Synthesis of functions

Autores: José Antonio Lozano Ruiz, Christoph Wirth

Synthesis of functions is called to the part of the synthesis of mechanisms that tries to so obtain the dimensions of the bars of a mechanism that allow to certain coordination between the input and output bars of the same one.

The problem consists of by means of a mechanism obtaining a functional relation f:

$$\theta_4 = f(\theta_2)$$

Where:

θ_2 is the parameter which specifies the position of the input bar.

θ_4 is the parameter which specifies the position of the output bar.

This problem of synthesis of functions can be raised of the following ways:

a. *Exact synthesis*:so that the positions of the input and output bars of the mechanism fulfil a certain functional relation.

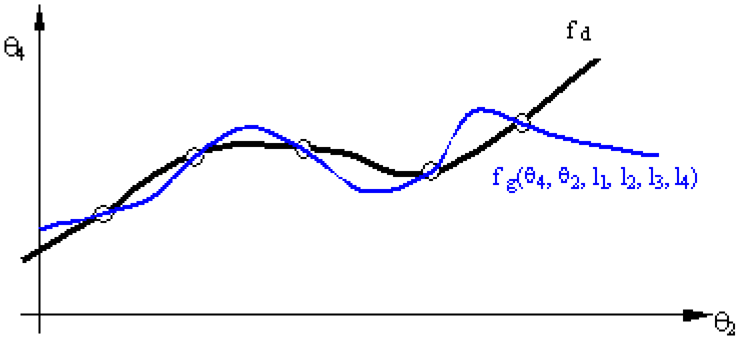
b. *Synthesis with precision points*: so that the mechanism is able to locate the input and output bars in two groups of specified positions.
c. *Synthesis for velocities and accelerations*: so that the speeds and accelerations of the input and output bars fulfil a certain correlation.

1. Synthesis with precision points

It is desired to obtain the dimensions of a mechanism, so that the positions of input and output bars agree with a finite number of points, called "*precision points*" of a certain functional relation f_d:

$$\theta_4 = f_d(\theta_2)$$

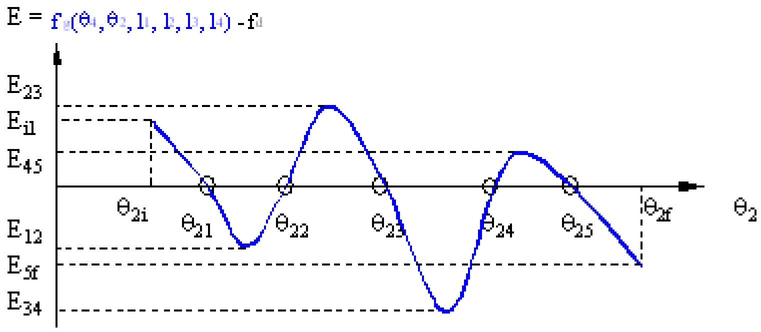
Where θ_2 and θ_4 are parameters that define the position of input and output bars of the mechanism, respectively.



Precision points en the synthesis of mechanisms.

Between the precision points certain errors known like structural errors are committed.The difference between the wished function f_d and the generated function f_g is called the *structural error function*.The first step of the problem of synthesis with precision points is to find the precision points that decrease the structural error:

Optimal spaced of Chebyshev



Structural error function.

FUNDAMENTAL THEOREM OF CHEBYSHEV:"If n independent parameters are involved in the design of a mechanism that generates a function that approximates to a wished function, then the greater absolute value of the structural error is decreased when there is n spaced points of precision on the way that the maximum values of n+1 errors between each pair of adjacent and terminal points are numerically equal, when sing changes".

The n points of precision for the synthesis of the mechanism that generates the approximate function f_g to the wished function f_d are:

$$\theta_{2j} = \frac{1}{2}(\theta_{2f} + \theta_{2i}) - \frac{1}{2}(\theta_{2f} - \theta_{2i})\cos\frac{\pi(2j-1)}{2n}$$

Where θ_{2j} belongs to the gap (θ_{2i} , θ_{2f}), for j=1,2,...,n.

A graphical construction that verifies the previous equation is the one that appears in the following figure

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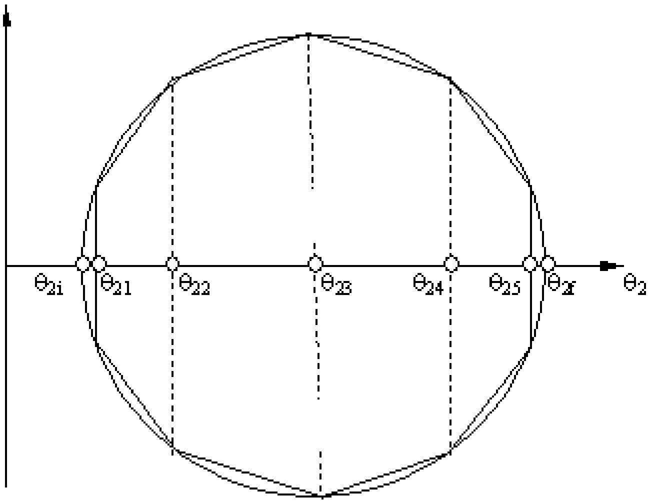


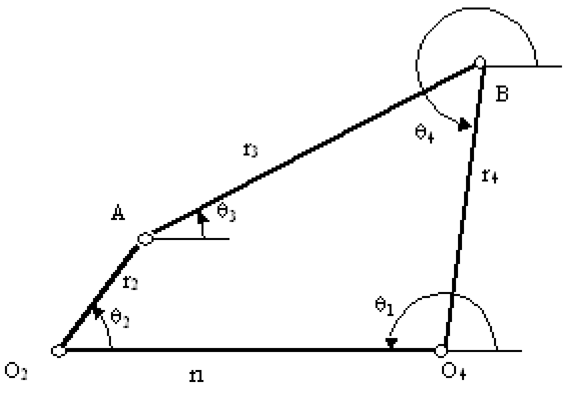
Synthesis of Bloch.

Once selected the precision points for the generation of the wished function fd, we will have a table of values to the precision points:

Precision points for the synthesis of the mechanism.

$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_4}$
θ_{2i}	θ_{4i}
θ_{21}	θ_{42}
θ_{22}	θ_{42}
...	...
θ_{2f}	θ_{2f}

The *Synthesis of Bloch*, consists simply of representing the bars of the mechanism by means of position vectors, all concatenated so that the end of a vector is the origin of the following one. The sum of all those vectors of position, for a same mechanism, must add zero at any moment.



Four bars mechanism by vector representation.

For the four bars mechanism of the previous figure, we can write the following vector equation:

$$\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 = 0$$

This equation is called ***loop equation*** of the mechanism, and it will be fulfilled for each one of the points of precision of the function that is desired to generate.

In polar complex notation, it follows that:

$$r_1 e^{j\theta_1} + r_2 e^{j\theta_2} + r_3 e^{j\theta_3} + r_4 e^{j\theta_4} = 0$$

In rectangular complex form and separating the real and imaginary components:

$$r_1 \cos\theta_1 + r_2 \cos\theta_2 + r_3 \cos\theta_3 + r_4 \cos\theta_4 = 0$$
$$r_1 \operatorname{sen}\theta_1 + r_2 \operatorname{sen}\theta_2 + r_3 \operatorname{sen}\theta_3 + r_4 \operatorname{sen}\theta_4 = 0$$

As sin θ_1=0 and cos θ_1= -1, clearing the terms associated to the third bar and squaring, becomes:

$$r_3^2 \cos^2\theta_3 = (r_1 - r_2\cos\theta_2 - r_4\cos\theta_4)^2 \quad r_3^2 \operatorname{sen}^2\theta_3 = (-r_2 \operatorname{sen}\theta_2 - r_4 \operatorname{sen}\theta_4)^2$$

Adding the two previous equations, developing the squared parentheses and adding the resulting terms, it follows that:

$$r_3^2 = r_1^2 + r_2^2 + r_4^2 - 2r_1r_2\cos\theta_2 - 2r_1r_4\cos\theta_4 + 2r_2r_4(\cos\theta_2\cos\theta_4 + \operatorname{sen}\theta_2 \operatorname{sen}\theta_4)$$

Replacing (cos $\theta_2$cos θ_4 + sin θ_2 sin θ_4)=cos(θ_2-θ_4), dividing by $2r_2r_4$ and rearranging, it follows that:

$$\frac{r_3^2 - r_1^2 - r_2^2 - r_4^2}{2r_2r_4} + \frac{r_1}{r_4}\cos\theta_2 + \frac{r_1}{r_2}\cos\theta_4 = \cos(\theta_2 - \theta_4)$$

Doing:

$$K_1 = \frac{r_1}{r_4} \quad K_2 = \frac{r_1}{r_2} \quad K_3 = \frac{r_3^2 - r_1^2 - r_2^2 - r_4^2}{2r_2r_4}$$

Finally becomes:

$$K_1\cos\theta_2 + K_2\cos\theta_4 + K_3 = \cos(\theta_2 - \theta_4)$$

This is called ***Equation of Freudenstein***.Specifying three precision points, a system of three equations with three unknowns K1, K2 and K3 is obtained:

$$K_1 \cos\theta_{21} + K_2 \cos\theta_{41} + K_3 = \cos(\theta_{21} - \theta_{41})$$
$$K_1 \cos\theta_{22} + K_2 \cos\theta_{42} + K_3 = \cos(\theta_{22} - \theta_{42})$$
$$K_1 \cos\theta_{23} + K_2 \cos\theta_{43} + K_3 = \cos(\theta_{23} - \theta_{43})$$

Calculating the values of K1, K2 and K3, the length of a bar is chosen, for example r1, the length of the three remaining bars of a four bars mechanism are obtained.

Go to Exercise 1