Four-bar linkages

Autores: José Antonio Lozano Ruiz, Christoph Wirth

Mechanisms consist of at least four links 1, 2, 3, 4 with the length l_1, l_2, l_3, l_4, (a, b, c, d) and the joints 12, 23, 34, 41 which connect them (Fig. 1.6). A kinematic pair comprising two contiguous links determines the joint. Lower kinematic pairs or sliding joints are in surface contact (e.g. shaft and bore) (Tab. 1.1), higher pairs in linear (e.g. cam plate and roller) or point contact (e.g. ball on a plate).Table 1.2 shows the most important planar links while Table 1.3 explains the differences between chain, mechanism, motor mechanism.

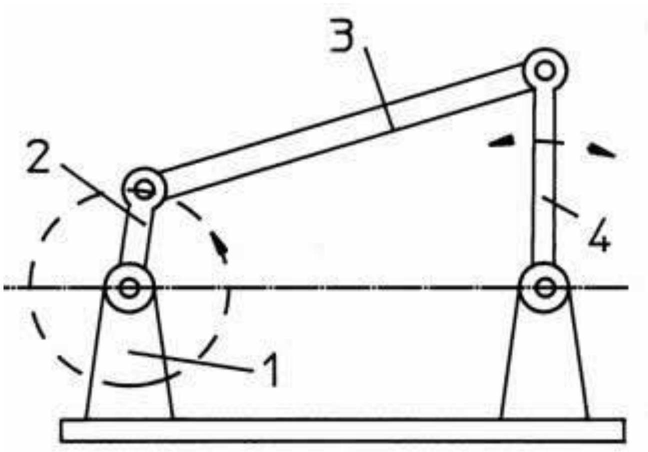


Fig. 1.6 Four-bar linkage (crank-and-rocker) 1 fixed block (housing), 2 crank, 3 coupler (connecting rod), 4 rocker

1. Symbols for joints

Symbol	Sketch of joint	Name of joint
		Turning joint

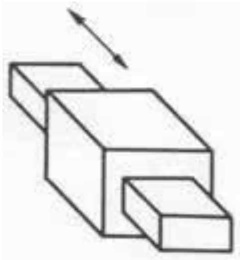
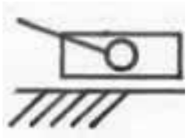
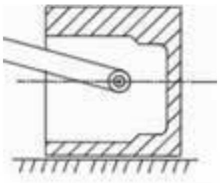
		Sliding joint
		Turning and sliding joint

Table 1.1 Planar joints

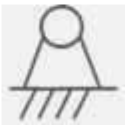
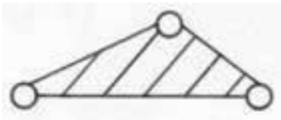
Symbol	Name of link
 , 	Fixed link or supported link
	Binary link with turning elements
	Ternary link with turning elements

Table 1.2 Planar links

2. Chain-mechanism-motor mechanism

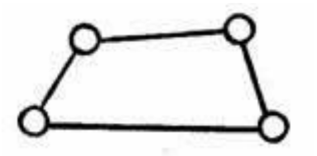
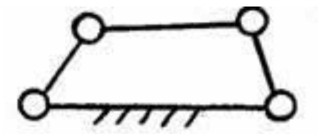
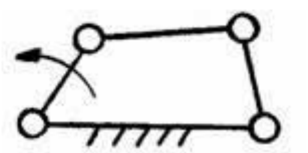
	A kinematic chain is a number of links with different length, which are connected by joints.
	The chain becomes a mechanism when one of the links is defined as the fixed link.
	The mechanism becomes a motor mechanism when one (or more) link are driven.

Table 1.3 Chain – mechanism – motor mechanism

Go to Exercise 1.1

3. Degree of mechanism freedom

The degree of freedom ***F*** of a mechanism is a function of the number ***n*** of links (including the fixed link), the number ***g*** of joints. For planar mechanisms that only have turning- and sliding joints F is given by **Grübler's equation**:

$$F = 3 (n - 1) - 2 g$$

$F = 1$ means forced motion of the mechanism by one drive or input. In general the degree of freedom of a mechanism F indicates the minimum number of drives or input impulses that a mechanism must receive to fulfil a function which is calculable in advance.

4. Capability of rotation for four-bar linkages

Only the shortest link of a four-bar linkage has the capability of rotation (except double crank and double rocker) if **Grashof's criterion** is met:

$$l_{min} + l_{max} \leq l' + l''$$

The sum of the lengths of the shortest and longest link must be less (at most equal) than the sum of the lengths of the other two links.

Go to Exercise 1.2

5. Four-bar turning-pair linkages

Depending on which of the four links is fixed there are four possible combinations of a four-bar linkage, but as two of these are similar only three mechanisms with different motions can be generated.

5.1 Crank-and-rocker (Beam-and-crank)

A crank-and-rocker mechanism (Fig. 1.7 a) converts rotary motion in to oscillating motion (1 fixed).

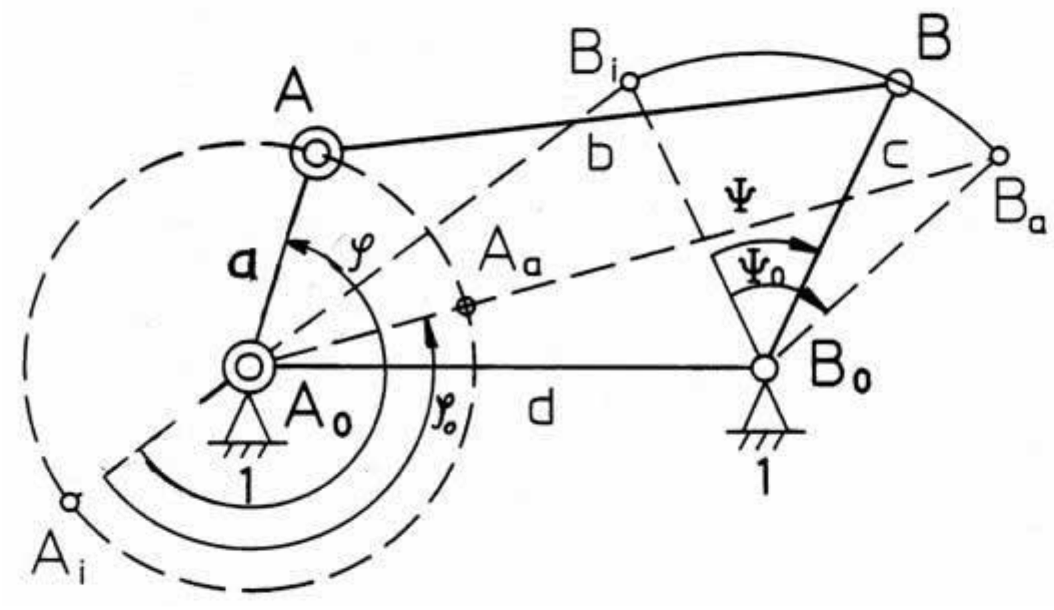


Fig. 1.7 a) Crank-and-rocker (Four-bar linkage) A crankpin, B rockerpin, A_0 crank bearing, B_0 rocker bearing Length of: a crank, b coupler, c rocker, d housing φ_0 crank angle, Ψ_0 oscillating angle

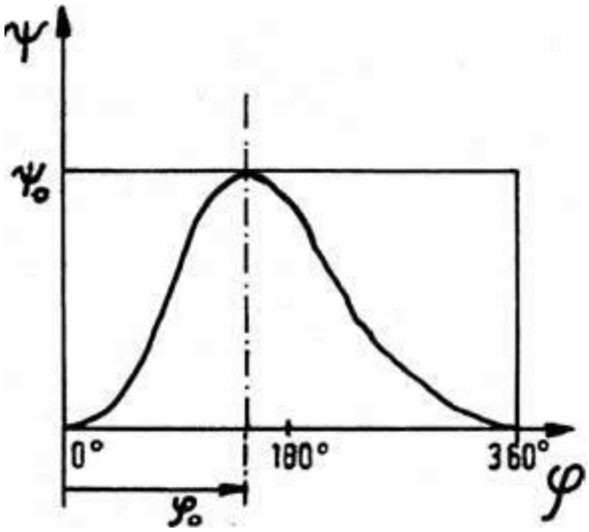


Fig. 1.7 b) The zero-transfer-function $\Psi = f(\varphi)$ of a crank-and-rocker

The **B**ottom **D**ead **C**enter position (**BDC**) can be calculated using the following formula:

$\boldsymbol{B_i A_o = b - a}$

The **T**op **D**ead **C**enter position (**TDC**) is defined as:

$\boldsymbol{B_a A_o = b + a}$

The zero-transfer-function $\Psi = f(\varphi)$ shows Fig. 1.7 b. Crank angle φ_0 is correlated with the oscillating angle Ψ_0 between **TDC**and **BDC**.

Crank-and-rocker mechanisms can be driven by the crank or the rocker (e.g. old sewing-machine).

Go to Exercise 1.3

5.2 Double crank (Drag- link)

If the shortest link (l_{min}) is fixed, you get a double crank (Fig. 1.8 a). One crank drives, the other is driven and both can rotate. As shown in Fig. 1.8 b), the driven crank is in some parts of the motion slower in other parts faster than the driving crank which normally runs at constant speed. The double crank mechanism converts constant rotary motion into a non constant one.

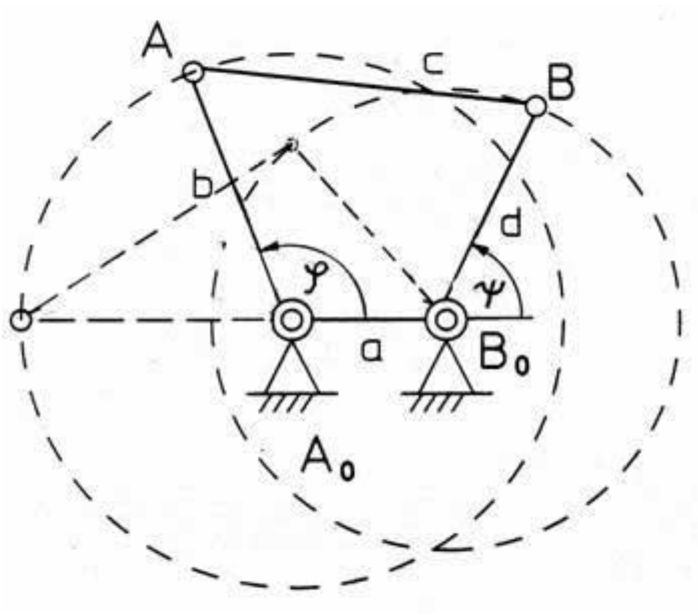


Fig. 1.8 a) Double crank (Four-bar linkage) A, B crankpins; A_0 , B_0 rocker bearings φ, Ψ crank angles

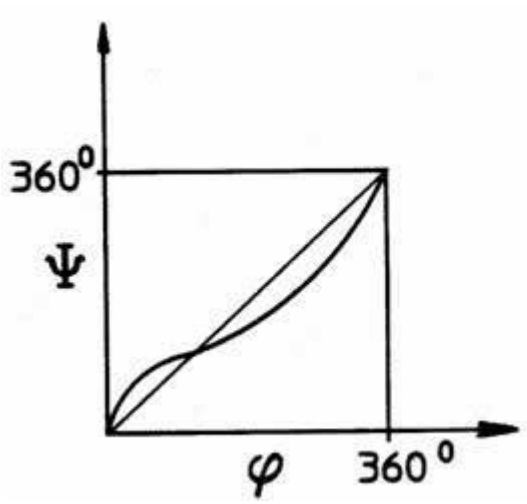


Fig. 1.8 b) The zero-transfer-function Ψ = f(φ) of a double crank

Go to Exercise 1.4

5.3 Double rocker

If the longest link (l_{max}) is fixed (Fig. 1.9) a non-revolving double rocker is obtained. Each rocker has two dead centers. A dead center appears if a rocker and the connecting link form a common line.

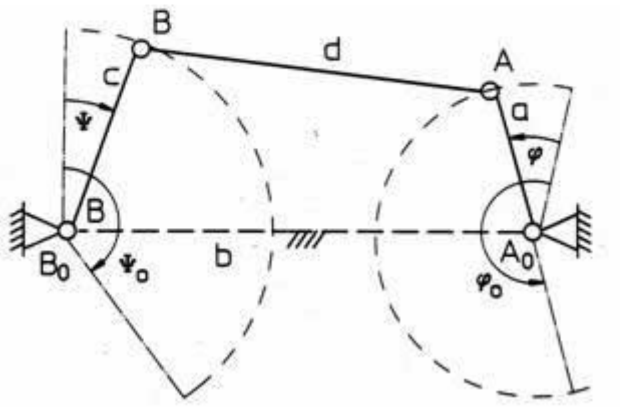


Fig. 1.9 Double rocker (Four-bar linkage) A, B rockerpins ;A_0 , B_0 rocker bearings φ, Ψ rocker angles

6. Coupler curves

The curve that describes any point of the connecting link (coupler) of a mechanism is called coupler curve *k.* This point must not be located between the length AB (Fig. 1.10), it only must be connected to the coupler.

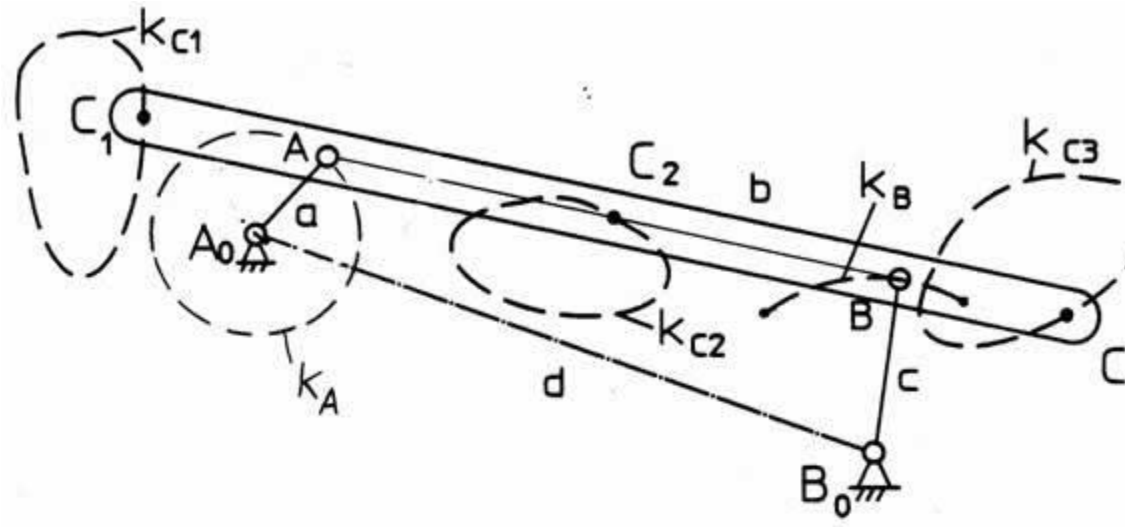


Fig. 1.10 Coupler curves of a four-bar linkage with turning joints

Simple curves are circles and parts of circles (points A , B in Fig. 1.10), other curves are shown for C_1, C_2 and C_3. The shape of the coupler curves depends on the construction of the mechanism, length of the links and coupler point.

Special curves with peaks and loops can be produced to guide workparts or tools.

7. Crank mechanism

If in a crank-and-rocker mechanism (Fig. 1.7) the length of the rocker *c* is infinite, the motion of *B* changes from an arc of a circle to a linear oscillation. In this case, the turning pair in B can be substituted to a sliding pair.

The most important crank mechanism is the slider crank (sliding-block linkage) which is used in piston engines, in presses and in hydraulic and pneumatic drives.

Slider crank (Sliding-block linkage)

The slider crank mechanism (Fig. 1.11) converts the oscillating motion of a piston *c* via the connecting rod *b* into rotary motion of the crank *a*, or vice versa.

The Grashof criterion (capability of rotation) can be defined or checked through the equation:

$$e \leq lmax - lmin ; \qquad (e \leq b - a)$$

The eccentricity (kinematic offset) *e* is the shortest distance between A_o and the centre line.

If the crank is driven a compressor for example is obtained. If the piston is driven a piston engine is received.

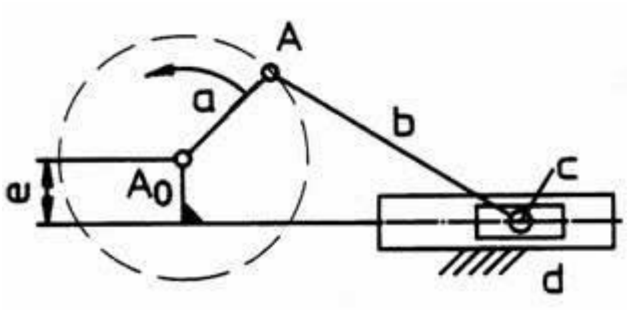


Fig. 1.11 Slider crank mechanism with 1 sliding joint and 3 turning joints

Go to Exercise 1.5