Synthesis of positions

Autores: José Antonio Lozano Ruiz, Christoph Wirth

The *synthesis of guidance of the rigid solid*, or simply *synthesis of guiding,* also is called *synthesis of positions*, tries the problem to locate a bar of a mechanism in specified positions including their situation and direction.The problem of the synthesis of positions has a high level of complexity than the other of the synthesis of trajectories.If in the guiding of the rigid body the direction is not specified, it is understood that the problem is reduced to a generation of trajectory.On the same way to the exposition of other kinds of synthesis, the methods that can be used depend of the demanded precision level and its complexity.Thus it is distinguished:

a. *Synthesis of exact guidance of the rigid solid,* where all a trajectory is specified and the direction of the body in each point of the trajectory of continuous form, and it is desired to obtain from exact form.

b. *Synthesis of approximated guidance of rigid solid*, starting off of the data of case a), is only tried to synthesise a mechanism that allows to obtain the trajectory and direction of the body of approximated way, admitting certain errors.
c. *Synthesis of guidance of rigid solid with precision positions.* In this case, only a mechanism which allows to locate the rigid solid in a few positions is tried to find.

1. Synthesis of guidance of Ridid body with three precision positions

This it is the simpler problem that it is possible to be raised in the synthesis of the rigid solid, that consist of finding the mechanism that allows to place one of it bars in three specified positions, including the orientation respect to a certain reference system. For example, the mechanism that allows to place a solid AB in three specified positions A_1B_1, A_2B_2, A_3B_3, (see figure 6.1.).

A mechanism that allows to obtain those three positions is the four bars mechanism using the coupler bar to generate these positions. Since by three points always a circumference, the central points A_0 and B_0 can be drawn up, (see figure 6,1), of the circumferences that pass through $A_1A_2A_3$ and $B_1B_2B_3$, they can be used as the fixed joints of the four bars mechanism A_0ABB_0.

As this synthesis has been considered, associating the movable joints of the four bars mechanism to the limits of solid AB, the solution is alone.In other words, we have fixed the form of the coupler bar exactly equal to the one of the solid that we want to locate. But this must not be thus.

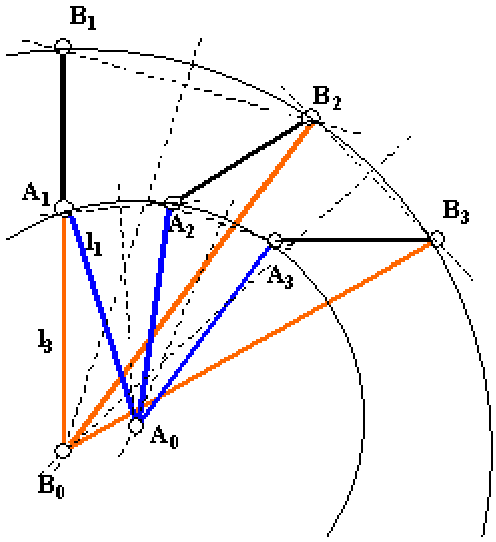

Figure 6.1. Precision positions. Four bars mechanism A_0ABB_0.

Let us suppose that the obtained solution does not satisfy us. Another four bars mechanism with a coupler bar reaching the same positions of solid AB, can be obtained. But the coupler bar has not the form of the solid AB. That is illustrated in figure 6.2, in which the synthesis of another four bars mechanisms A_0ACC_0 is allowed, that generates the same positions of given solid AB in figure 6,1, but using the coupler bar AC.

A graphical method to solve the synthesis of guidance of the rigid solid with three positions of precision is informed by itself in figures 6.1. and 6.2, with no need of greater explanations.

The analytical methods need to raise the equations of a circumference based on the position of their centre and the radius of circumference:

$$(x - x_0)^2 + (y - y_0)^2 = r^2 \qquad \text{Ec.6.1.}$$

Where (x,y) is the position of any point of the circumference. (x_0, y_0) is the centre of the circumference. And r is the radius of the same one.

Having three precision positions for the synthesis of guidance of the rigid solid, by means of three positions $A_1A_2A_3$ and $B_1B_2B_3$ of two points A and B of the solid, (see figure 6,1).

For A_1,A_2 and A_3, (figure 6,1), using expression 6.1., three equations can be raised:

$$(x_{A1} - x_{A0})^2 + (y_{A1} - y_{A0})^2 = r^2$$
$$(x_{A2} - x_{A0})^2 + (y_{A2} - y_{A0})^2 = r^2$$
$$(x_{A2} - x_{A0})^2 + (y_{A2} - y_{A0})^2 = r^2 \qquad \text{Ecs.6.2.}$$

Solving this system of three equations 6.2, their three unknowns can be obtained. (x_{A0}, y_{A0}) is the position of the fixed joint A_0 of the four bars mechanism, and r_A is the radius of the circumference that passes through the three A_i points, that agree with the length of the movable bar of the mechanism l_2.

By the same way, using the points B_1, B_2 and B_3 of figure 6,1, (or C_1, C_2 and C_3 of figure 6,2), can be operated to obtain the fixed joint B_0 (or C_0 in figure 6,2.); as well as the length of the bar l_3 of the four bar mechanism.And with this it would be concluded the synthesis of guidance of solid AB with three precision positions.

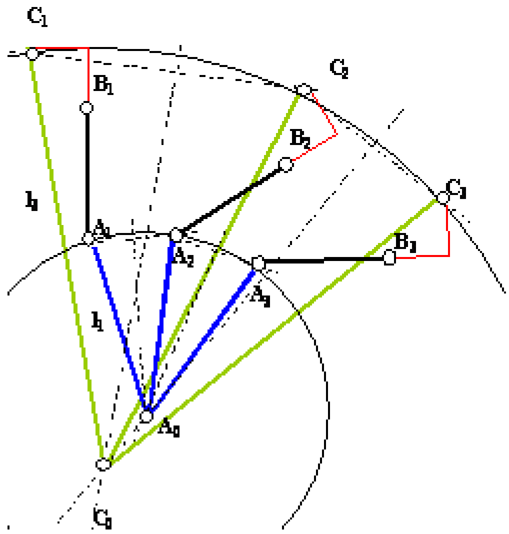

Figure 6.2. Precision Positions. Four bars mechanism A_0ACC_0.

2. Guidance of Rigid solid with more than three precision positions

The procedure of synthesis exposed in the previous section is based on the property of which when a rigid solid occupies three arbitrary positions on a plane, a circumference always exists that passes through the three positions A_1, A_2 and A_3 of each point A of the solid.When solid saying occupies four arbitrary positions on a plane, it does not exist, in general, a circumference that passes through the four positions, A_1, A_2, A_3 and A_4 of each point A of the solid.Only special points of the solid have the property that, for four positions of this one, they are located on a circumference.These special points will solve the synthesis of guidance of the rigid solid for four precision positions.Finding two of those special points, we can use them like movable joints of a four bars mechanism, and the centres of the two circumferences that describe those two selected special points, can be used like the fixed joints of the same four bars mechanism.

Curve of circular points is the geometric place of the points of a movable plane, which are on a circumference, for four positions of the plane.***Curve of centres*** is called to the geometric place of the centres of the circumferences that contain the four positions A_1, A_2, A_3 and A_4 of each point A of the curve of circular points.

For four given positions of a plane, (or solid), with flat movement, the curves of circular points and centres are unique and both are known like ***curves of Burmester.***

Equation of the Curves of Burmester according Kaufman

Having four positions in a rigid solid, defined by A_1B_1, A_2B_2, A_3B_3 and A_4B_4, and the $\theta_{12}\theta_{13}\theta_{14}$ turning angles of the solid between the first position and the others, (see figure 6,3). Considering K_1 a point of the curve of circular points, in common to the first position, and M the corresponding centre of the circumference that passes through K_1, K_2, K_3 and K_4. (K_2, K_3 and K_4 are the corresponding positions of K_1 to the positions of A_22B_2, A_3B_3 and A_44B_4). From figure 6.3., it is possible to be obtained:

$$r_j - r_1 = \delta_{1j} = w_j - w_1 + z_j - z_1 \qquad \text{Ec.6.3.}$$

Or, which is equivalent

$$(e^{i\theta 1j} - 1)z_1 + (e^{i\gamma 1j} - 1)w_1 = \delta_{1j} \quad j=2,3,4. \qquad \text{Ec.6.4.}$$

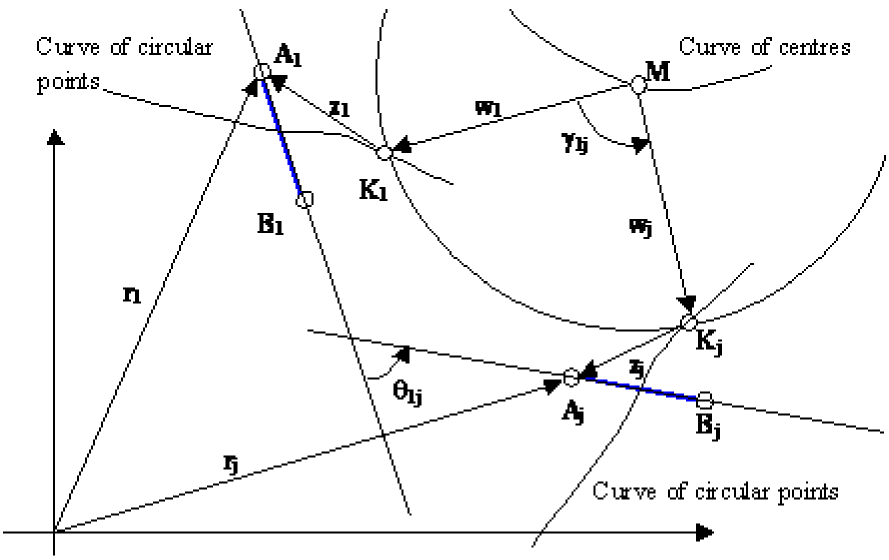

Figure 6.3. Curves of Burmester.

Having four positions, three complex equations are obtained with the unknowns z_1, w_1 y γ_{12}, γ_{13} y γ_{14}. Considering like unknowns only z_1 y w_1, the determinant of the matrix of coefficients and independence terms must be zero in order the resultant linear system of equations had a solution.

$$\begin{vmatrix} e^{i\theta 12} - 1 & e^{i\gamma 12} - 1 & \delta_{12} \\ e^{i\theta 13} - 1 & e^{i\gamma 13} - 1 & \delta_{13} \\ e^{i\theta 14} - 1 & e^{i\gamma 14} - 1 & \delta_{14} \end{vmatrix} = 0 \qquad \text{Ec.6.5.}$$

Equation 6.5, means the condition of compatibility of the system and is a complex equation, equivalent to two simple equations with three unknowns. If a value to one of these unknowns is given, for example γ_{12}, then γ_{13} y γ_{14} can be calculated.

After calculated γ_{1j}, also the K1 point of the curve of circular can be calculated, (figure 6,3), by means of:

$$\overline{OK_1} = r_1 - z_1 \qquad \text{Ec.6.6.}$$

And point M of the curve of centres, (figure 6,3), is calculated by:

$$\overline{OM_1} = r_1 - z_1 - w_1 \qquad \text{Ec.6.7.}$$

Varying γ_{12} from 0 to 2π others points of the curves of Burmester can be found.

Go to Exercise 3