Mecánica de Sólidos y Sistemas Estructurales

Sólido deformable (III)

Mariano Vázquez Espí

Ondara, 21 de noviembre de 2007.

El modelo cable $\sigma - \varepsilon$

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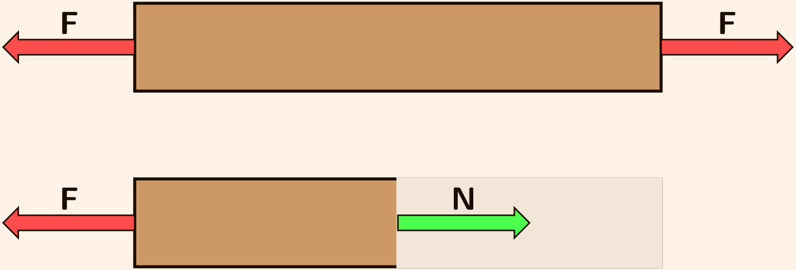
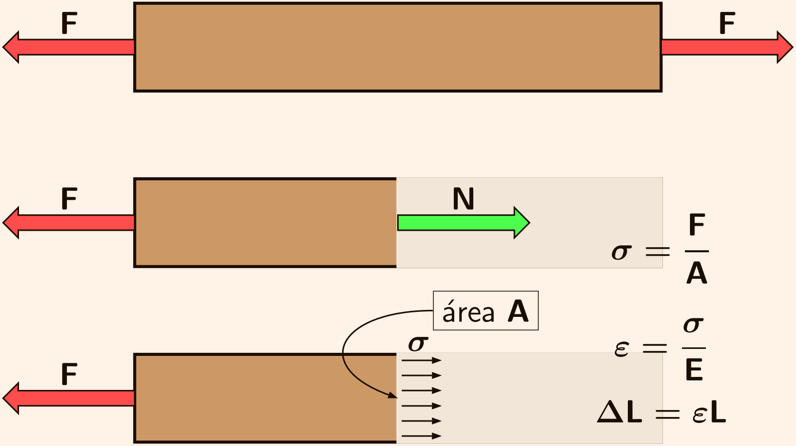


El modelo cable $\sigma - \varepsilon$

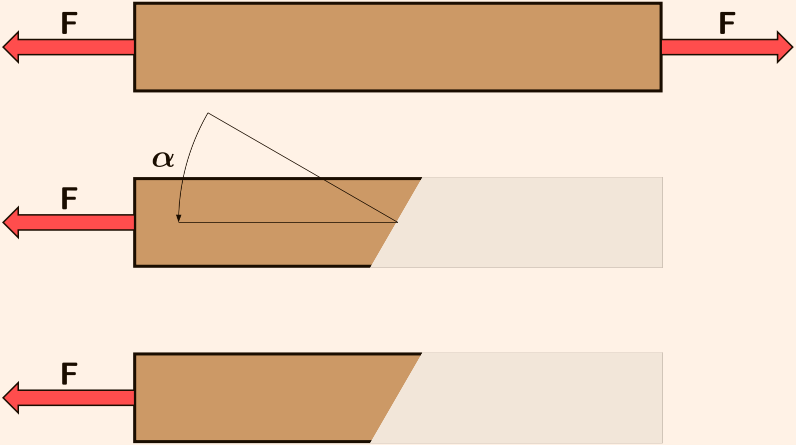


$$\sigma = \frac{F}{A}$$

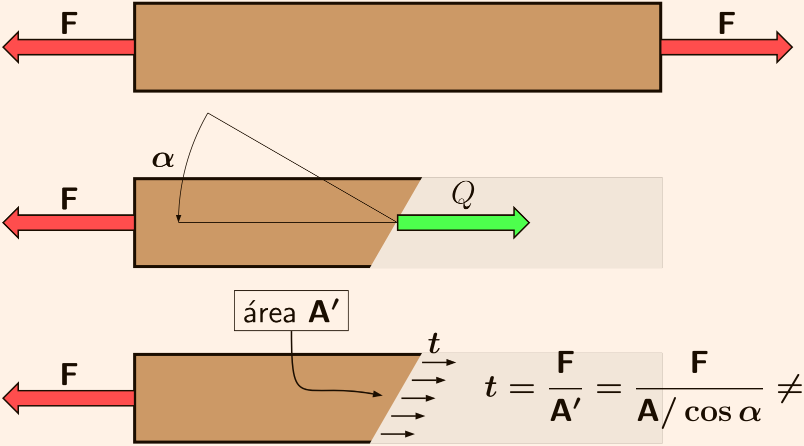
$$\varepsilon = \frac{\sigma}{E}$$

$$\Delta L = \varepsilon L$$

El modelo cable $\sigma - \varepsilon$

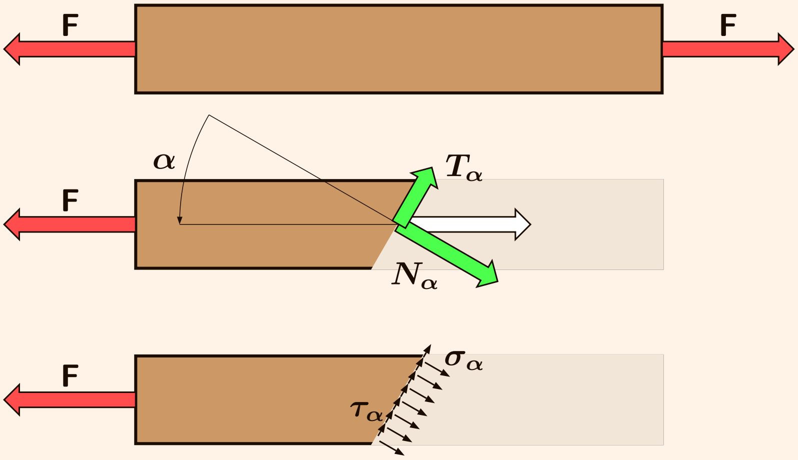


El modelo cable $\sigma - \varepsilon$

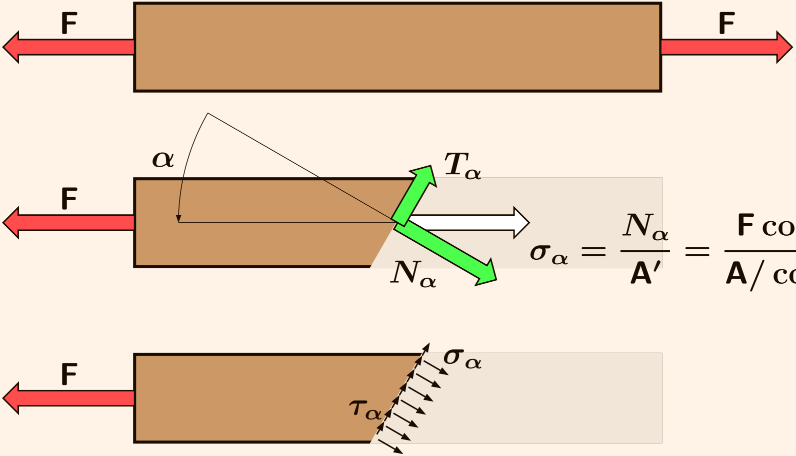


$$t = \frac{\mathbf{F}}{\mathbf{A}'} = \frac{\mathbf{F}}{\mathbf{A}/\cos\alpha} \neq \boldsymbol{\sigma}$$

El modelo cable $\sigma - \varepsilon$

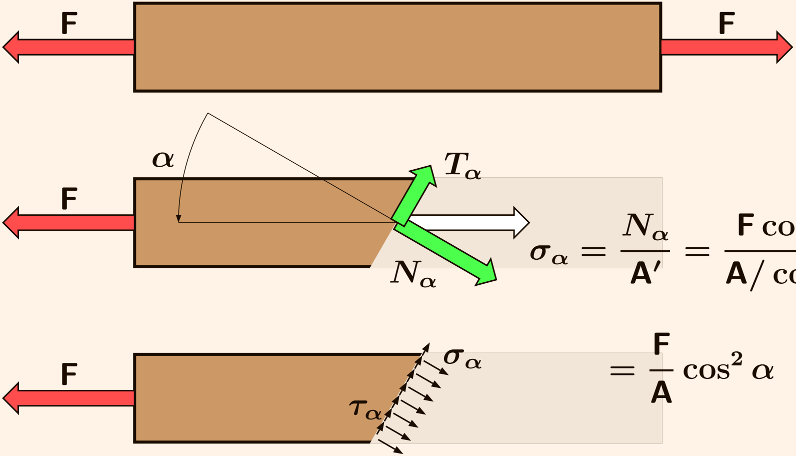


El modelo cable $\sigma - \varepsilon$



$$\sigma_\alpha = \frac{N_\alpha}{\mathbf{A}'} = \frac{\mathbf{F}\cos\alpha}{\mathbf{A}/\cos\alpha}$$

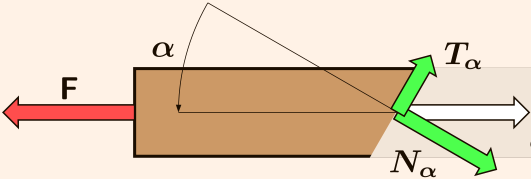
El modelo cable $\sigma - \varepsilon$



$$\boldsymbol{\sigma_\alpha} = \frac{N_\alpha}{\mathsf{A}'} = \frac{\mathsf{F}\cos\alpha}{\mathsf{A}/\cos\alpha}$$

$$= \frac{\mathsf{F}}{\mathsf{A}}\cos^2\alpha$$

El modelo cable $\sigma - \varepsilon$

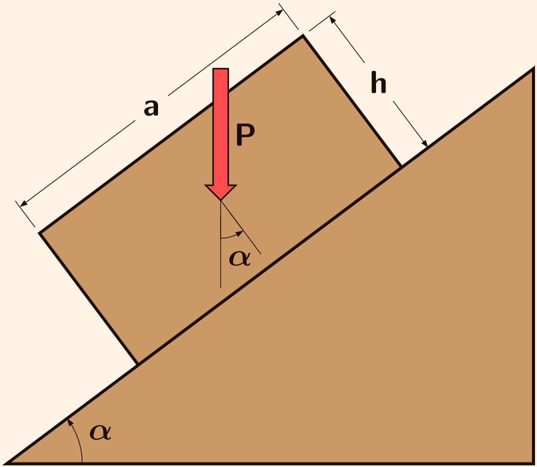


$$\sigma_\alpha = \frac{N_\alpha}{\mathsf{A}'} = \frac{\mathsf{F}\cos\alpha}{\mathsf{A}/\cos\alpha}$$

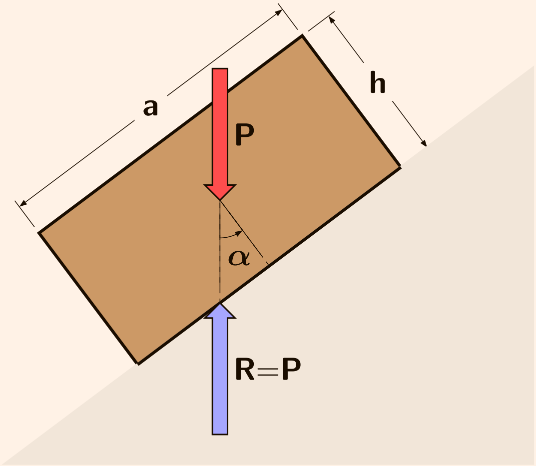
$$= \frac{\mathsf{F}}{\mathsf{A}}\cos^2\alpha$$

$$\tau_\alpha = \frac{\mathsf{F}}{\mathsf{A}}\cos\alpha\sin\alpha$$

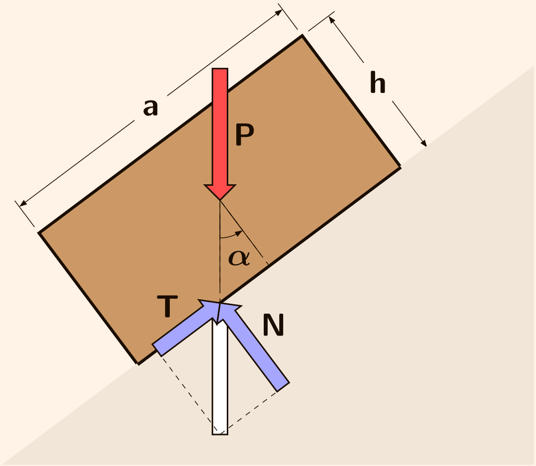
Plano inclinado



Plano inclinado



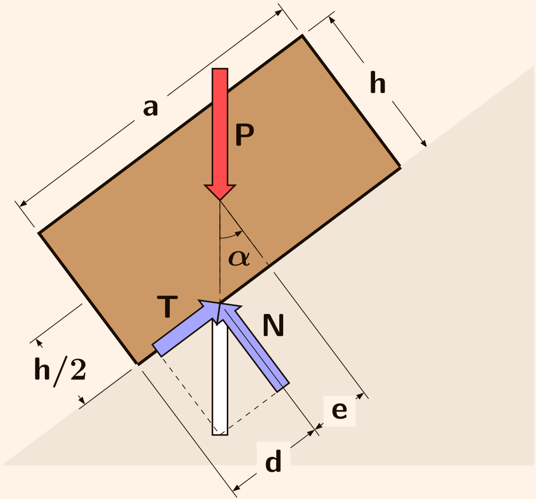
Plano inclinado



$$\mathsf{N} = \mathsf{P}\cos\alpha$$

$$\mathsf{T} = \mathsf{P}\sin\alpha$$

Plano inclinado



$$\mathbf{N} = \mathbf{P}\cos\alpha$$

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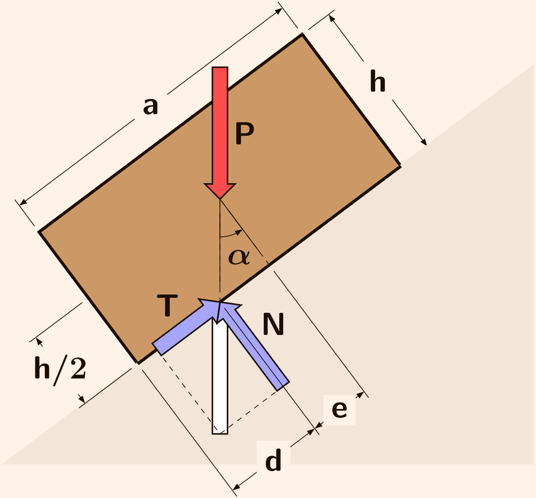
Plano inclinado

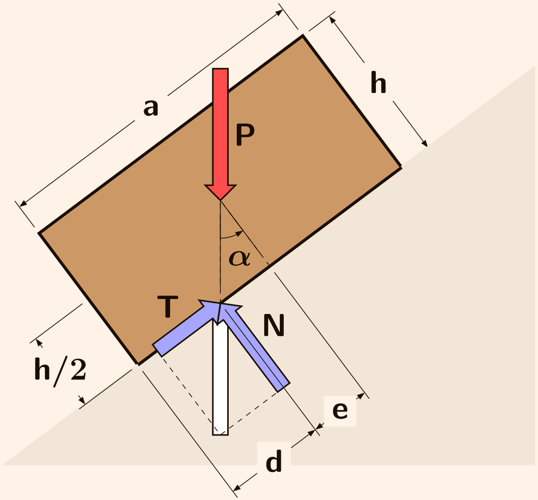


$$\mathsf{N} = \mathsf{P}\cos\alpha$$

$$\mathsf{T} = \mathsf{P}\sin\alpha$$

$$\mathsf{e} = \frac{\mathsf{h}}{2}\tan\alpha$$

Plano inclinado



$$\mathbf{N} = \mathbf{P}\cos\alpha$$

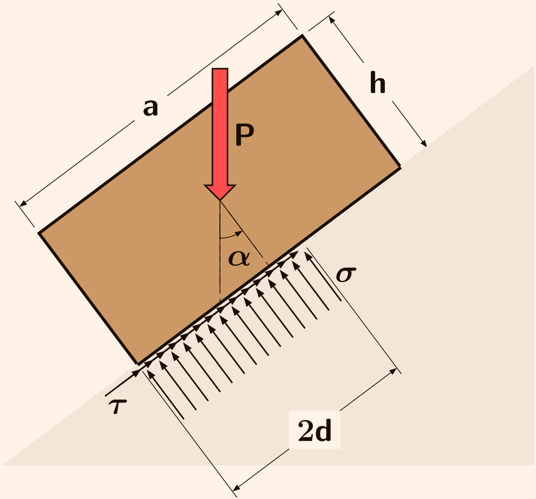
$$\mathbf{T} = \mathbf{P}\sin\alpha$$

$$\mathbf{e} = \frac{\mathbf{h}}{2}\tan\alpha$$

$$\mathbf{d} = \frac{\mathbf{a}}{2} - \mathbf{e}$$

área $\mathbf{A} = 2\mathbf{d}\cdot\mathbf{b}$

Plano inclinado



$$\mathsf{N} = \mathsf{P}\cos\alpha$$

$$\mathsf{T} = \mathsf{P}\sin\alpha$$

$$\mathsf{e} = \frac{\mathsf{h}}{2}\tan\alpha$$

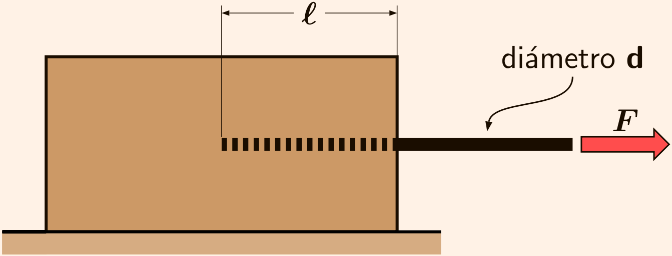
$$\mathsf{d} = \frac{\mathsf{a}}{2} - \mathsf{e}$$

área $\mathsf{A} = 2\mathsf{d}\cdot\mathsf{b}$

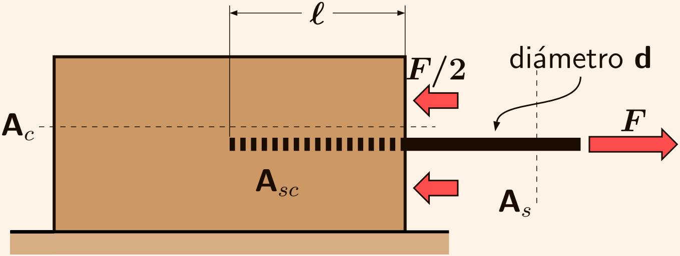
$$\sigma = \frac{\mathsf{N}}{\mathsf{A}}$$

$$\tau = \frac{\mathsf{T}}{\mathsf{A}}$$

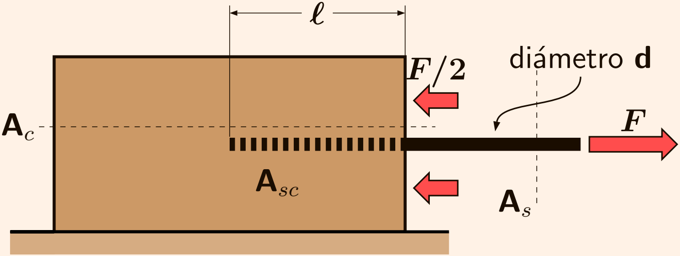
Adherencia hormigón—acero

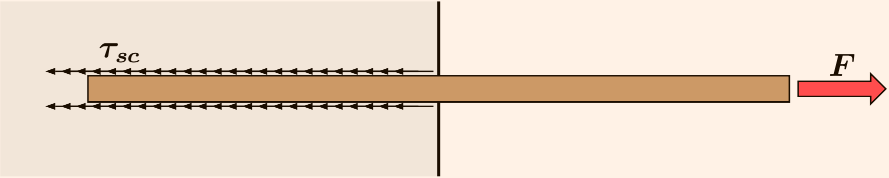


Adherencia hormigón—acero

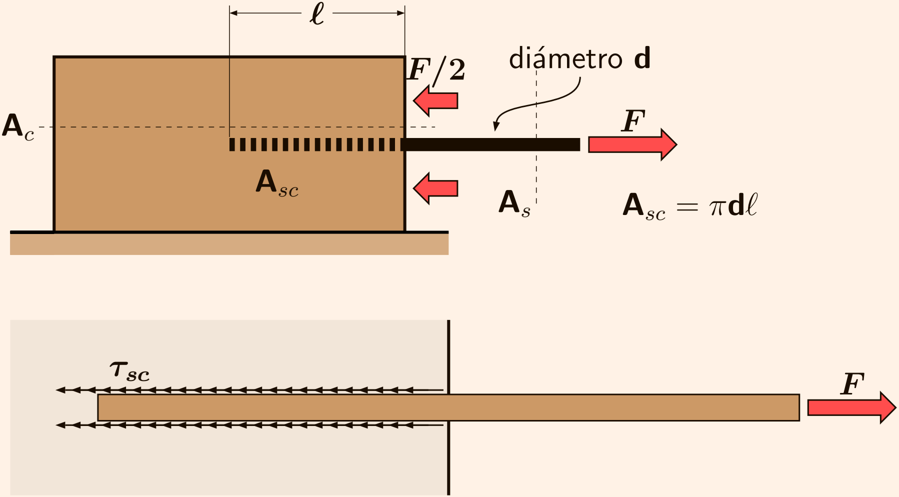


Adherencia hormigón—acero

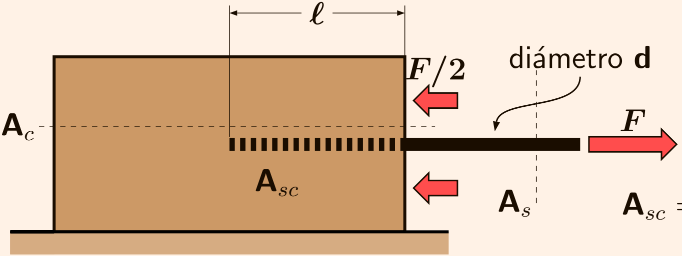




Adherencia hormigón—acero

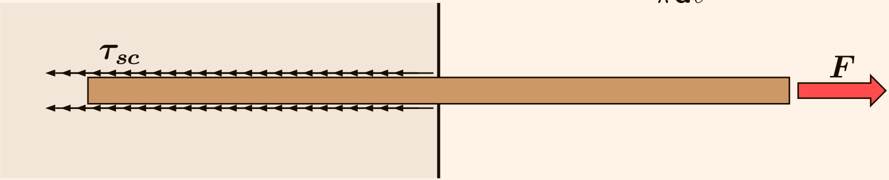


Adherencia hormigón—acero

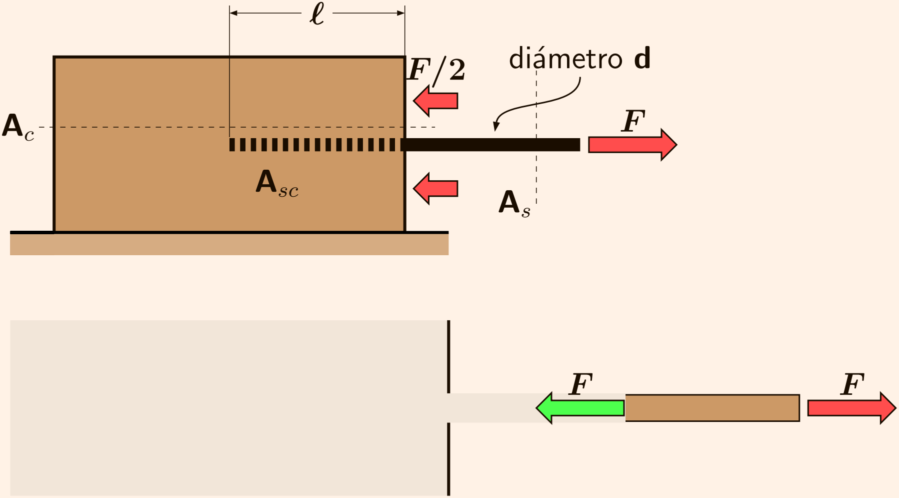


$$\mathbf{A}_{sc} = \pi \mathbf{d} \ell$$

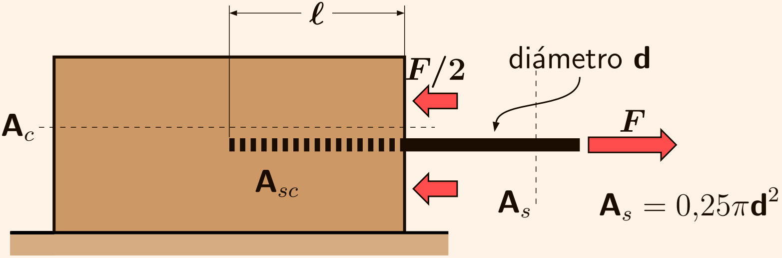
$$\tau_{sc} = \frac{F}{\pi \mathbf{d} \ell} \leq \mathbf{f}_{sc}$$

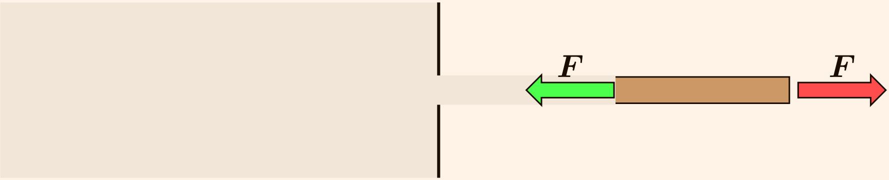


Adherencia hormigón—acero

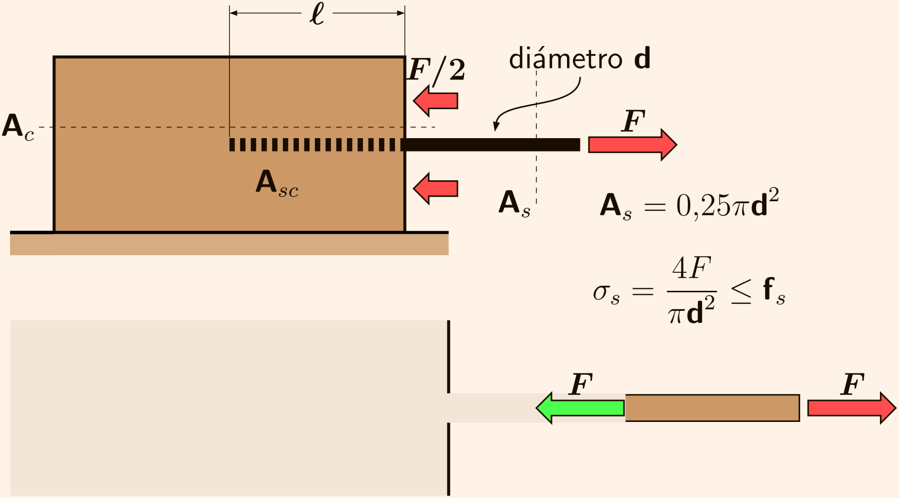


Adherencia hormigón—acero





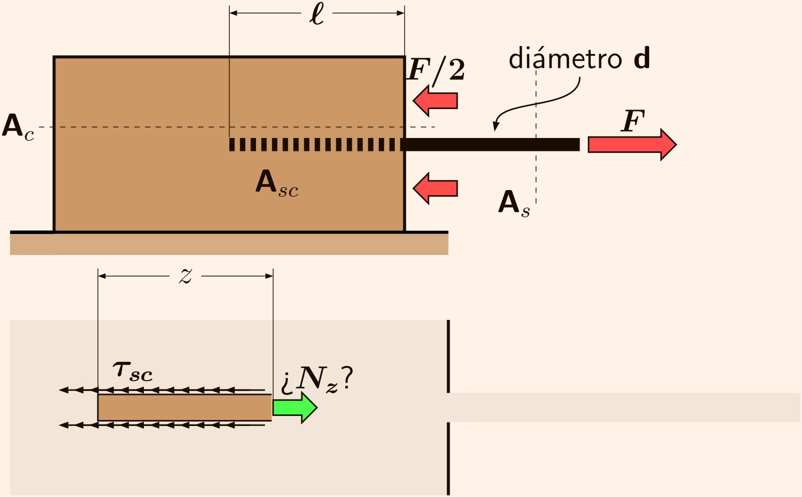
Adherencia hormigón—acero



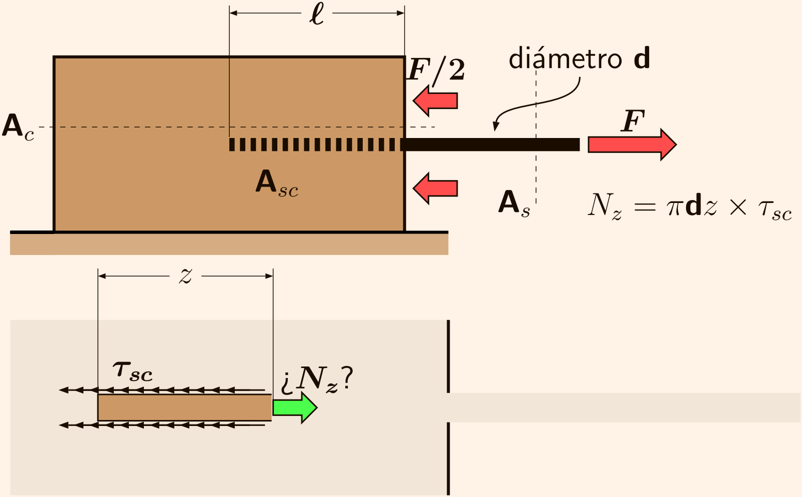
$$\mathbf{A}_s = 0{,}25\pi\mathbf{d}^2$$

$$\sigma_s = \frac{4F}{\pi\mathbf{d}^2} \le \mathbf{f}_s$$

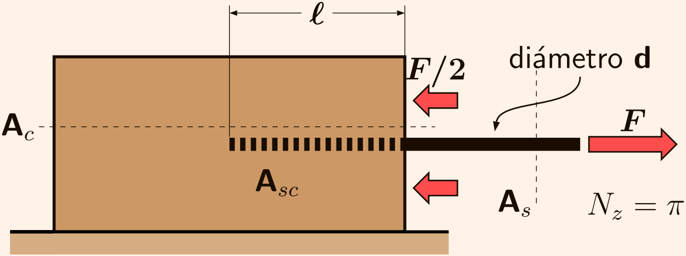
Adherencia hormigón—acero



Adherencia hormigón—acero



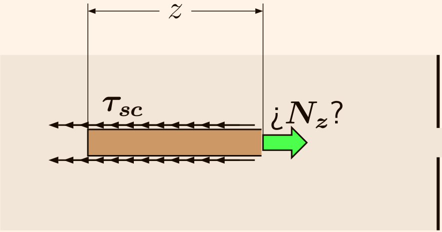
Adherencia hormigón—acero



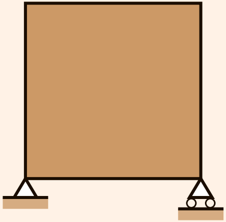
$$N_z = \pi \mathbf{d} z \times \tau_{sc}$$

$$\sigma_z = \frac{\pi \mathbf{d} z}{0{,}25 \pi \mathbf{d}^2} \tau_{sc}$$

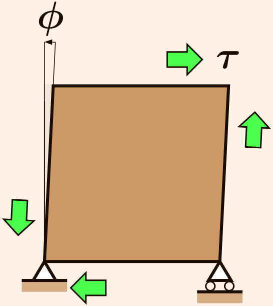
$$= \frac{4 \tau_{sc}}{\mathbf{d}} z$$



Ensayo de cizalladura



Ensayo de cizalladura



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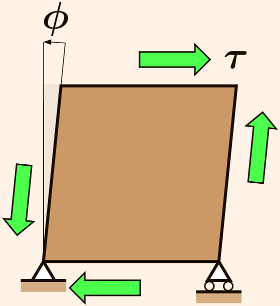

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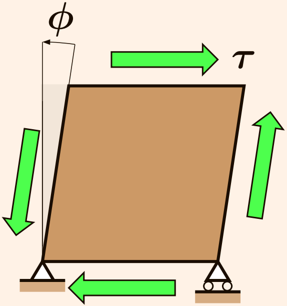
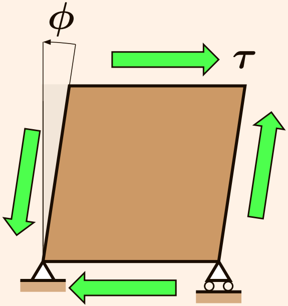


Ensayo de cizalladura

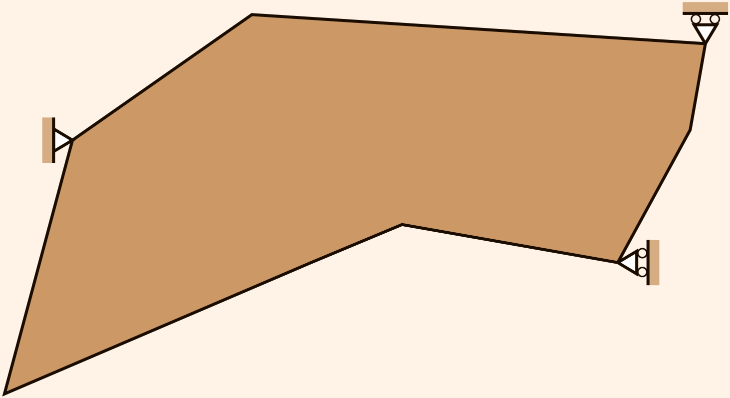


Ley de Hooke: $\begin{cases} \dfrac{\sigma}{\varepsilon} = \mathbf{E} \\ \dfrac{\tau}{\phi} = \mathbf{G} \end{cases}$

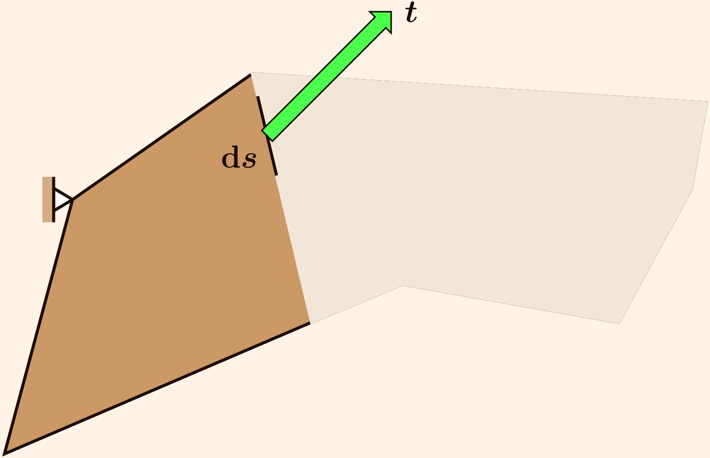
Tensiones *seguras* — Material:	Acero laminado	Hormigón	Fábrica de ladrillo	Madera
normal **f** (N/mm^2)	180	9	1	10
tangencial $\mathbf{f}_\tau$ (N/mm^2)	100	0,4	0,1*	1
$\mathbf{f}_\tau/\mathbf{f}$	0,56	0,04	0,1*	0,1

* Se obtiene más resistencia considerando el rozamiento.

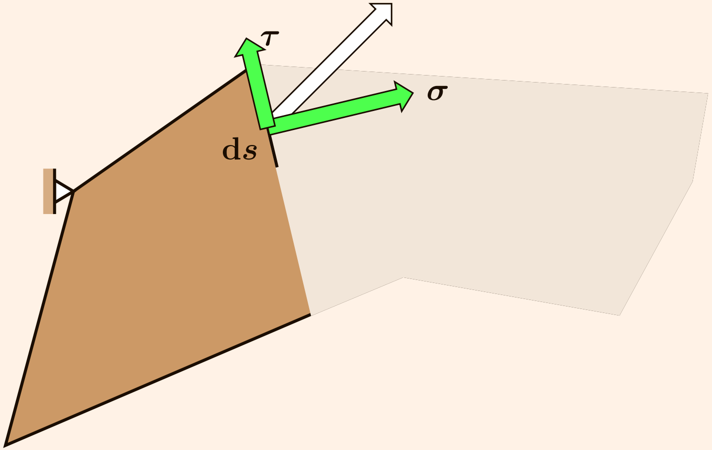
Tensiones en cortes



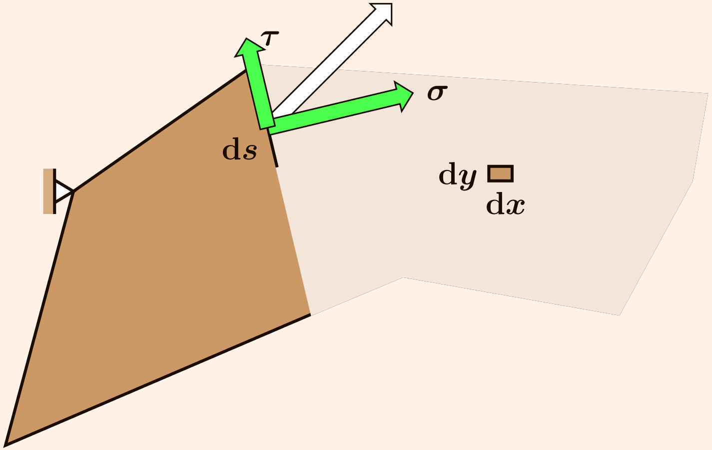
Tensiones en cortes



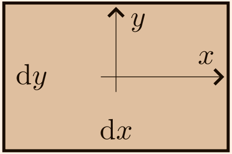
Tensiones en cortes



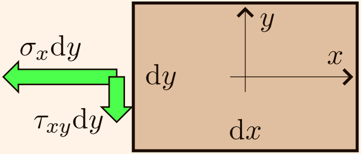
Tensiones en cortes



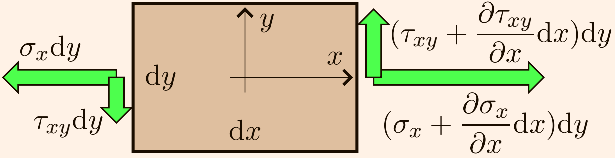
Ecuaciones diferenciales de equilibrio



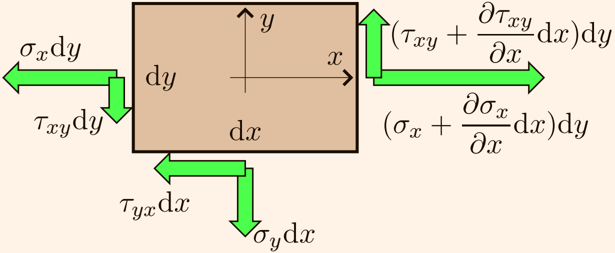
Ecuaciones diferenciales de equilibrio



Ecuaciones diferenciales de equilibrio



Ecuaciones diferenciales de equilibrio



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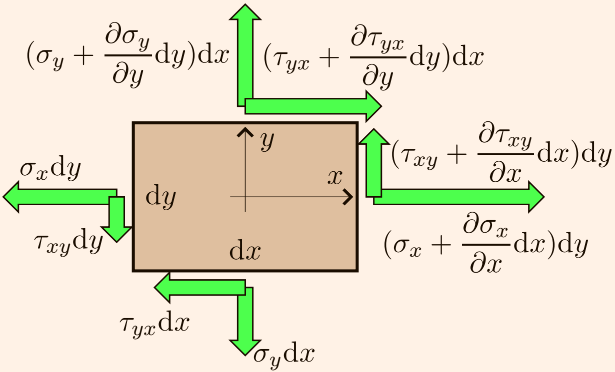

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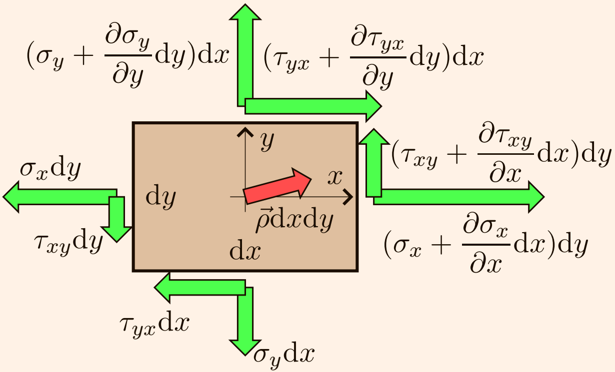

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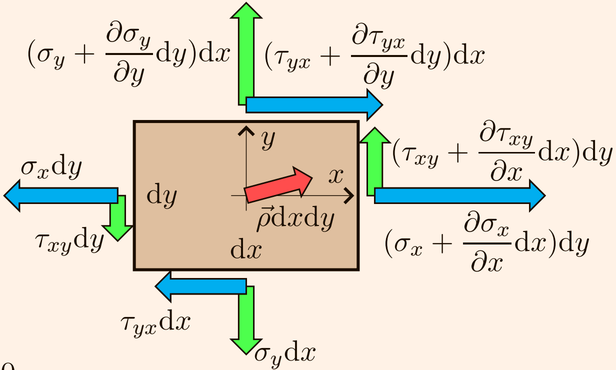
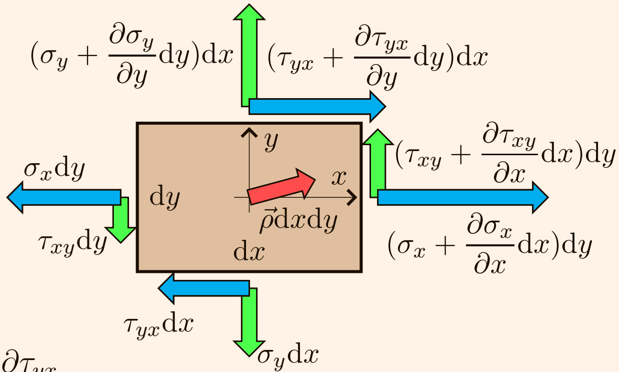


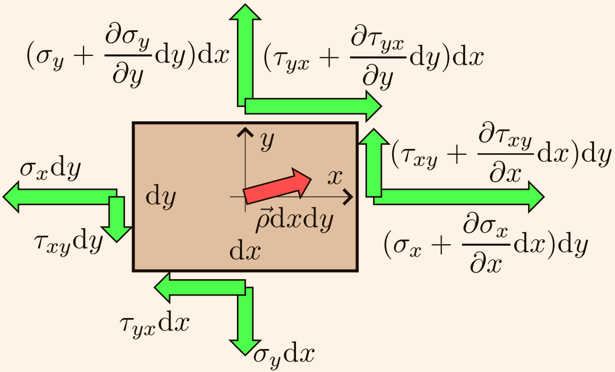
$$\sum F_x = 0$$

Ecuaciones diferenciales de equilibrio



$$\sum F_x = 0 \Rightarrow \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \rho_x = 0$$

Ecuaciones diferenciales de equilibrio

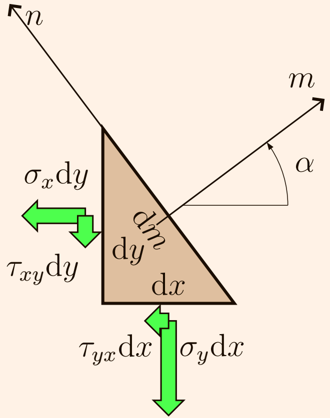


$$\sum F_x = 0 \Rightarrow \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \rho_x = 0 \qquad \sum F_y = 0 \Rightarrow \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \rho_y = 0$$

$$\sum M_{(0,0)} = 0 \Rightarrow \tau_{xy} = \tau_{yx}$$

Tensiones en una dirección cualquiera

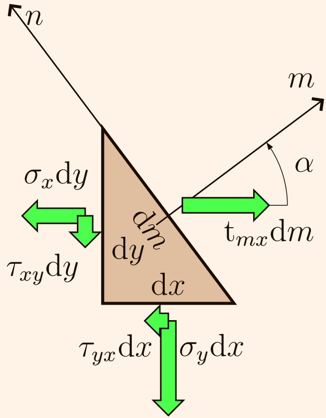
$$\cos\alpha = \frac{\mathrm{d}y}{\mathrm{d}m} \qquad \sin\alpha = \frac{\mathrm{d}x}{\mathrm{d}m}$$



Tensiones en una dirección cualquiera

$$\cos\alpha = \frac{\mathrm{d}y}{\mathrm{d}m} \qquad \sin\alpha = \frac{\mathrm{d}x}{\mathrm{d}m}$$

$$\mathrm{t}_{mx}\mathrm{d}m = \sigma_x\mathrm{d}y + \tau_{yx}\mathrm{d}x$$



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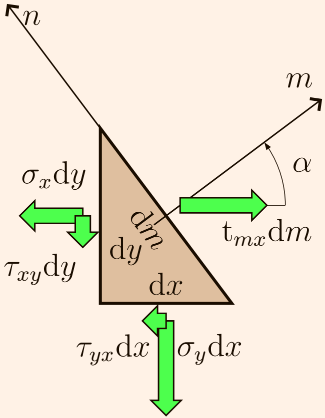
$$\cos\alpha = \frac{\mathrm{d}y}{\mathrm{d}m} \qquad \sin\alpha = \frac{\mathrm{d}x}{\mathrm{d}m}$$

$$\mathrm{t}_{mx}\mathrm{d}m = \sigma_x \mathrm{d}y + \tau_{yx}\mathrm{d}x$$

$$\mathrm{t}_{mx} = \sigma_x \cos\alpha + \tau_{yx}\sin\alpha$$

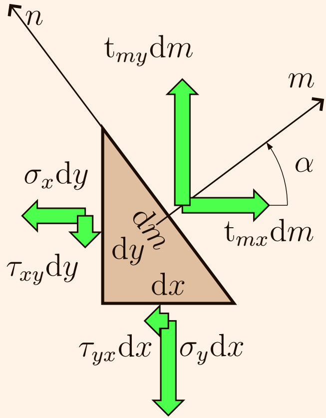


Tensiones en una dirección cualquiera

$$\cos\alpha = \frac{\mathrm{d}y}{\mathrm{d}m} \qquad \sin\alpha = \frac{\mathrm{d}x}{\mathrm{d}m}$$

$$\mathrm{t}_{mx} = \sigma_x \sin\alpha + \tau_{yx}\cos\alpha$$

$$\mathrm{t}_{my}\mathrm{d}m = \sigma_y \mathrm{d}x + \tau_{xy}\mathrm{d}y$$



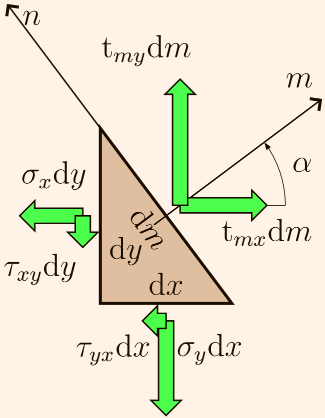
Tensiones en una dirección cualquiera

$$\cos\alpha = \frac{\mathrm{d}y}{\mathrm{d}m} \qquad \sin\alpha = \frac{\mathrm{d}x}{\mathrm{d}m}$$

$$\mathrm{t}_{mx} = \sigma_x \sin\alpha + \tau_{yx}\cos\alpha$$

$$\mathrm{t}_{my}\mathrm{d}m = \sigma_y \mathrm{d}x + \tau_{xy}\mathrm{d}y$$

$$\mathrm{t}_{my} = \tau_{xy}\cos\alpha + \sigma_y \sin\alpha$$

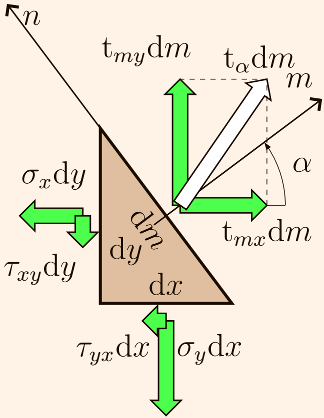


Tensiones en una dirección cualquiera

$$\cos\alpha = \frac{\mathrm{d}y}{\mathrm{d}m} \qquad \sin\alpha = \frac{\mathrm{d}x}{\mathrm{d}m}$$

$$\mathrm{t}_{mx} = \sigma_x \sin\alpha + \tau_{yx}\cos\alpha$$

$$\mathrm{t}_{my} = \tau_{xy}\cos\alpha + \sigma_y \sin\alpha$$



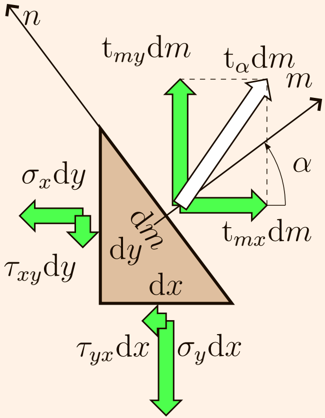
Tensiones en una dirección cualquiera

$$\cos\alpha = \frac{\mathrm{d}y}{\mathrm{d}m} \qquad \sin\alpha = \frac{\mathrm{d}x}{\mathrm{d}m}$$

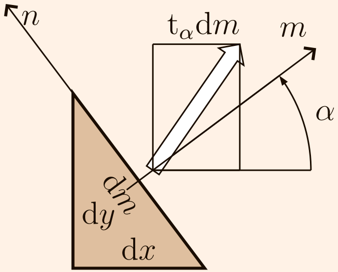
$$\mathrm{t}_{mx} = \sigma_x \sin\alpha + \tau_{yx}\cos\alpha$$

$$\mathrm{t}_{my} = \tau_{xy}\cos\alpha + \sigma_y \sin\alpha$$

$$\{\mathrm{t}_{mx}\ \mathrm{t}_{my}\} = \{\cos\alpha\ \ \sin\alpha\} \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix}$$

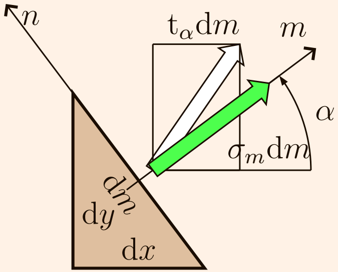


Tensiones en una dirección cualquiera



Tensiones en una dirección cualquiera

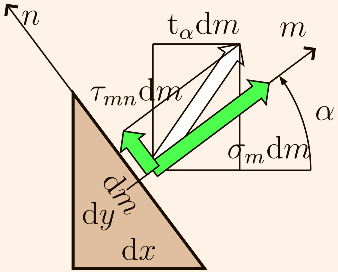
$$\sigma_m = \mathrm{t}_{mx} \cos\alpha + \mathrm{t}_{my} \sin\alpha$$



Tensiones en una dirección cualquiera

$$\sigma_m = \mathrm{t}_{mx} \cos\alpha + \mathrm{t}_{my} \sin\alpha$$

$$\tau_{mn} = -\mathrm{t}_{mx} \sin\alpha + \mathrm{t}_{my} \cos\alpha$$



Tensiones en una dirección cualquiera

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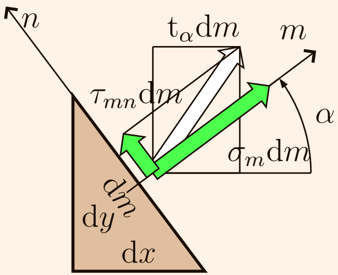
$$\tau_{mn} = -\mathrm{t}_{mx} \sin\alpha + \mathrm{t}_{my} \cos\alpha$$

$$\sigma_m = \sigma_x \cos^2\alpha + \sigma_y \sin^2\alpha + 2\tau_{xy} \sin\alpha \cos\alpha$$

$$\tau_{mn} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\alpha + \tau_{xy} \cos 2\alpha$$



Direcciones principales de tensión

$$\textbf{Dado} \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix},$$

$$\textbf{¿existirá} \begin{bmatrix} \sigma_m & 0 \\ 0 & \sigma_n \end{bmatrix} \textbf{ para alguna dirección } mn\textbf{?}$$

Direcciones principales de tensión

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$$\textbf{¿existirá} \quad \begin{bmatrix} \sigma_m & 0 \\ 0 & \sigma_n \end{bmatrix} \quad \textbf{para alguna dirección } mn\textbf{?}$$

$$\tau_{mn} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\beta + \tau_{xy} \cos 2\beta = 0$$

$$\tan 2\beta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Direcciones principales de tensión

Dado $\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix}$,

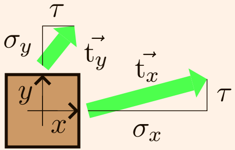
¿existirá $\begin{bmatrix} \sigma_m & 0 \\ 0 & \sigma_n \end{bmatrix}$ **para alguna dirección** mn**?**

$$\tan 2\beta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\begin{aligned} \sigma_a &= \frac{\sigma_x + \sigma_y}{2} + \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2} \\ \sigma_b &= \frac{\sigma_x + \sigma_y}{2} - \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2} \end{aligned}$$

Circunferencia de Mohr: direcciones principales

$$\boxed{\sigma_{a,b} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}}$$



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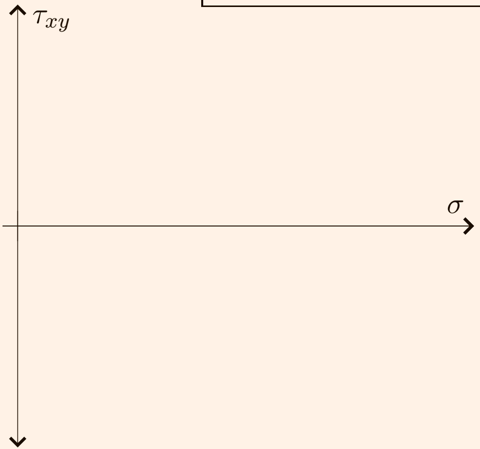

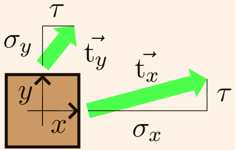

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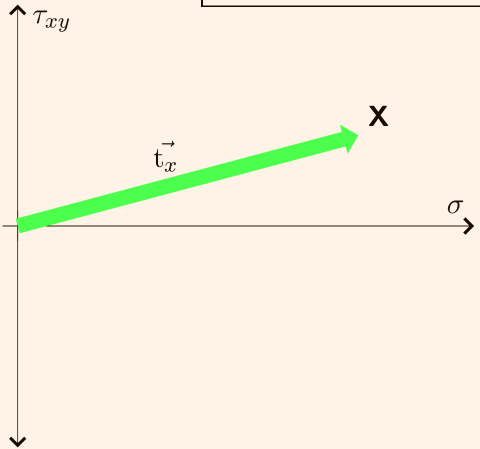

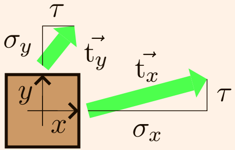

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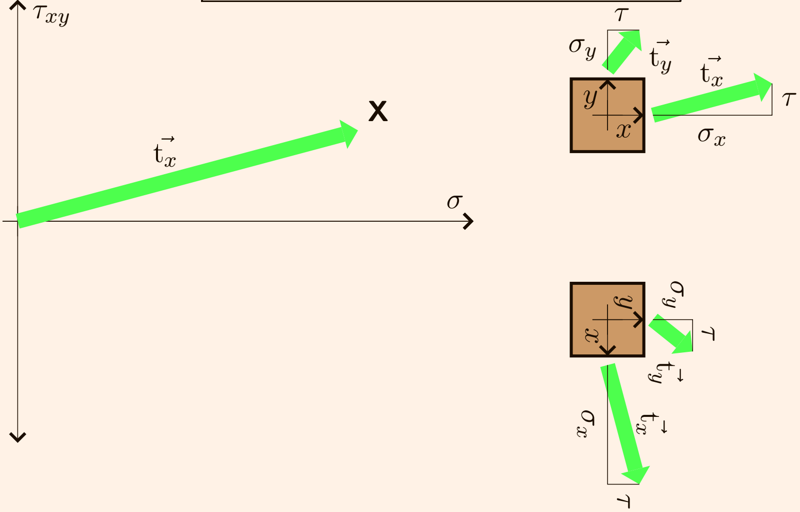

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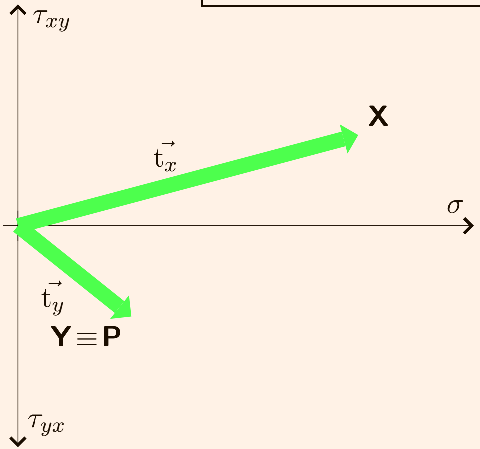

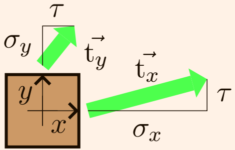

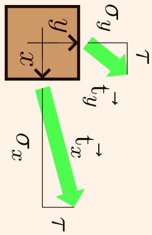

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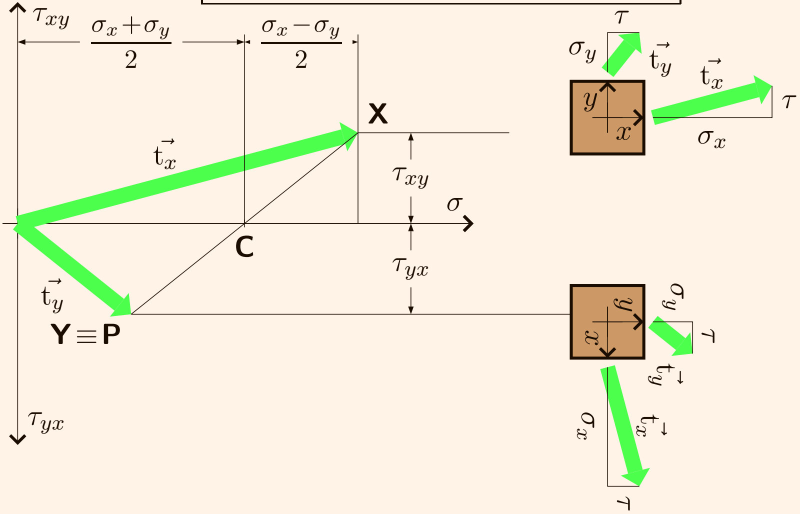
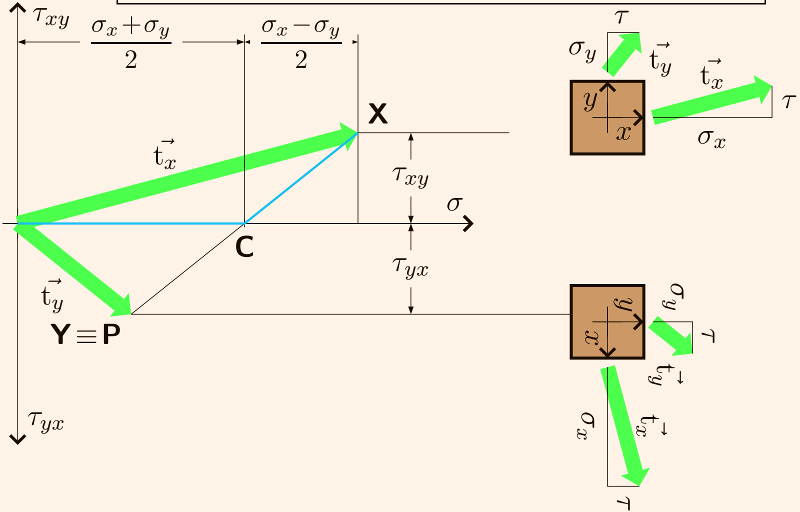


Circunferencia de Mohr: direcciones principales

$$\sigma_{a,b} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2} = \bar{OC} \pm \bar{CX}$$



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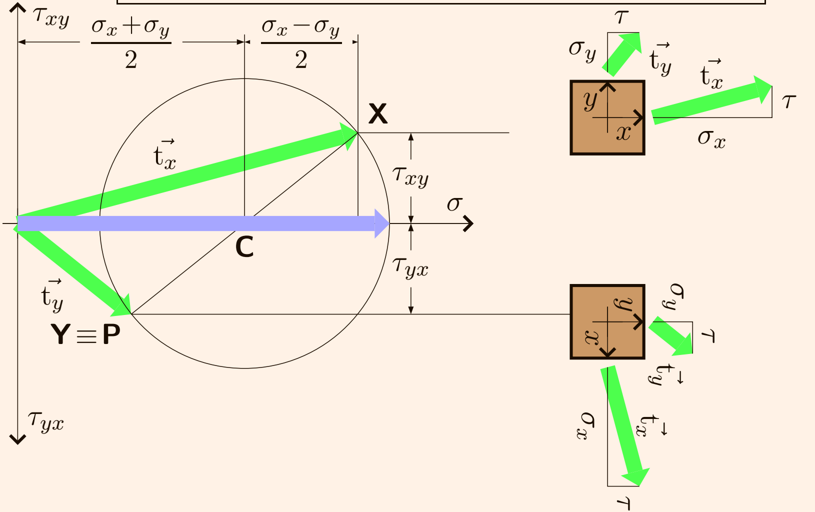

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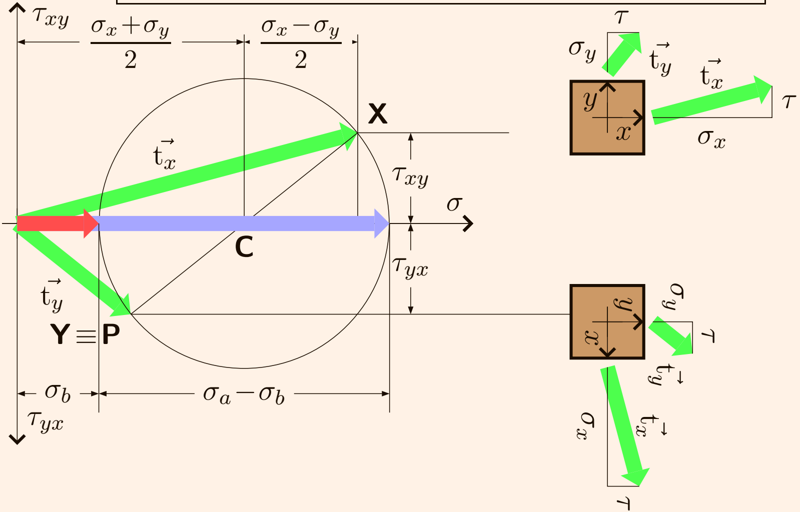
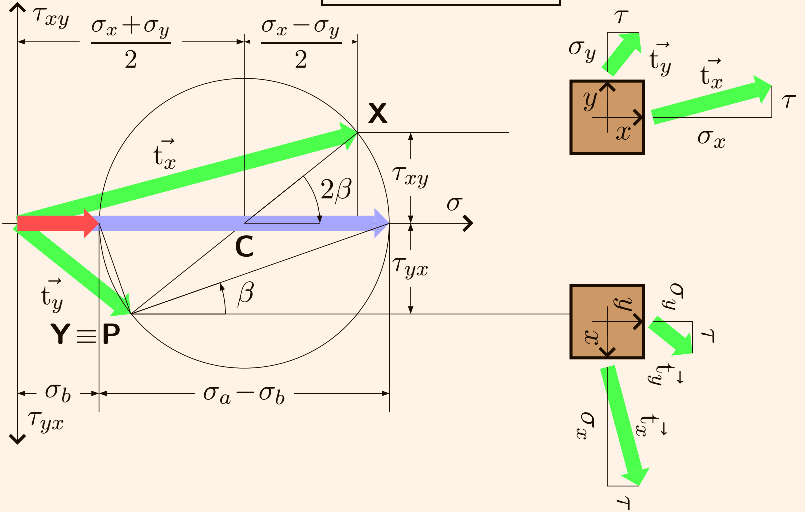


Circunferencia de Mohr: direcciones principales

$$\tan 2\beta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$



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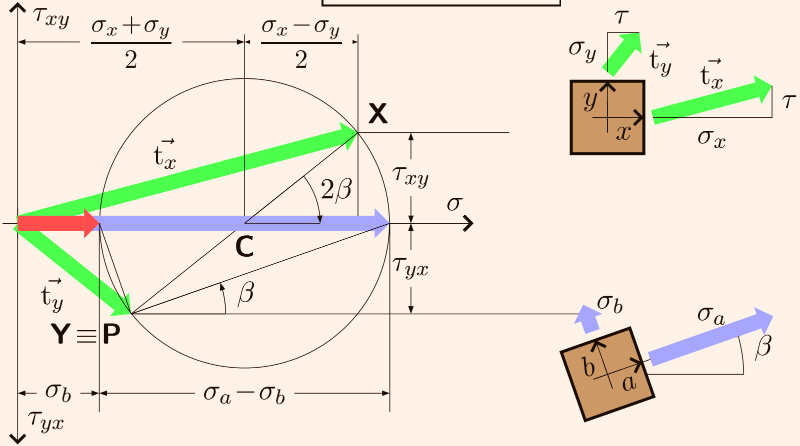

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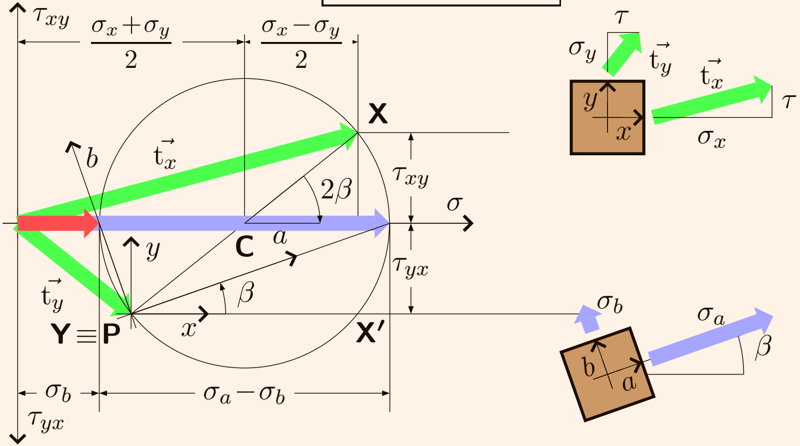
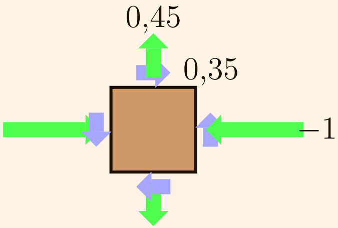
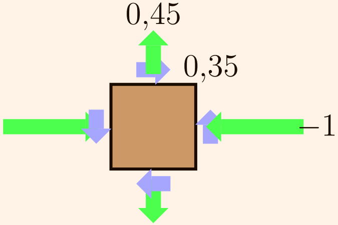



0,45
0,35
−1

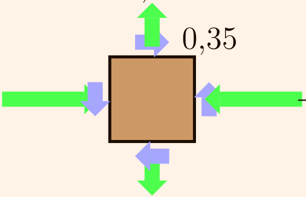


$$\tan(2\beta) = \frac{2\times 0{,}35}{-1-0{,}45} = -0{,}48 \;\Rightarrow$$

$$\Rightarrow\; \beta = 77{,}12^{o}$$

$$2\beta = 154{,}24 \qquad 2\beta = -205{,}76$$

¡ojo! $2\beta \neq -25{,}76$



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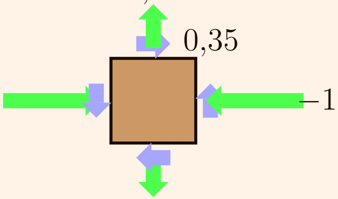
$$\frac{\sigma_x+\sigma_y}{2} = -0{,}28 \quad \frac{\sigma_x-\sigma_y}{2} = -0{,}73$$



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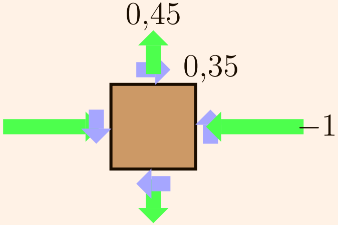
$$\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = 0{,}81$$



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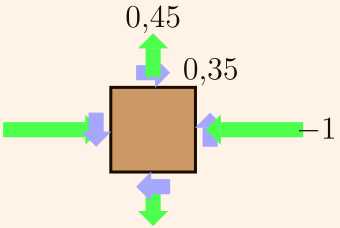

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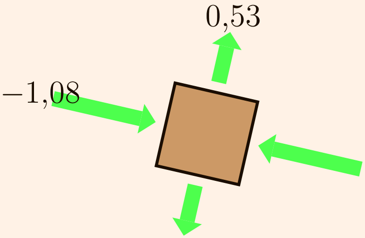

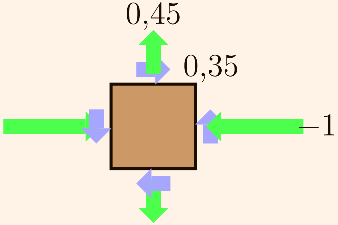
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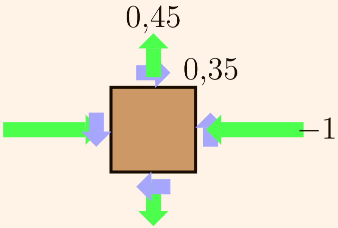
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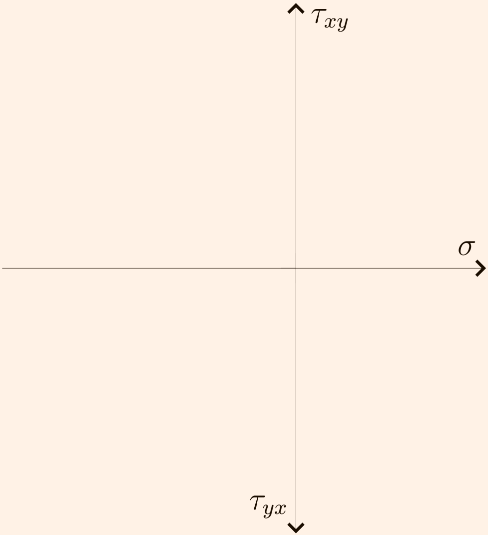
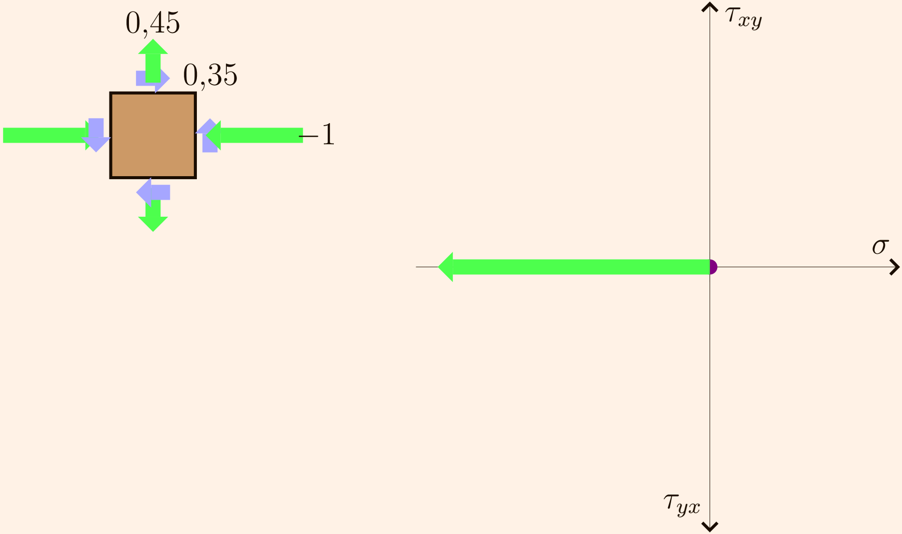

τ_{xy}
σ
τ_{yx}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τ_{xy}
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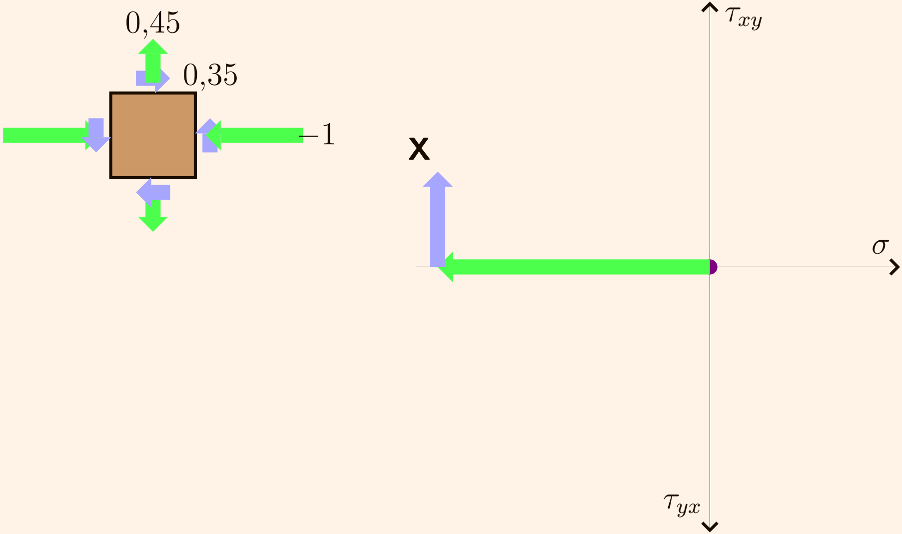
τ_{yx}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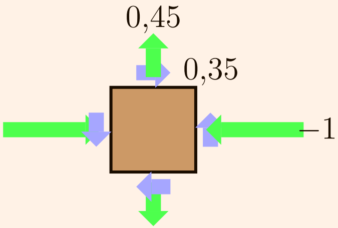
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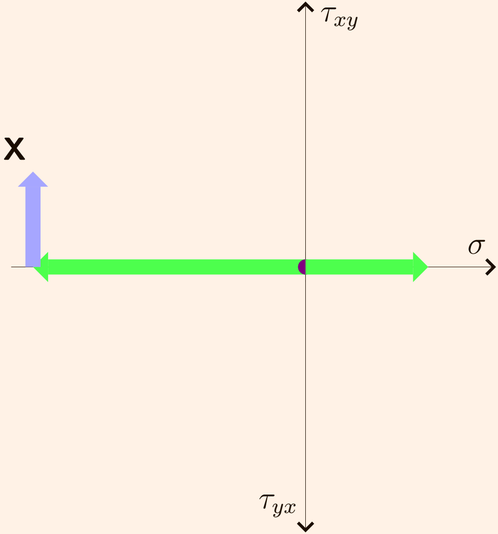
X
τ_{xy}
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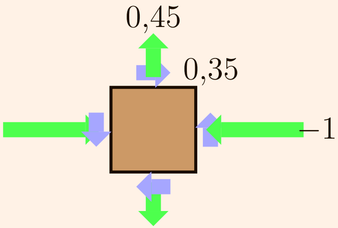

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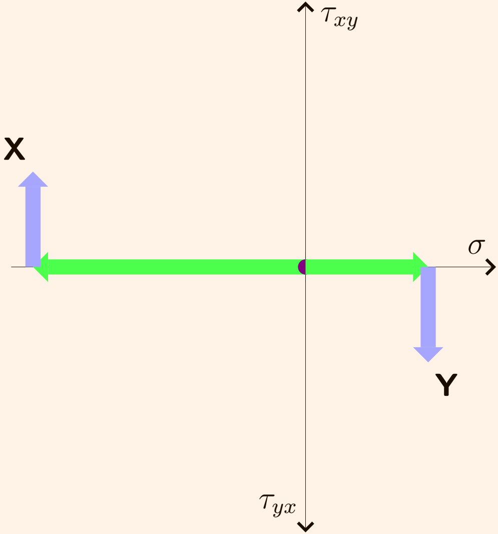
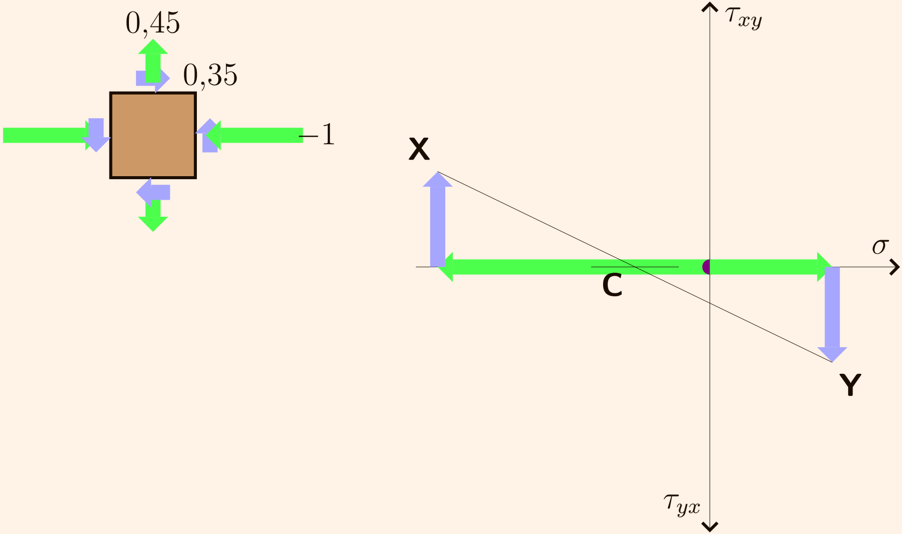


τ_{xy}
X
σ
Y
τ_{yx}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τ_{xy}
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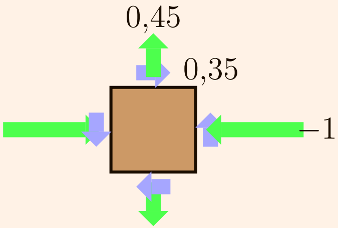

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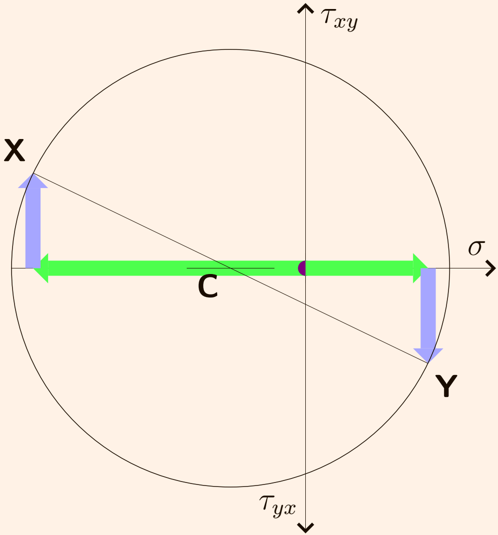

τ_{xy}
X
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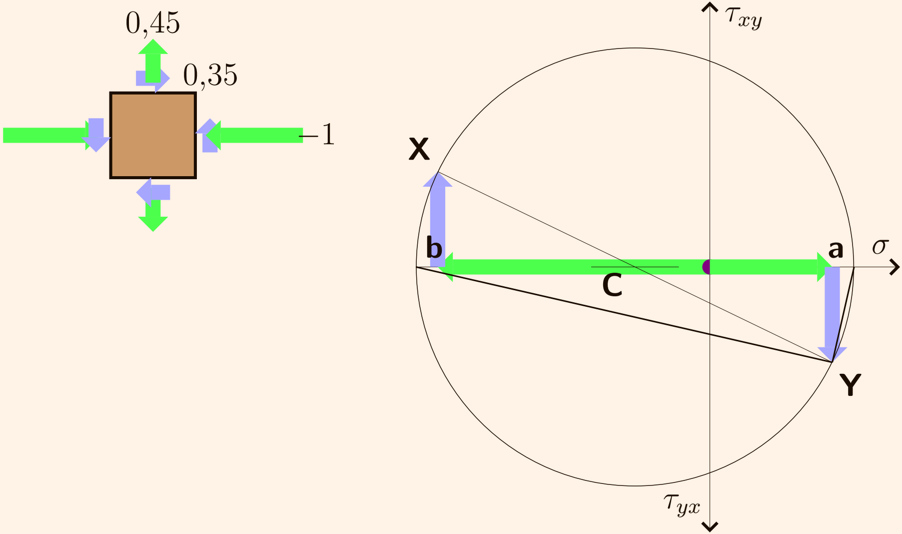
τ_{yx}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τ_{xy}
X
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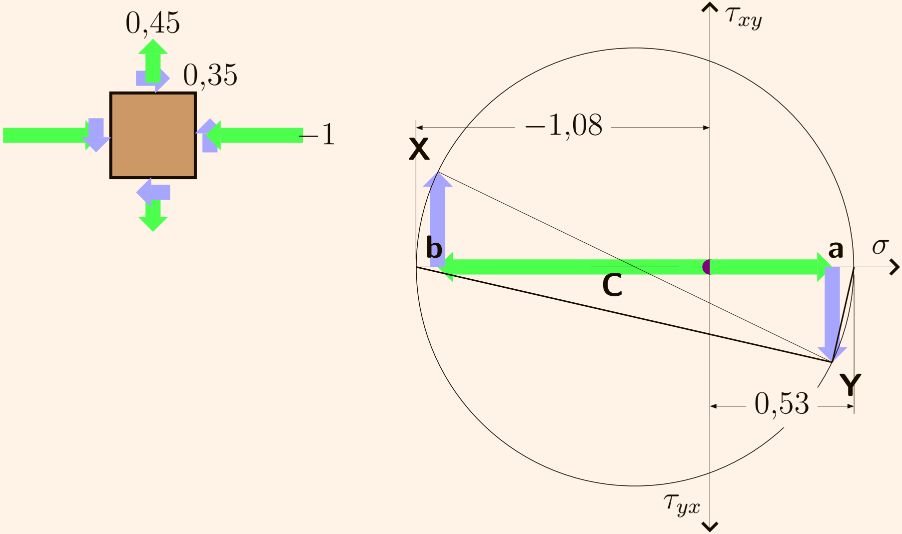
0,45
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τ_{xy}
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X
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a
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C
Y
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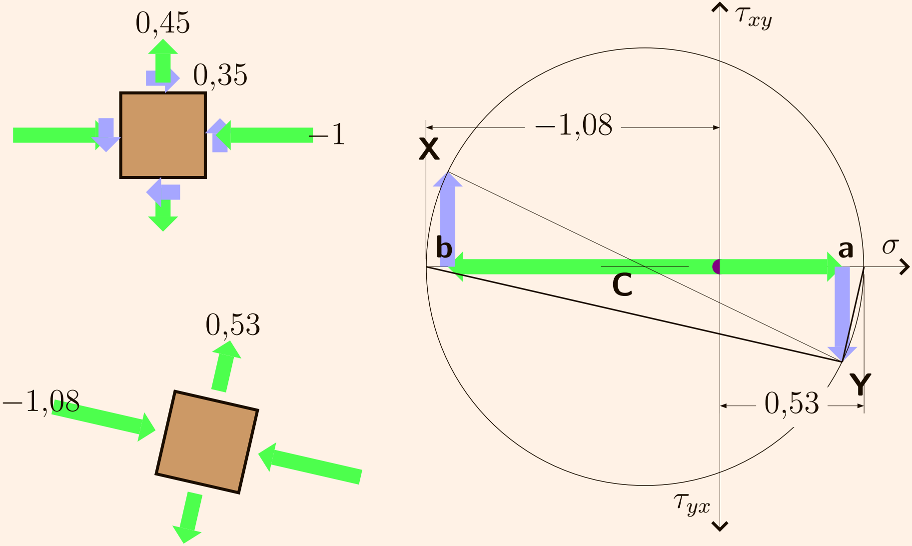
τ_{yx}


0,45
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0,53
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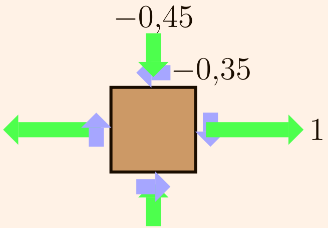
τ_{xy}
−1,08
X
b
a
σ
C
Y
0,53
τ_{yx}


−0,45
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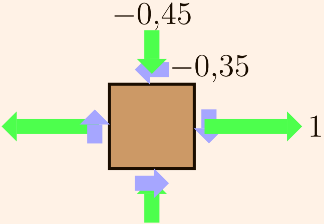
1

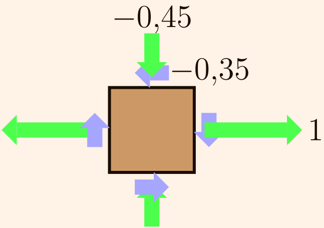


$$\tan(2\beta) = \frac{2\times(-0{,}35)}{1-0{,}45} = -0{,}48 \;\Rightarrow$$

$$\Rightarrow\; \beta = 167{,}12^{\text{o}}$$

$$2\beta = 334{,}24 \qquad 2\beta = -25{,}76$$

¡ojo! $2\beta \neq 154{,}24$



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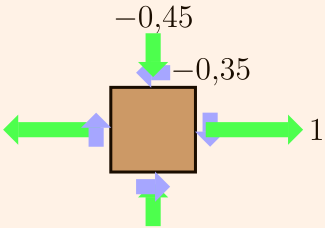

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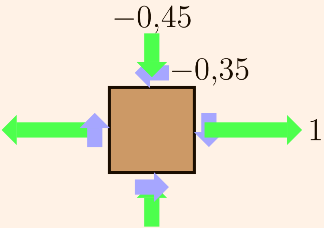
$$\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = 0{,}81$$



$$\tan(2\beta) = \frac{2\times(-0{,}35)}{1-0{,}45} = -0{,}48 \;\Rightarrow$$

$$\Rightarrow\; \beta = 167{,}12^{\text{o}}$$

$$2\beta = 334{,}24 \qquad 2\beta = -25{,}76$$

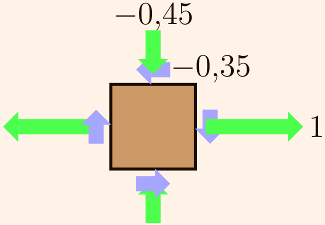
¡ojo! $2\beta \neq 154{,}24$

$$\frac{\sigma_x+\sigma_y}{2} = 0{,}28 \qquad \frac{\sigma_x-\sigma_y}{2} = 0{,}73$$

$$\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = 0{,}81$$

$$\sigma_a = 0{,}28+0{,}81 = 1{,}08$$

$$\sigma_b = 0{,}28-0{,}81 = -0{,}53$$



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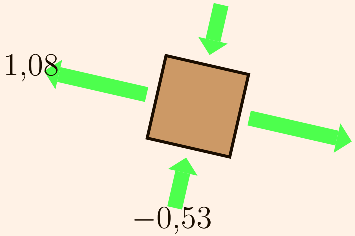
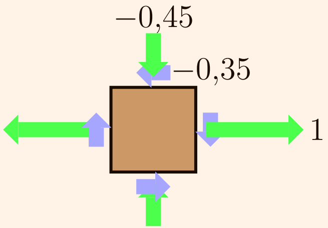
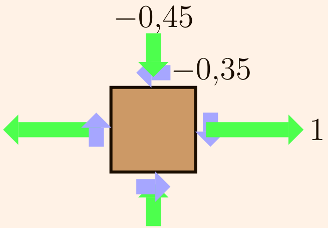
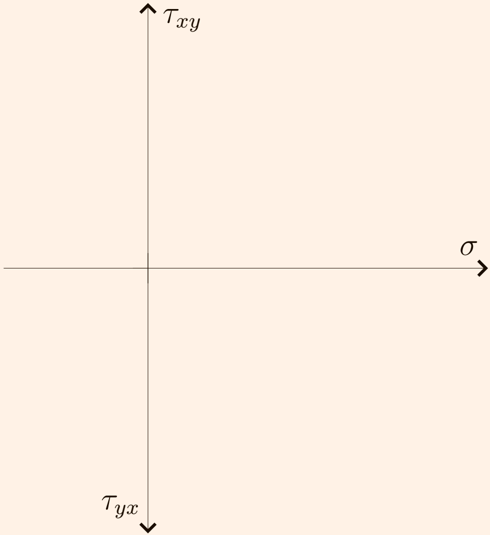
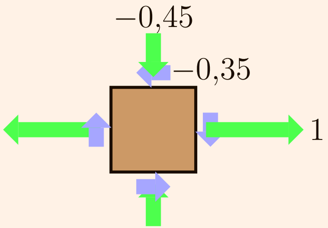
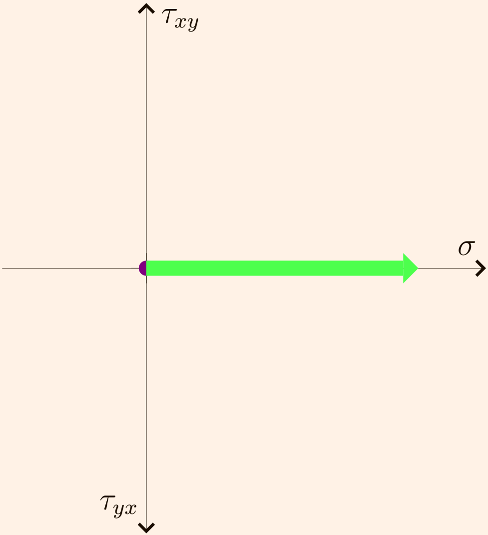


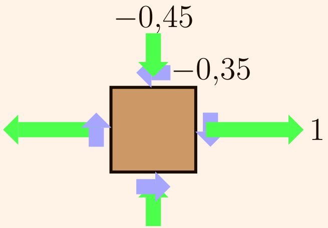

−0,45
−0,35
1


−0,45
−0,35
1

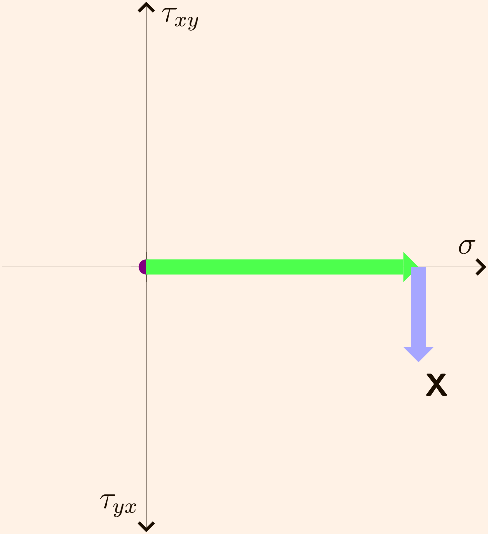

τ_{xy}
σ
τ_{yx}


−0,45
−0,35
1

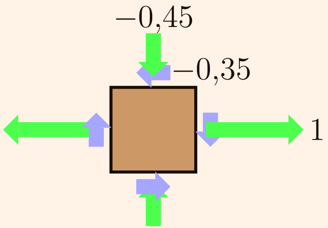

τ_{xy}
σ
τ_{yx}

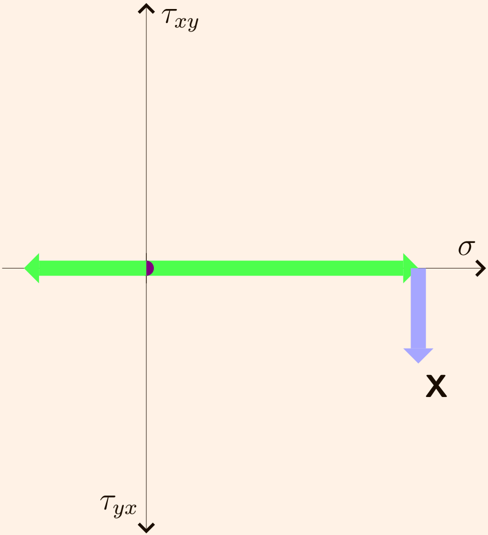


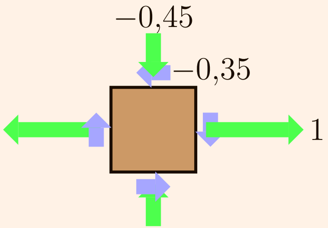
−0,45
−0,35
1

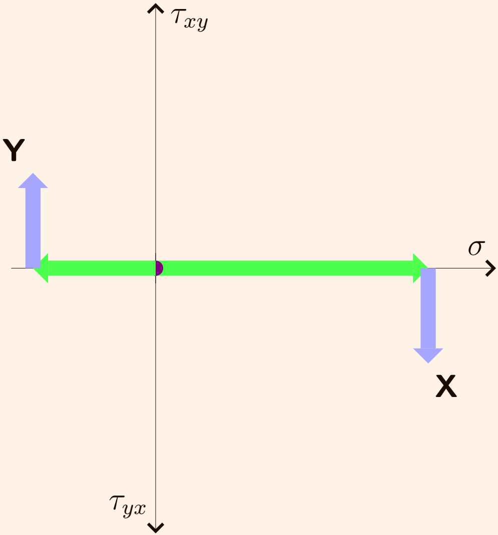


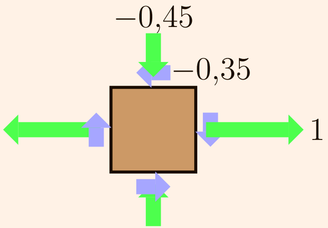
τ_{xy}
σ
X
τ_{yx}

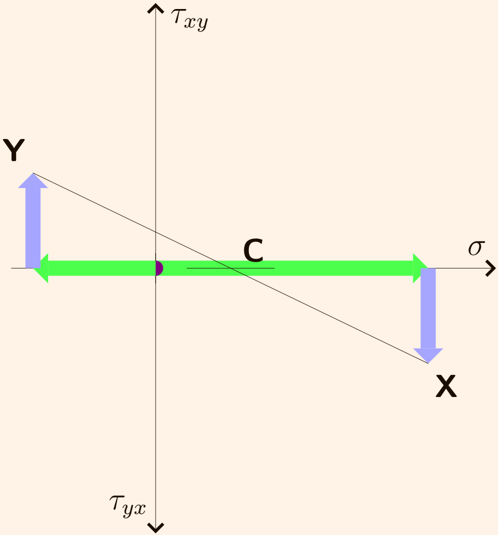

−0,45
−0,35
1

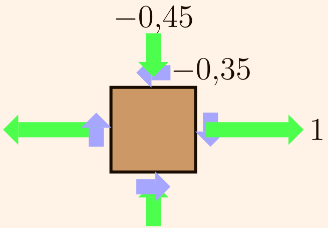

τ_{xy}
σ
X
τ_{yx}

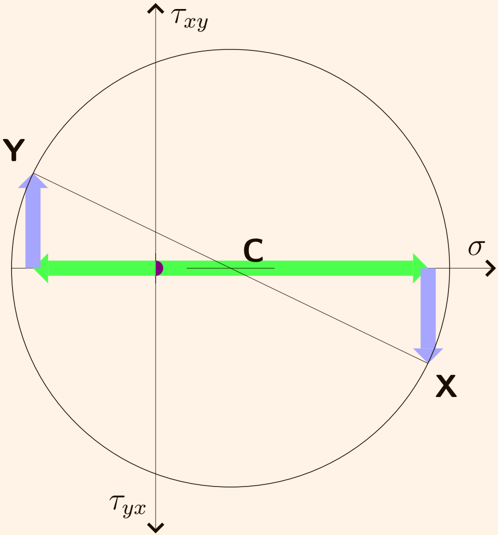

−0,45
−0,35
1

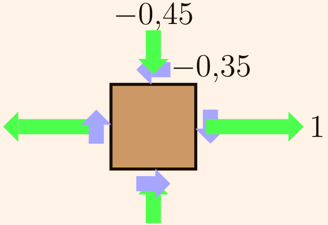

τ_{xy}
Y
σ
X
τ_{yx}


−0,45
−0,35
1

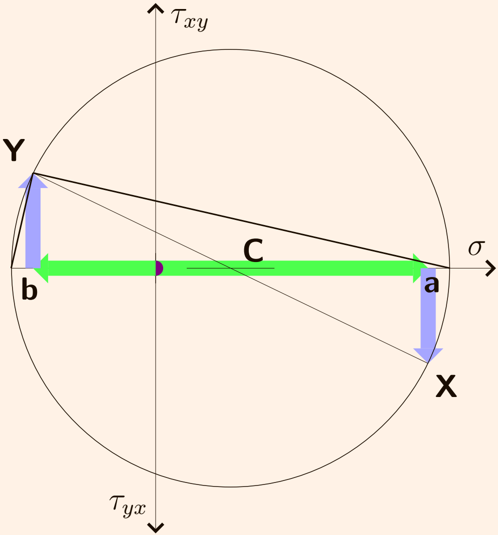

τ_{xy}
Y
C
σ
X
τ_{yx}


−0,45
−0,35
1

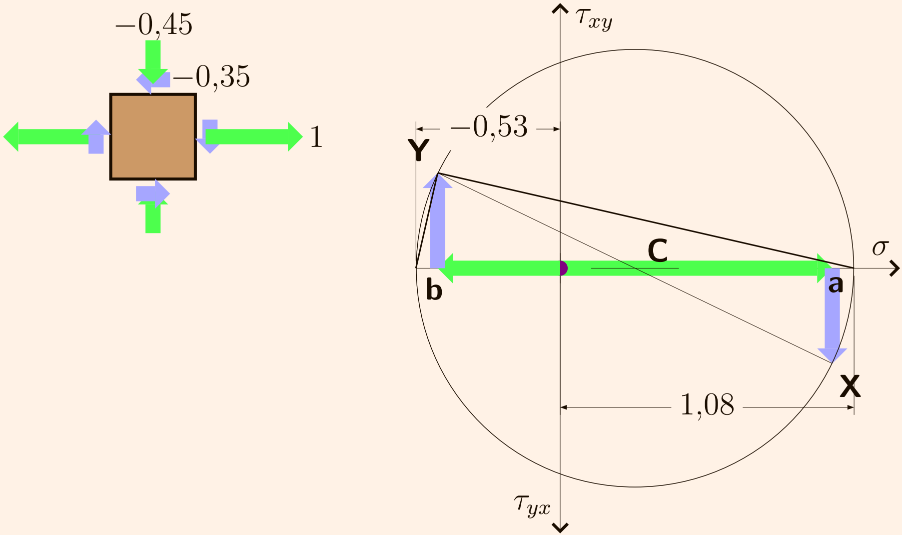

τ_{xy}
Y
C
σ
X
τ_{yx}

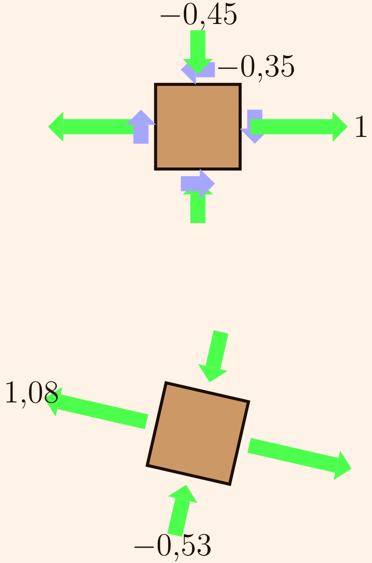


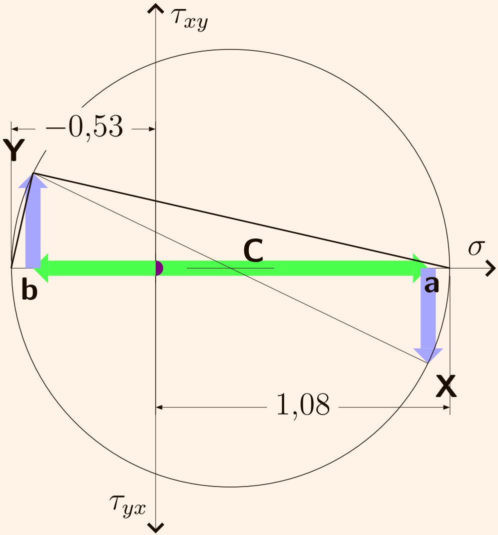
−0,45
−0,35
1

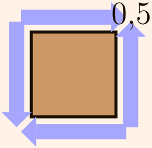


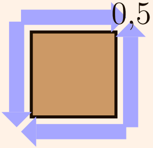
τ_{xy}
Y
C
σ
b
a
X
τ_{yx}


−0,45
−0,35
1
τ_{xy}
−0,53
Y
C
σ
b
a
X
1,08
τ_{yx}


−0,45
−0,35
1
1,08
−0,53

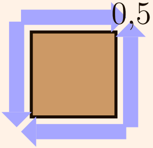

τ_{xy}
−0,53
Y
C
σ
b
a
X
1,08
τ_{yx}


0,5



$$\tan(2\beta) = \frac{2\times 0{,}5}{0-0} = \infty \;\Rightarrow$$

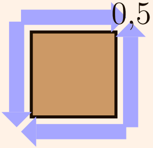
$$\Rightarrow\; \beta = 45^{\text{o}}$$



$$\tan(2\beta) = \frac{2 \times 0{,}5}{0 - 0} = \infty \;\Rightarrow$$

$$\Rightarrow \;\beta = 45^{\text{o}}$$

$$\frac{\sigma_x + \sigma_y}{2} = 0 \qquad \frac{\sigma_x - \sigma_y}{2} = 0$$



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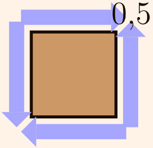
$$\frac{\sigma_x+\sigma_y}{2} = 0 \qquad \frac{\sigma_x-\sigma_y}{2} = 0$$

$$\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = 0{,}5$$



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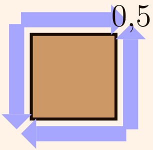
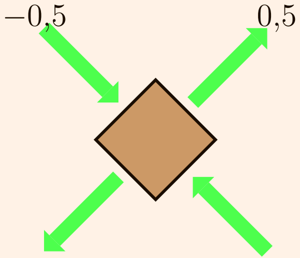
$$\sigma_a = 0 + 0{,}5 = 0{,}5$$

$$\sigma_b = 0 - 0{,}5 = -0{,}5$$





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$$\sigma_a = 0 + 0{,}5 = 0{,}5$$

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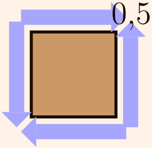
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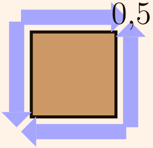
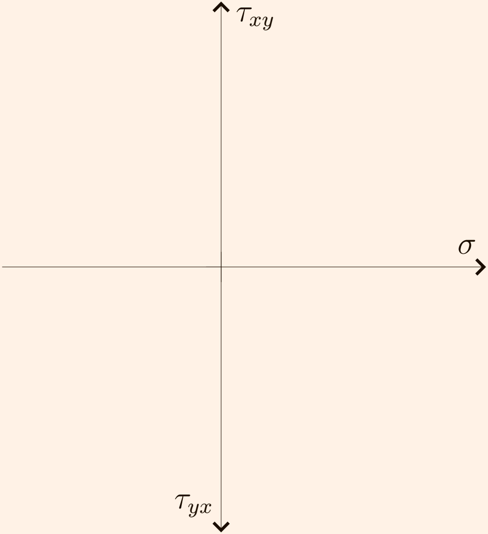
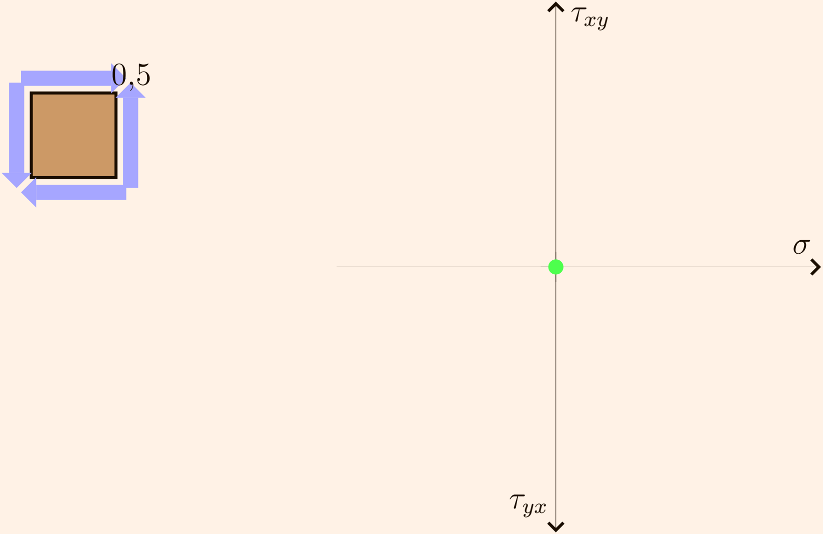
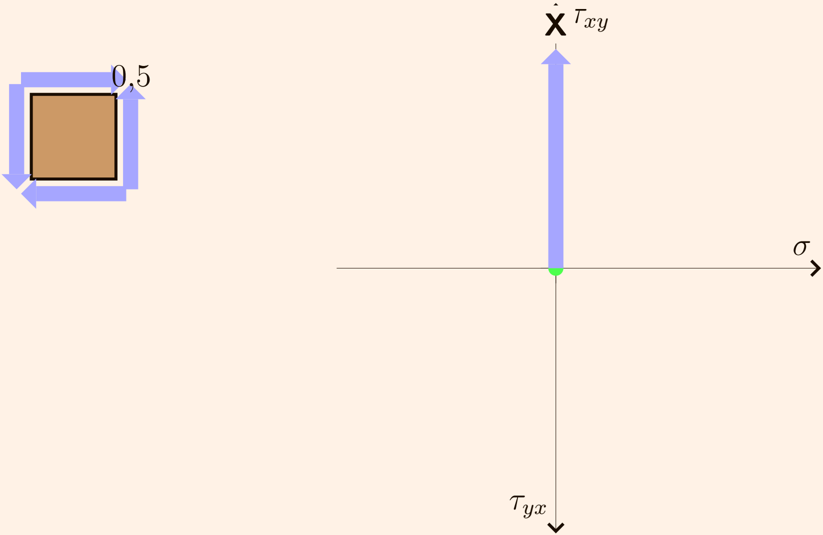
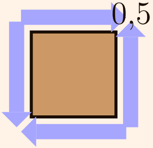
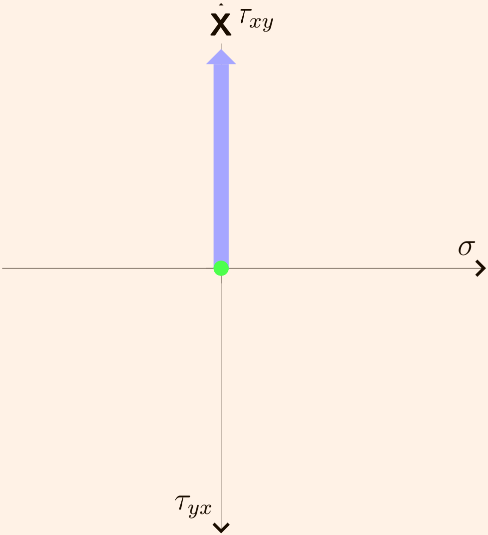

0,5

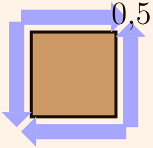

τ_{xy}
σ
τ_{yx}


0,5
τ_{xy}
σ
τ_{yx}


0,5
X
τ_{xy}
σ
τ_{yx}


0,5


X
τ_{xy}
σ
τ_{yx}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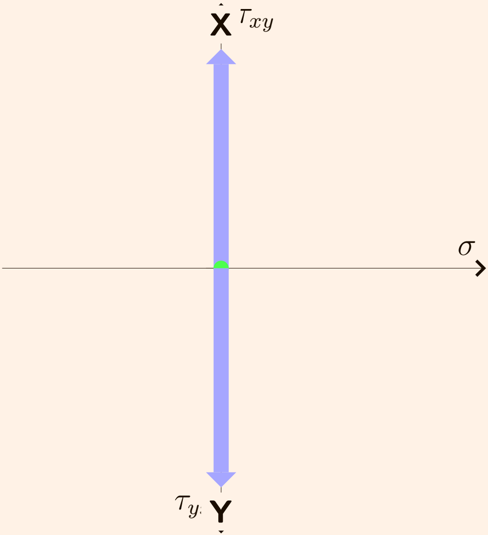
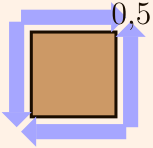
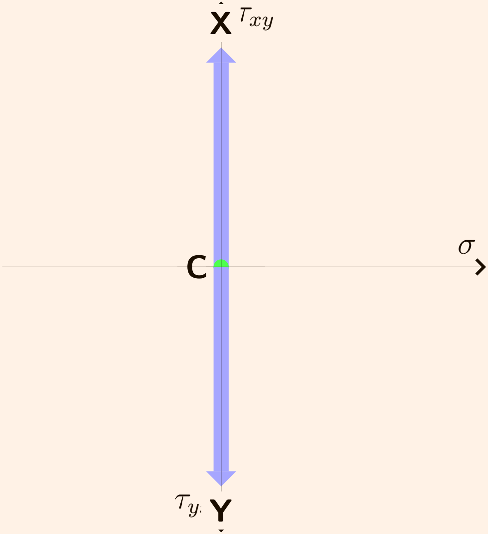
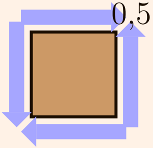

X
τ_{xy}
σ
Y


0,5


X
τ_{xy}
σ
C
Y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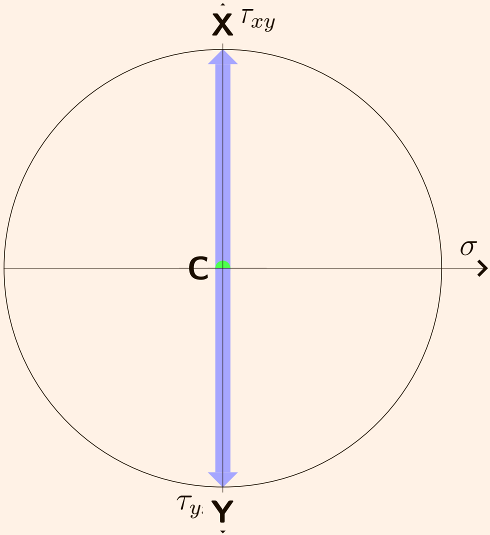
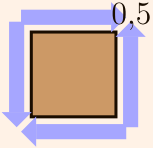

X
τ_{xy}
C
σ
Y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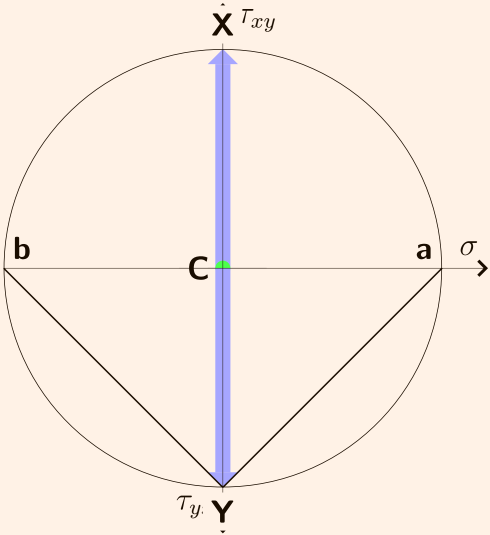
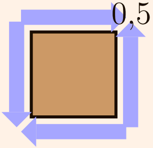

X
τ_{xy}
b
C
a
σ
Y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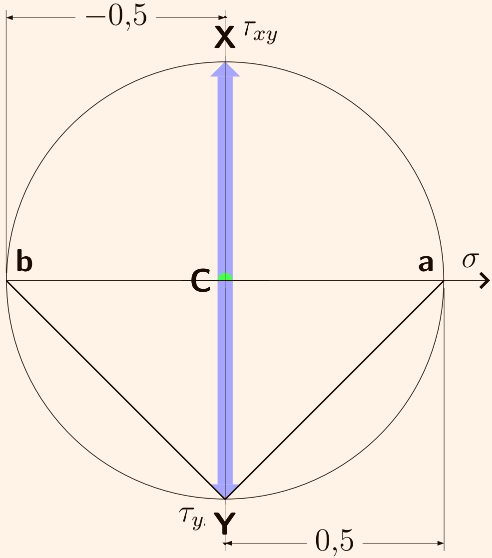
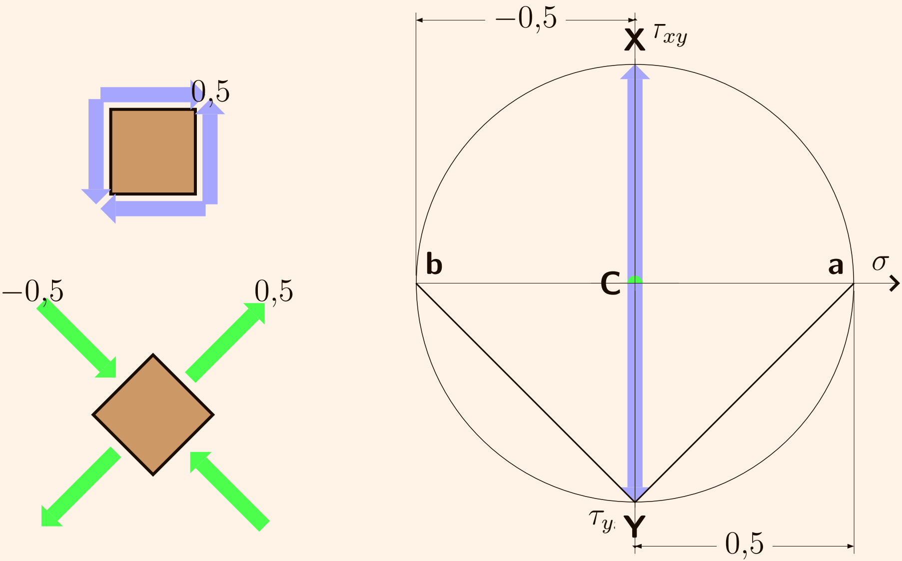
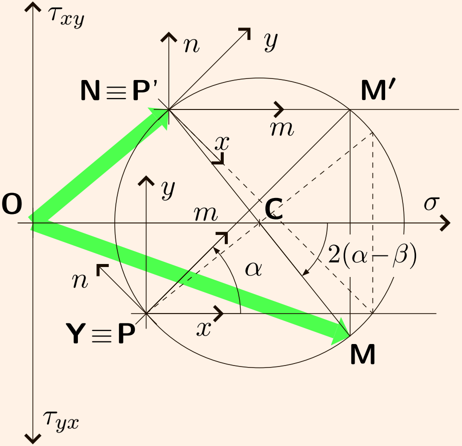
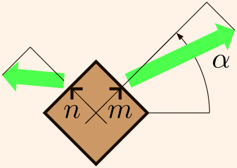
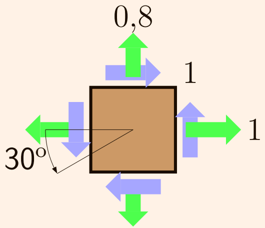

−0,5
X
τ_{xy}
b
C
a
σ
Y
0,5

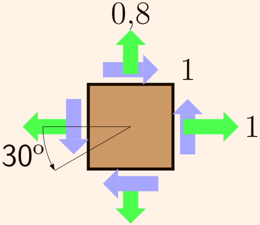

0,5
−0,5
0,5
−0,5
τ_{xy}
X
b
C
a
σ
τ_{y}
Y
0,5

Circunferencia de Mohr: una dirección cualquiera



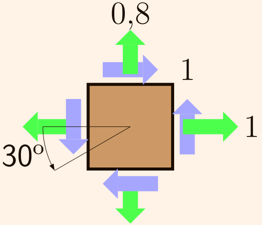



0,8
1
1
30º



$$\alpha = 30 \quad \cos\alpha = 0{,}87 \quad \sin\alpha = 0{,}5$$

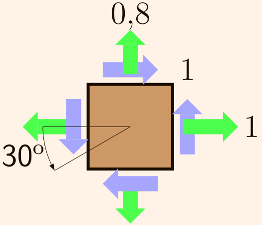
$$2\alpha = 60 \quad \cos 2\alpha = 0{,}5 \quad \sin 2\alpha = 0{,}87$$



$$\alpha = 30 \quad \cos\alpha = 0{,}87 \quad \sin\alpha = 0{,}5$$

$$2\alpha = 60 \quad \cos 2\alpha = 0{,}5 \quad \sin 2\alpha = 0{,}87$$

$$\sigma_m = \sigma_x \cos^2\alpha + \sigma_y \sin^2\alpha + 2\tau_{xy} \sin\alpha\cos\alpha$$
$$= 1{,}82$$

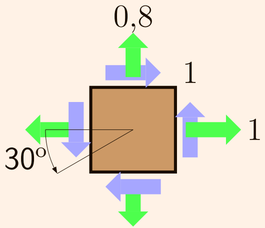


$$\alpha = 30 \quad \cos\alpha = 0{,}87 \quad \sin\alpha = 0{,}5$$

$$2\alpha = 60 \quad \cos 2\alpha = 0{,}5 \quad \sin 2\alpha = 0{,}87$$

$$\sigma_m = \sigma_x \cos^2\alpha + \sigma_y \sin^2\alpha + 2\tau_{xy}\sin\alpha\cos\alpha$$
$$= 1{,}82$$

$$\tau_{mn} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\alpha + \tau_{xy}\cos\alpha$$
$$= 0{,}41$$



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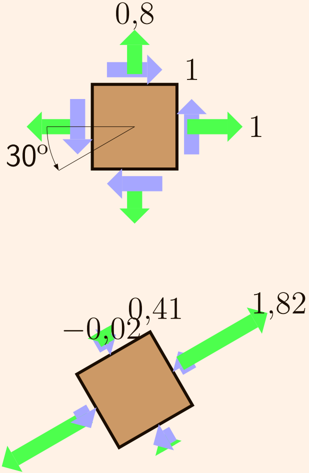
$$= 1{,}82$$

$$\tau_{mn} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\alpha + \tau_{xy} \cos\alpha$$
$$= 0{,}41$$

$$\sigma_n = \sigma_x \sin^2\alpha + \sigma_y \cos^2\alpha - 2\tau_{xy} \cos\alpha \sin\alpha$$
$$= -0{,}02$$



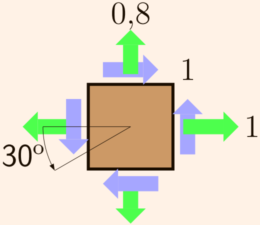
$$\alpha = 30 \quad \cos\alpha = 0{,}87 \quad \sin\alpha = 0{,}5$$

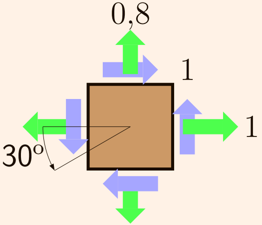
$$2\alpha = 60 \quad \cos 2\alpha = 0{,}5 \quad \sin 2\alpha = 0{,}87$$

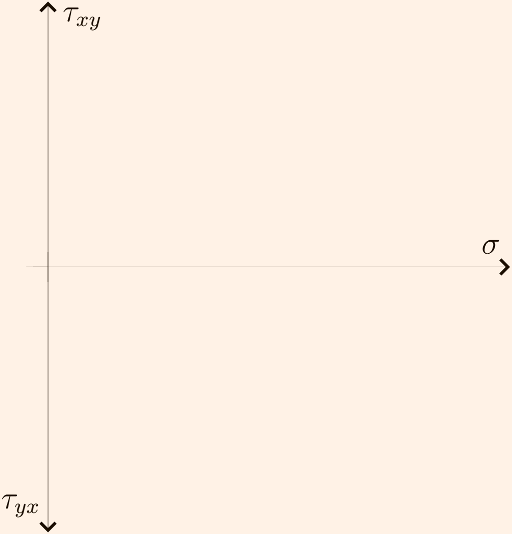
$$\sigma_m = \sigma_x \cos^2\alpha + \sigma_y \sin^2\alpha + 2\tau_{xy}\sin\alpha\cos\alpha = 1{,}82$$

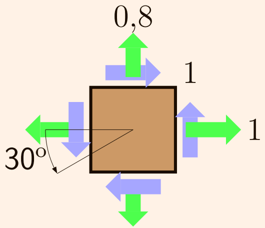
$$\tau_{mn} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\alpha + \tau_{xy}\cos\alpha = 0{,}41$$

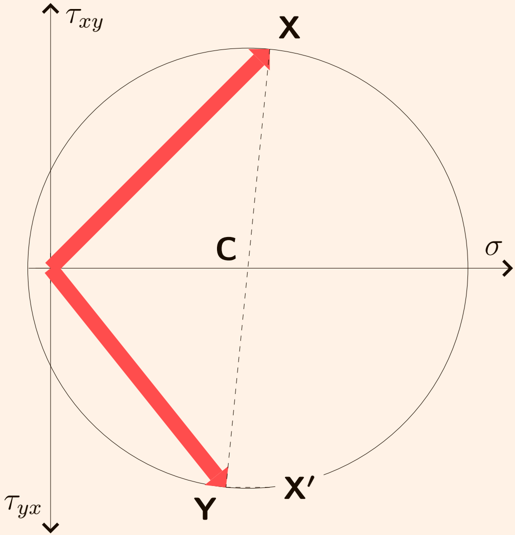
$$\sigma_n = \sigma_x \sin^2\alpha + \sigma_y \cos^2\alpha - 2\tau_{xy}\cos\alpha\sin\alpha = -0{,}02$$

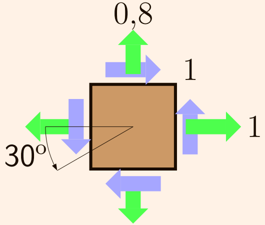

0,8
1
1
30º

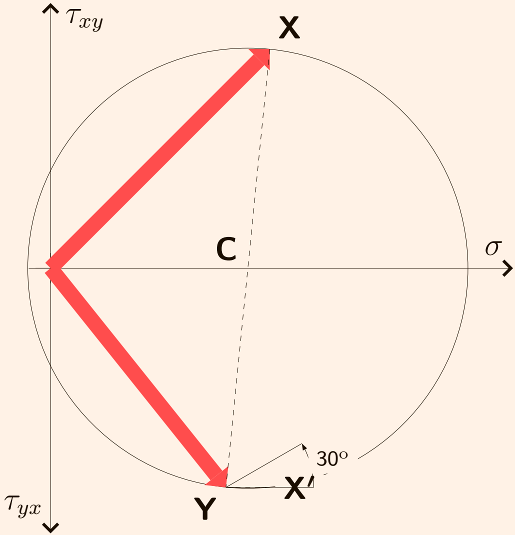

0,8
1
1
30º

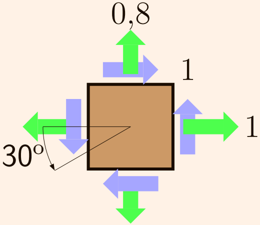

τ_{xy}
σ
τ_{yx}

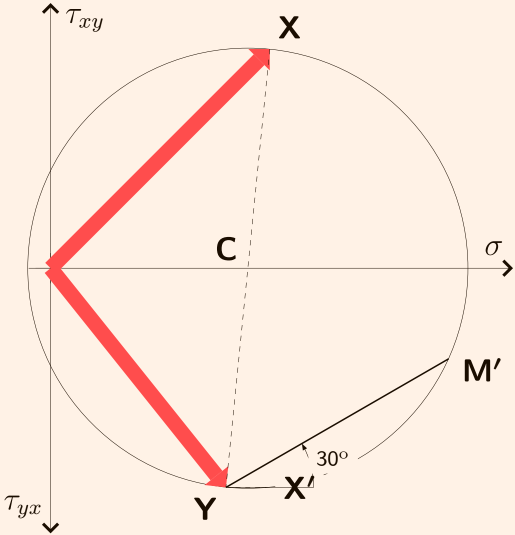

0,8
1
1
30º

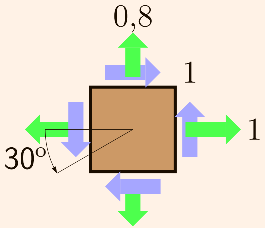

τ_{xy}
X
C
σ
X′
τ_{yx}
Y

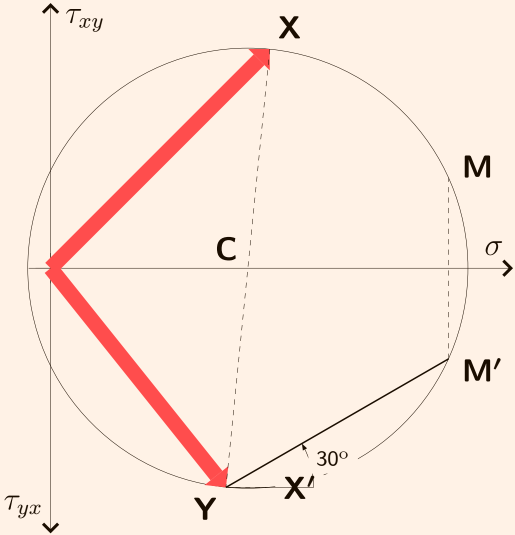

0,8
1
1
30º

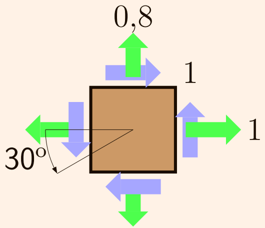

τ_{xy}
X
C
σ
30º
X′
Y
τ_{yx}

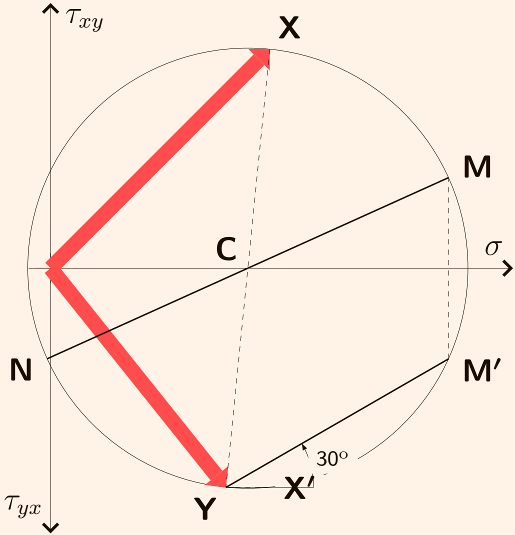

0,8
1
1
30º

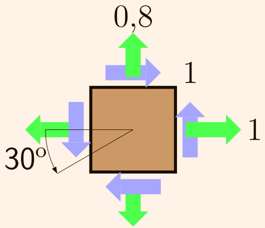

τ_{xy}
X
C
σ
M′
30º
X′
Y
τ_{yx}

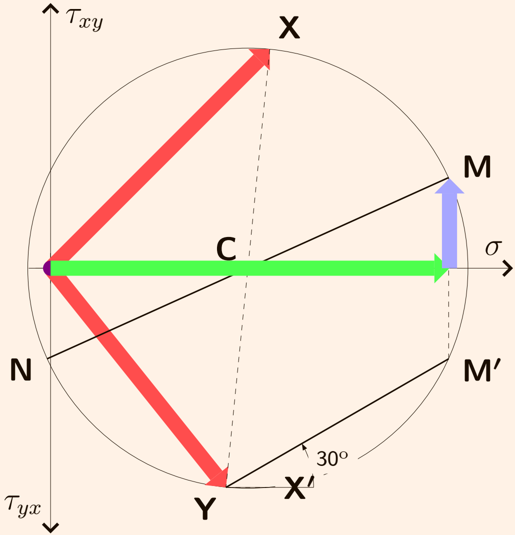

0,8
1
1
30º

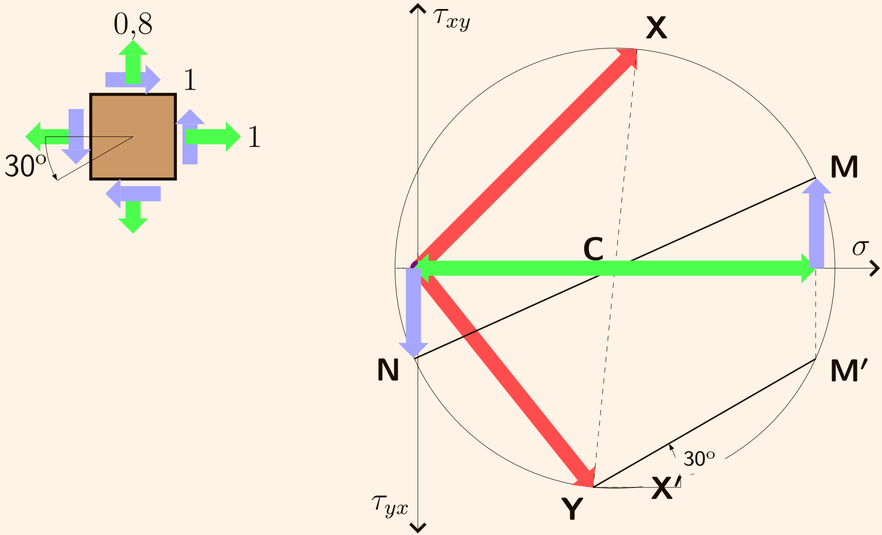

τ_{xy}
X
M
C
σ
M′
30º
X′
Y
τ_{yx}

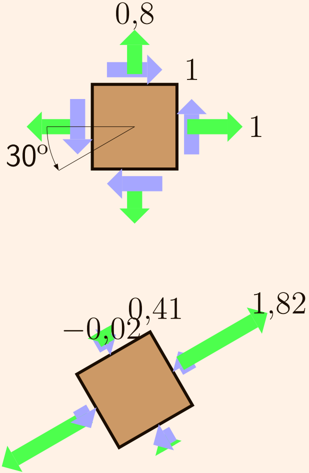

0,8
1
1
30º

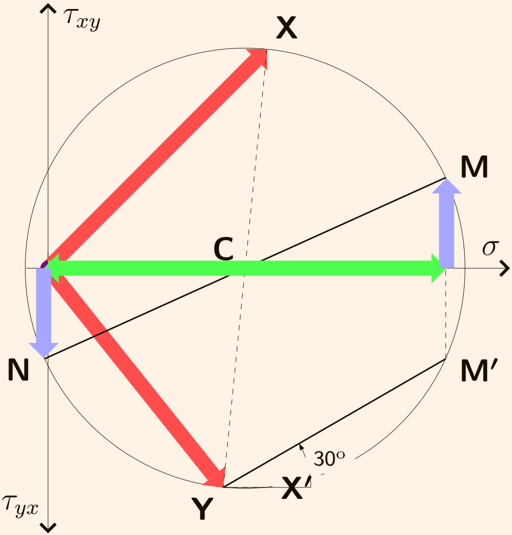

τ_{xy}
X
M
C
σ
N
M′
30º
X′
τ_{yx}
Y

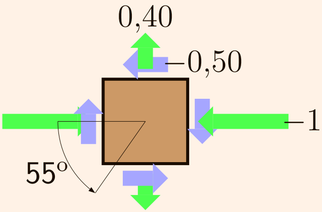

0,8
1
1
30º

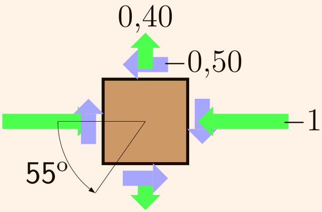

τ_{xy}
X
M
C
σ
N
M′
30º
X′
τ_{yx}
Y


0,8
1
1
30º
τxy
X
M
C
σ
N
M′
30º
X′
Y
τyx


0,8
1
1
30º
0,41
1,82
−0,02

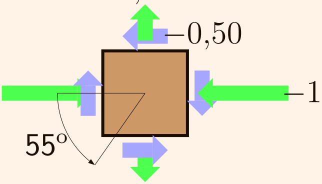

τ_{xy}
X
M
C
σ
N
M′
30º
X′
Y
τ_{yx}


0,40
−0,50
−1
55º



$$\alpha = 55 \quad \cos\alpha = 0{,}57 \quad \sin\alpha = 0{,}82$$

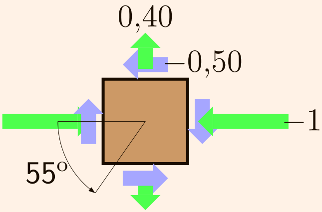
$$2\alpha = 110 \quad \cos 2\alpha = -0{,}34 \quad \sin 2\alpha = 0{,}94$$



$$\alpha = 55 \quad \cos\alpha = 0{,}57 \quad \sin\alpha = 0{,}82$$

$$2\alpha = 110 \quad \cos 2\alpha = -0{,}34 \quad \sin 2\alpha = 0{,}94$$

$$\sigma_m = \sigma_x \cos^2\alpha + \sigma_y \sin^2\alpha + 2\tau_{xy} \sin\alpha \cos\alpha$$
$$= -0{,}53$$

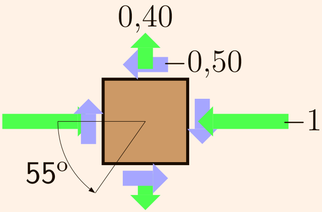


$$\alpha = 55 \quad \cos\alpha = 0{,}57 \quad \sin\alpha = 0{,}82$$

$$2\alpha = 110 \quad \cos 2\alpha = -0{,}34 \quad \sin 2\alpha = 0{,}94$$

$$\begin{aligned}\sigma_m &= \sigma_x \cos^2\alpha + \sigma_y \sin^2\alpha + 2\tau_{xy}\sin\alpha\cos\alpha \\ &= -0{,}53\end{aligned}$$

$$\begin{aligned}\tau_{mn} &= -\frac{\sigma_x - \sigma_y}{2}\sin 2\alpha + \tau_{xy}\cos\alpha \\ &= 0{,}83\end{aligned}$$



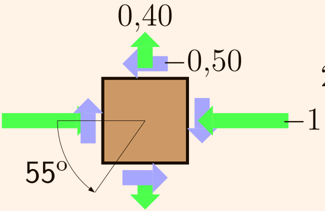
$$\alpha = 55 \quad \cos\alpha = 0{,}57 \quad \sin\alpha = 0{,}82$$

$$2\alpha = 110 \quad \cos 2\alpha = -0{,}34 \quad \sin 2\alpha = 0{,}94$$

$$\sigma_m = \sigma_x \cos^2\alpha + \sigma_y \sin^2\alpha + 2\tau_{xy}\sin\alpha\cos\alpha$$
$$= -0{,}53$$

$$\tau_{mn} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\alpha + \tau_{xy}\cos\alpha$$
$$= 0{,}83$$

$$\sigma_n = \sigma_x \sin^2\alpha + \sigma_y \cos^2\alpha - 2\tau_{xy}\cos\alpha\sin\alpha$$
$$= -0{,}07$$



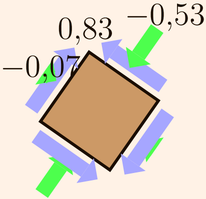
$$\alpha = 55 \quad \cos\alpha = 0{,}57 \quad \sin\alpha = 0{,}82$$

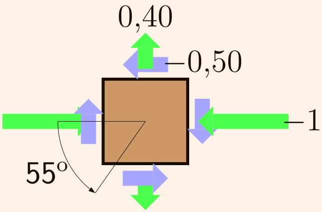
$$2\alpha = 110 \quad \cos 2\alpha = -0{,}34 \quad \sin 2\alpha = 0{,}94$$

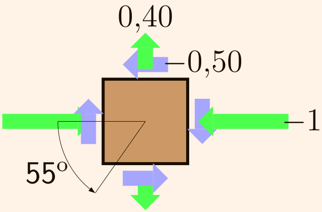
$$\begin{aligned}\sigma_m &= \sigma_x \cos^2\alpha + \sigma_y \sin^2\alpha + 2\tau_{xy}\sin\alpha\cos\alpha \\ &= -0{,}53\end{aligned}$$

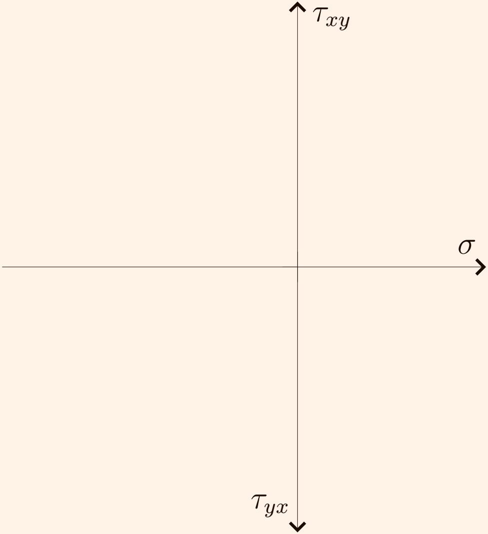
$$\begin{aligned}\tau_{mn} &= -\frac{\sigma_x - \sigma_y}{2}\sin 2\alpha + \tau_{xy}\cos\alpha \\ &= 0{,}83\end{aligned}$$

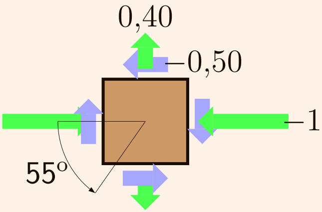
$$\begin{aligned}\sigma_n &= \sigma_x \sin^2\alpha + \sigma_y \cos^2\alpha - 2\tau_{xy}\cos\alpha\sin\alpha \\ &= -0{,}07\end{aligned}$$

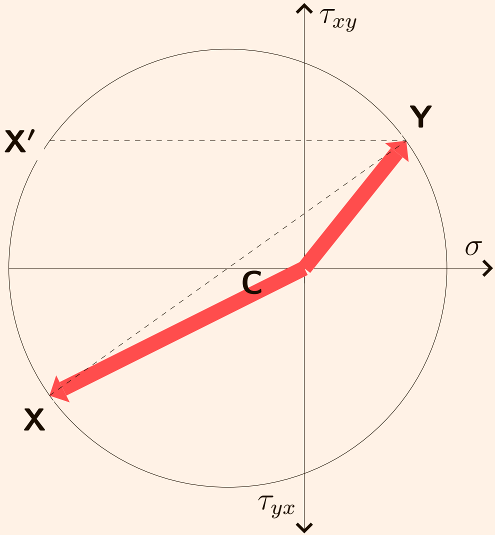


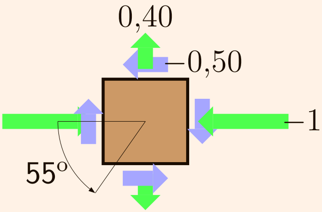

0,40
−0,50
−1
55º

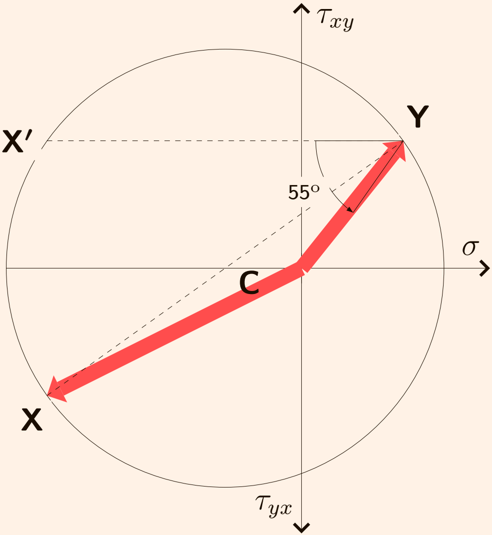

0,40
−0,50
−1
55º

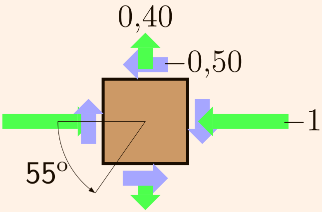

τ_{xy}
σ
τ_{yx}

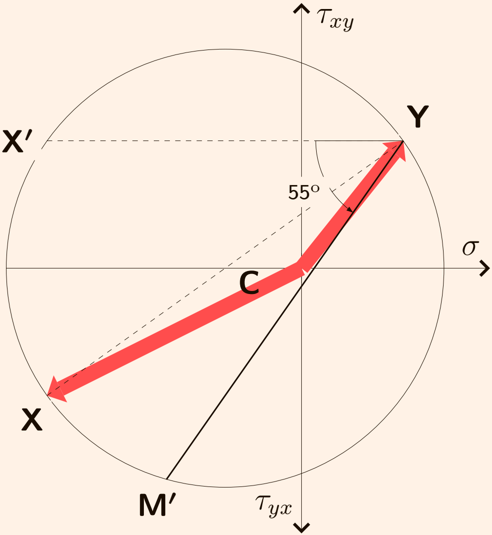

0,40
−0,50
−1
55o

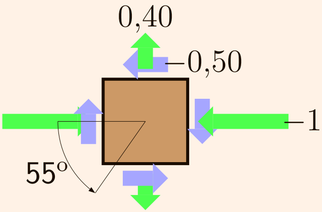

τxy
Y
X′
σ
C
X
τyx

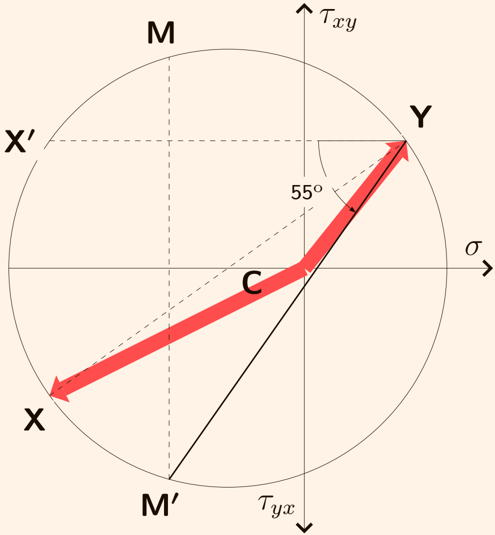

0,40
−0,50
−1
55º

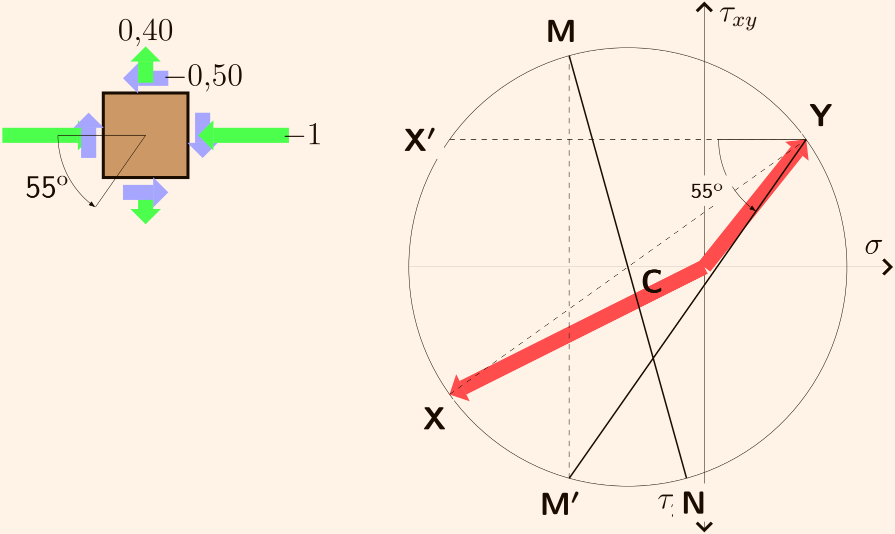

τ_{xy}
Y
X′
55º
σ
C
X
τ_{yx}

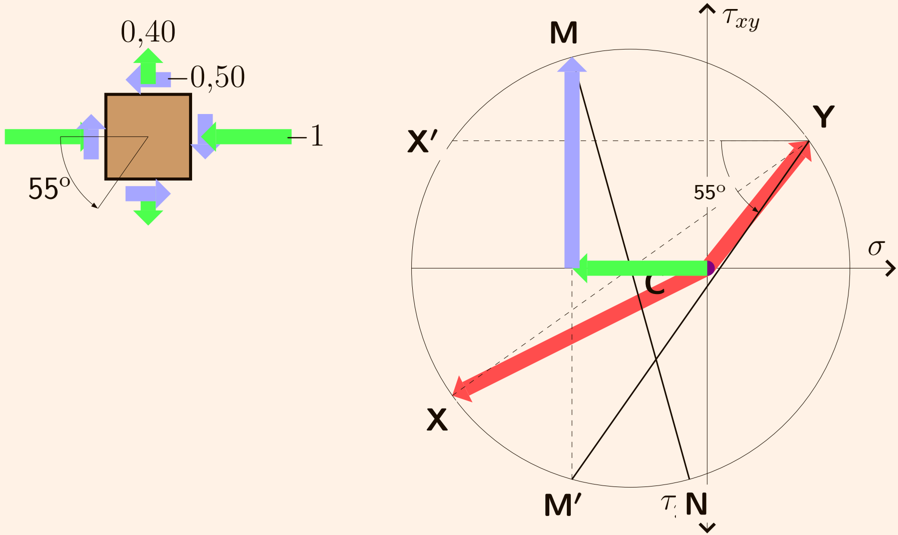

0,40
−0,50
−1
55º

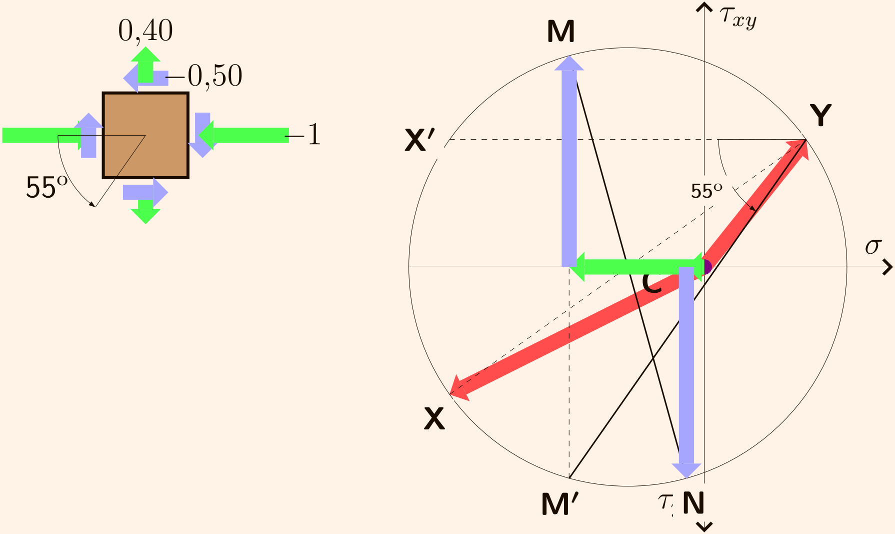

τ_{xy}
Y
X′
55º
σ
C
X
M′
τ_{yx}


0,40
−0,50
−1
55º

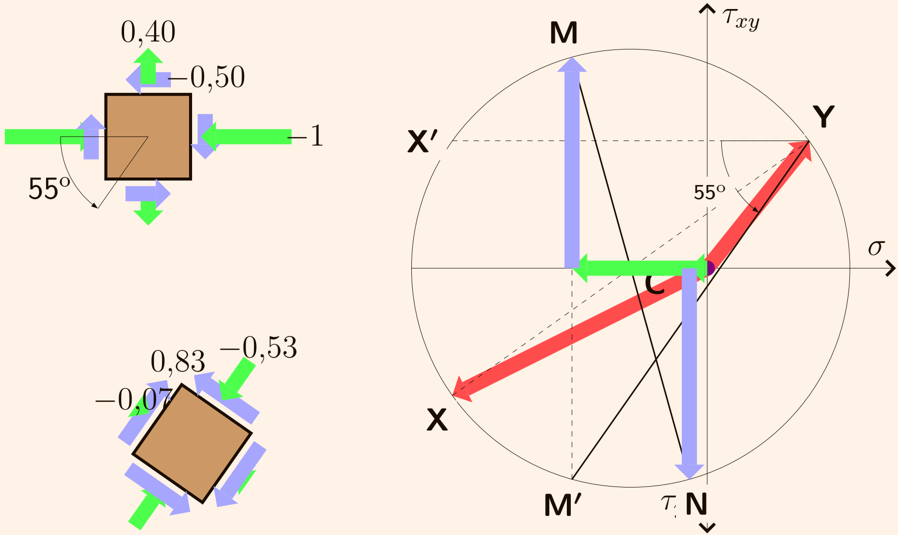

τ_{xy}
M
Y
X′
55º
σ
C
X
M′
τ_{yx}


0,40
−0,50
−1
55º
M
τ_{xy}
X′
Y
55º
σ
C
X
M′
N

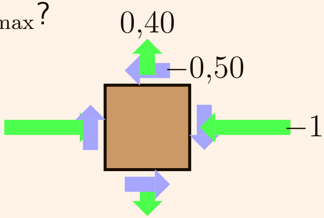

0,40
−0,50
−1
55°
M
τ_{xy}
Y
X′
55°
σ
C
X
M′
N



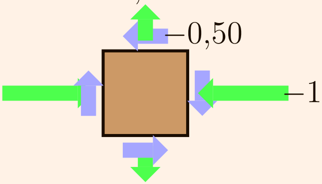
0,40
−0,50
−1
55º
M
τ_{xy}
Y
X′
55º
σ
C
X
M′
N

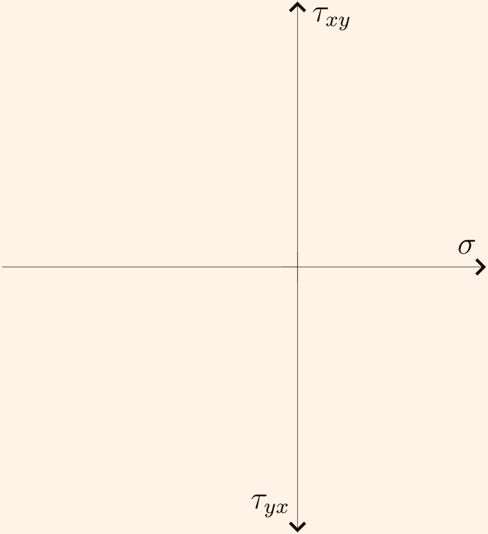

0,40
−0,50
−1
55º
0,83
−0,53
−0,07
M
X′
Y
55º
σ
C
X
M′
N
τ_xy

¿$\tau_{\max}$?

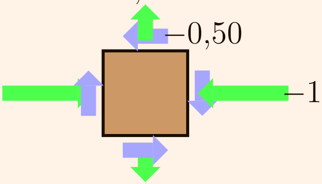


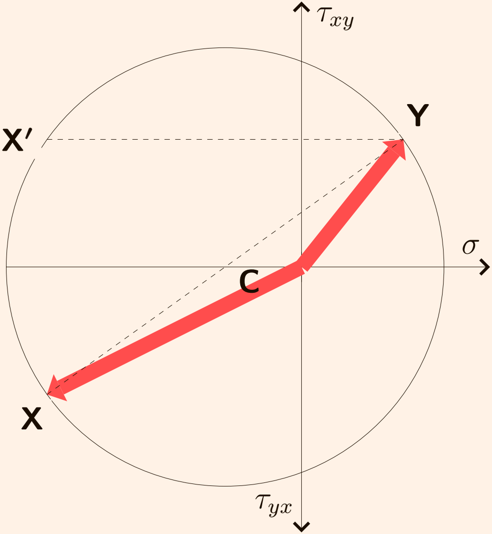
¿$\tau_{\max}$?



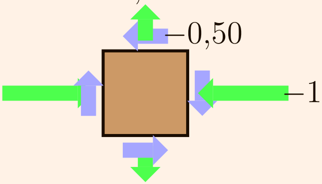


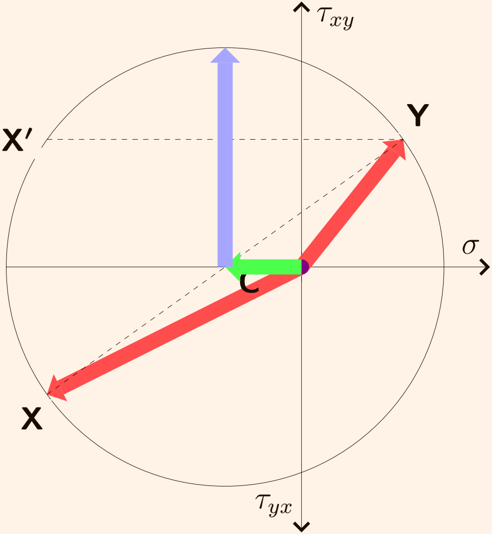
¿$\tau_{\max}$?



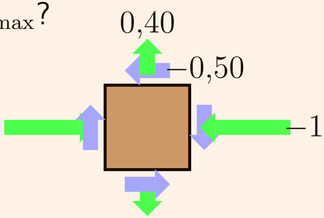


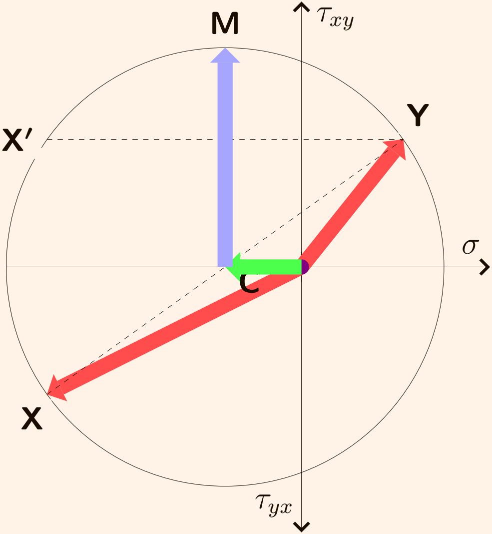
¿$\tau_{\max}$?



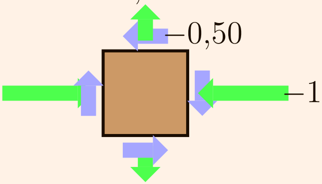


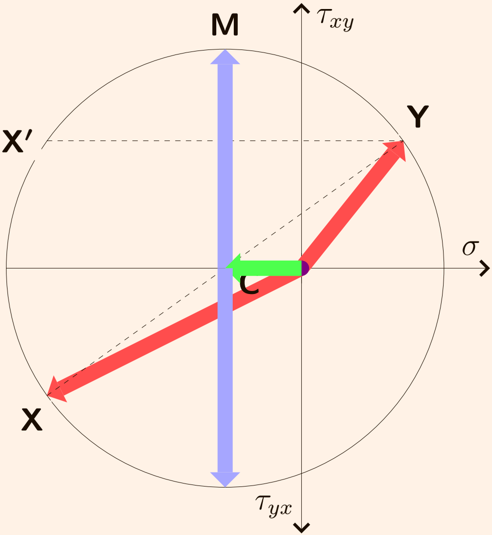
¿$\tau_{\max}$?



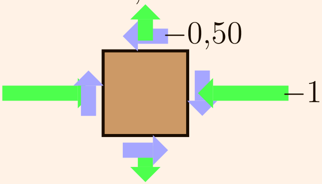


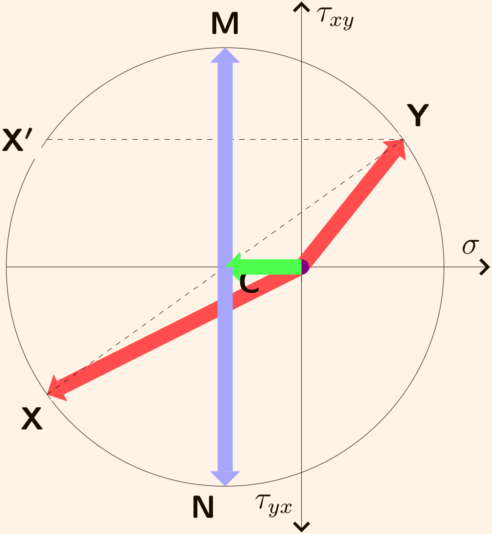
¿$\tau_{\max}$?



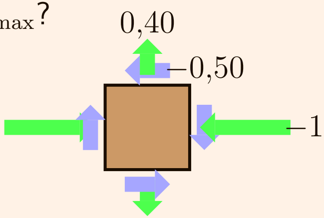


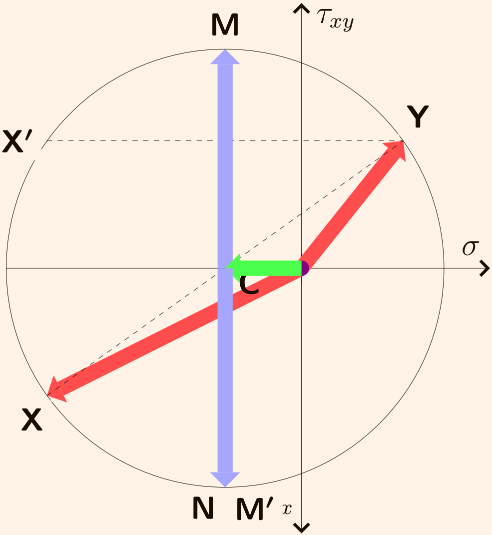
¿$\tau_{\max}$?



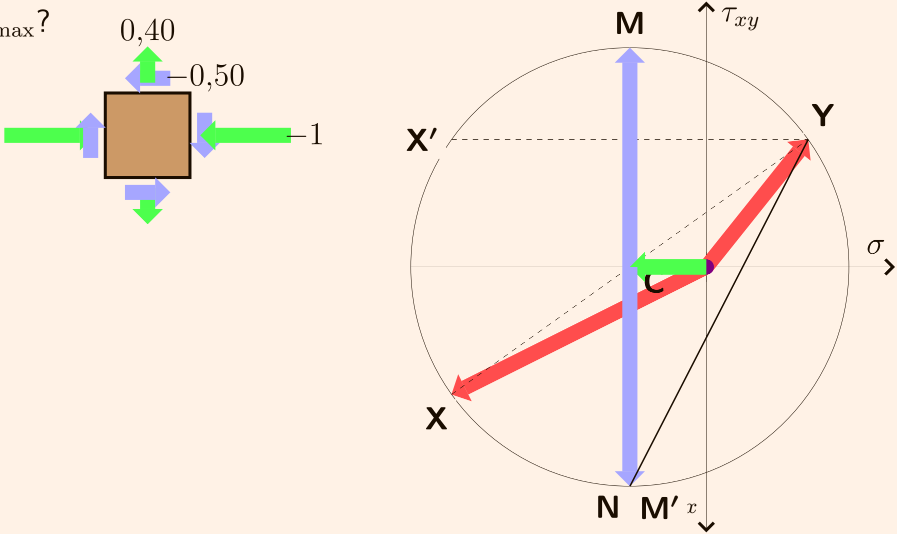


¿$\tau_{\max}$?

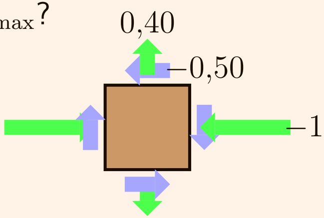


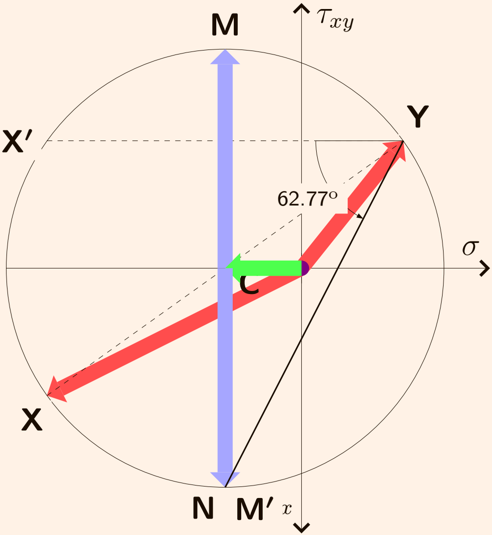


¿$\tau_{\max}$?

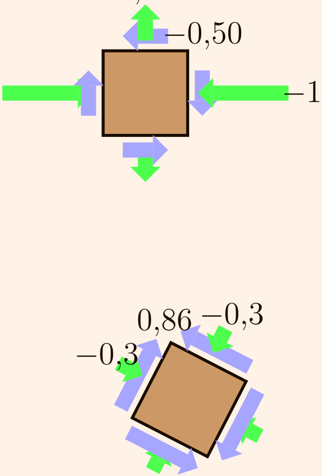


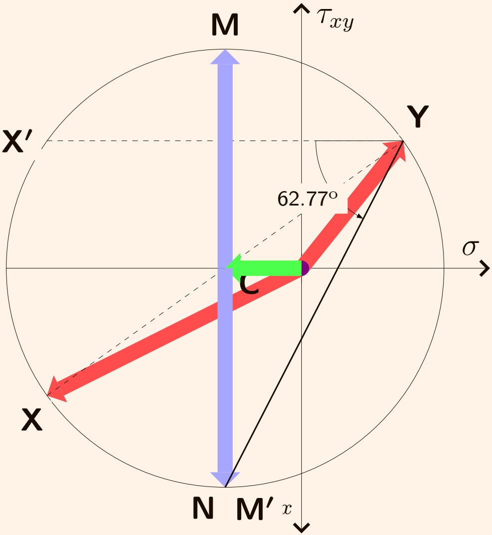
¿$\tau_{\max}$?



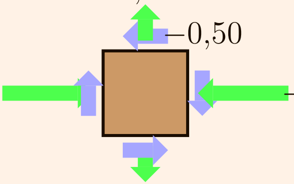


¿$\tau_{\max}$?

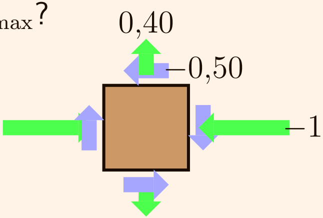




¿$\tau_{\max}$?

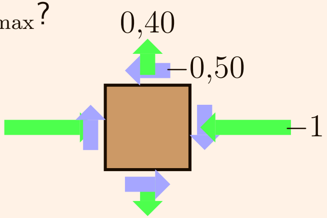


¿$\tau_{\max}$?



$$\text{centro} = \frac{\sigma_x + \sigma_y}{2} = -0{,}3$$

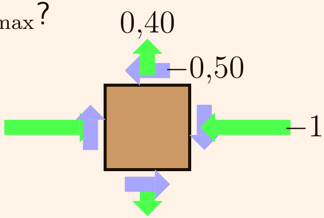
¿$\tau_{\max}$?



$$\text{centro} = \frac{\sigma_x + \sigma_y}{2} = -0{,}3$$

$$\text{radio} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 0{,}86$$

¿$\tau_{\max}$?



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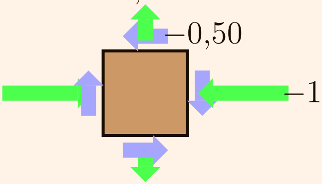
$$\sigma_m = \sigma_n = \text{centro} = -0{,}3$$

$$\tau_{\max} = \text{radio} = 0{,}86$$

¿$\tau_{\max}$?



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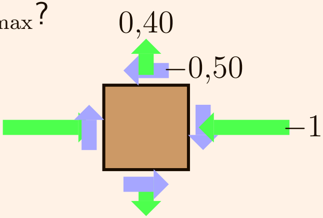
$$\sigma_m = \sigma_n = \text{centro} = -0{,}3$$

$$\tau_{\max} = \text{radio} = 0{,}86$$

$$\beta = 107{,}77^{\text{o}}$$

$$\alpha = \beta \pm 45^{\text{o}} = 62{,}77$$

¿$\tau_{\max}$?



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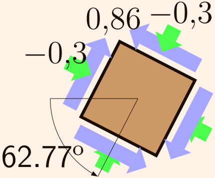

Mecánica de Sólidos y Sistemas Estructurales

Sólido deformable (III)

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