^2 \cos(\theta(t)) \right.$$

$$\left. - 2.5 \cos(\alpha(t)) \left(\frac{d^2}{dt^2} \alpha(t) \right) \cos(\theta(t)) + 5.0 \cos(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) \sin(\theta(t)) \left(\frac{d}{dt} \theta(t) \right) \right.$$

$$\left. + 2.5 \sin(\alpha(t)) \cos(\theta(t)) \left(\frac{d}{dt} \theta(t) \right)^2 + 2.5 \sin(\alpha(t)) \sin(\theta(t)) \left(\frac{d^2}{dt^2} \theta(t) \right) \right) \sin(\alpha(t)) \sin(\theta(t))$$

$$+ 12.50000000 \left(-3 \sin(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) - 2.5 \cos(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) \cos(\theta(t)) \right.$$

$$\left. + 2.5 \sin(\alpha(t)) \sin(\theta(t)) \left(\frac{d}{dt} \theta(t) \right) \right) \cos(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) \sin(\theta(t))$$

$$+ 12.50000000 \left(-3 \sin(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) - 2.5 \cos(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) \cos(\theta(t)) \right.$$

$$\left. + 2.5 \sin(\alpha(t)) \sin(\theta(t)) \left(\frac{d}{dt} \theta(t) \right) \right) \sin(\alpha(t)) \cos(\theta(t)) \left(\frac{d}{dt} \theta(t) \right)$$

$$- 12.50000000 \left(-3 \sin(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right)^2 + 3 \cos(\alpha(t)) \left(\frac{d^2}{dt^2} \alpha(t) \right) - 2.5 \cos(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right)^2 \cos(\theta(t)) \right.$$

$$- 2.5 \sin(\alpha(t)) \left(\frac{d^2}{dt^2} \alpha(t) \right) \cos(\theta(t)) + 5.0 \sin(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) \sin(\theta(t)) \left(\frac{d}{dt} \theta(t) \right)$$

$$- 2.5 \cos(\alpha(t)) \cos(\theta(t)) \left(\frac{d}{dt} \theta(t) \right)^2 - 2.5 \cos(\alpha(t)) \sin(\theta(t)) \left(\frac{d^2}{dt^2} \theta(t) \right) \Bigg) \cos(\alpha(t)) \sin(\theta(t))$$

$$+ 12.50000000 \Bigg(3 \cos(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) - 2.5 \sin(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) \cos(\theta(t))$$

$$- 2.5 \cos(\alpha(t)) \sin(\theta(t)) \left(\frac{d}{dt} \theta(t) \right) \Bigg) \sin(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) \sin(\theta(t))$$

$$- 12.50000000 \Bigg(3 \cos(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) - 2.5 \sin(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) \cos(\theta(t))$$

$$- 2.5 \cos(\alpha(t)) \sin(\theta(t)) \left(\frac{d}{dt} \theta(t) \right) \Bigg) \cos(\alpha(t)) \cos(\theta(t)) \left(\frac{d}{dt} \theta(t) \right)$$

$$- 31.25000000 \cos(\theta(t)) \left(\frac{d}{dt} \theta(t) \right)^2 \sin(\theta(t)) + 31.25000000 \cos(\theta(t))^2 \left(\frac{d^2}{dt^2} \theta(t) \right)$$

$$- 5 \left(-3 \sin(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) - 2.5 \cos(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) \cos(\theta(t)) + 2.5 \sin(\alpha(t)) \sin(\theta(t)) \left(\frac{d}{dt} \theta(t) \right) \right)$$

$$\left(2.5 \cos(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) \sin(\theta(t)) + 2.5 \sin(\alpha(t)) \cos(\theta(t)) \left(\frac{d}{dt} \theta(t) \right) \right)$$

$$- 5 \left(3 \cos(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) - 2.5 \sin(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) \cos(\theta(t)) - 2.5 \cos(\alpha(t)) \sin(\theta(t)) \left(\frac{d}{dt} \theta(t) \right) \right) \left(2.5 \right.$$

$$\left. \sin(\alpha(t)) \left(\frac{d}{dt} \alpha(t) \right) \sin(\theta(t)) - 2.5 \cos(\alpha(t)) \cos(\theta(t)) \left(\frac{d}{dt} \theta(t) \right) \right) + 122.50 \cos(\theta(t)) \Bigg]$$

Paso 9. Asignación de valores numéricos a los parámetros que queden sun asignar para poder proceder a la integración numérica.

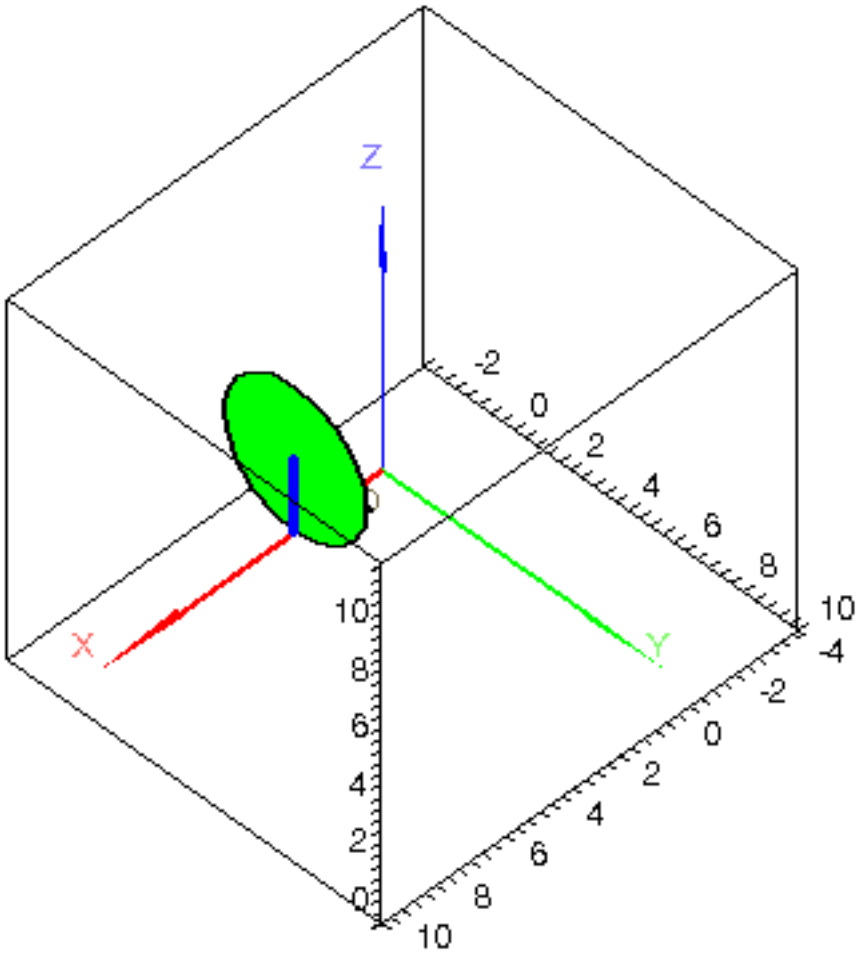
```
> g:=9.8:M:=15:m:=5:     R:=2.5:d:=3:
```

Paso 10. Integración numérica del problema mediante la función fint asignando el resultado a la variable res.

```
> res:=fint([0,1,0,0]):
```

Paso 11. Procedemos a realizar una animación del movimiento del conjunto por medio de la función dibu3.

```
> dibu3(2,50);
```




```
>
```