Synthesis of trajectories

Autores: José Antonio Lozano Ruiz, Christoph Wirth

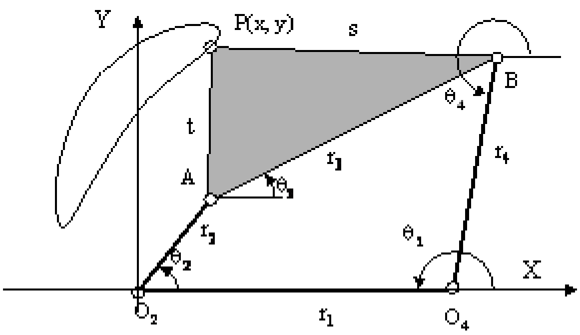
Part of the synthesis of mechanisms that studies if the trajectories described by points pertaining to the bars of a mechanism, during the movement of this one, fit with other specified trajectories.Depending on the requirements the following problems can be considered:

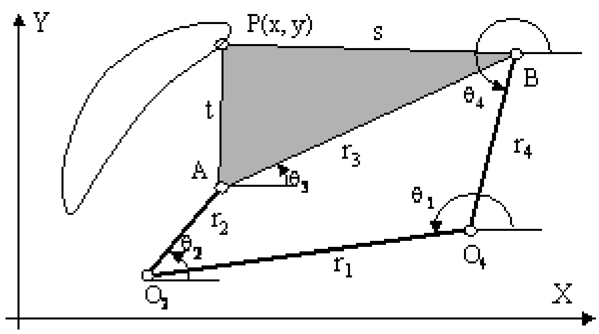
- Generation of a trajectory exactly.
- Generation of a trajectory " approximately ".
- That a point of a bar passes, during the motion of the mechanism, by a fixed number of precision points belonging to a given trajectory.
- Generation of special trajectories, such as trajectories with double points, points of backward movement, symmetrical respect to an axis, with almost circular sections, almost rectilinear sections...;

1. ***Study of the trajectory of coupler of the four bars mechanism***

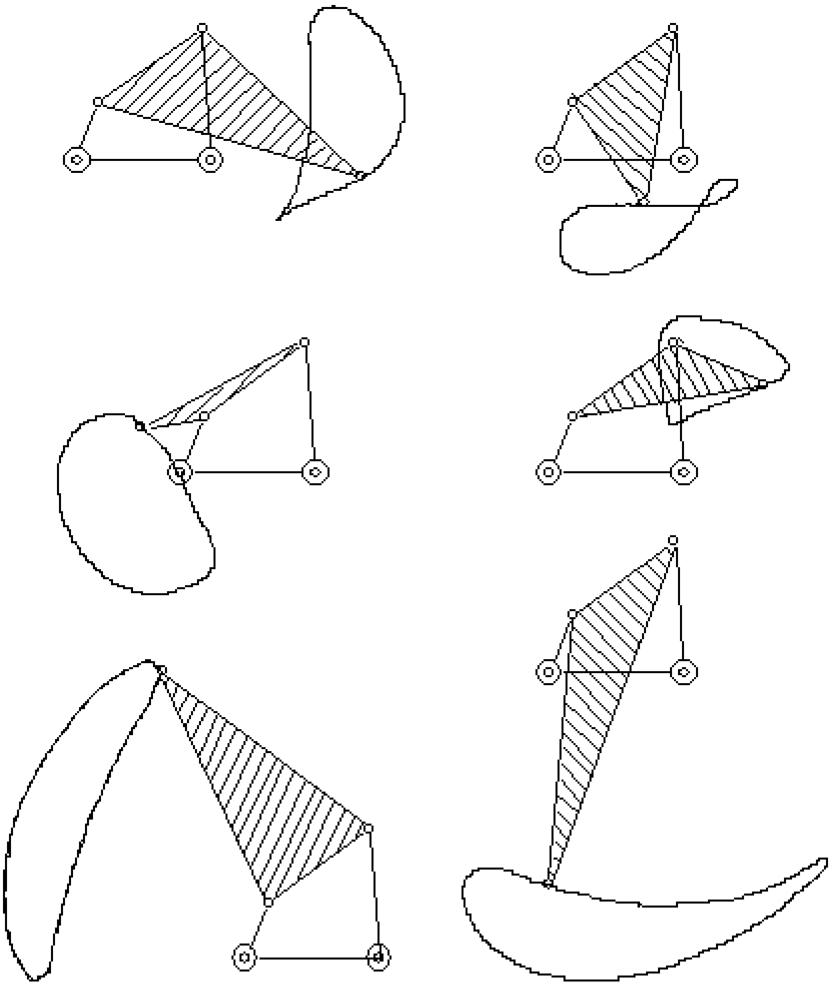
$G(x, y, r1, r2, r3, r4, s, t) = 0$

$G(x, y, r1, r2, r3, r4, s, t, x'O2, y'O2, y'O4) = 0$

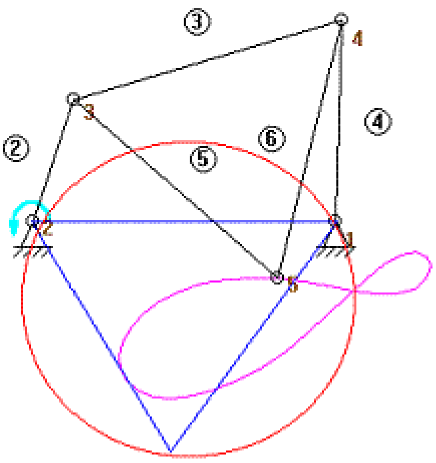




Kinds of trajectories of four bars mechanism.



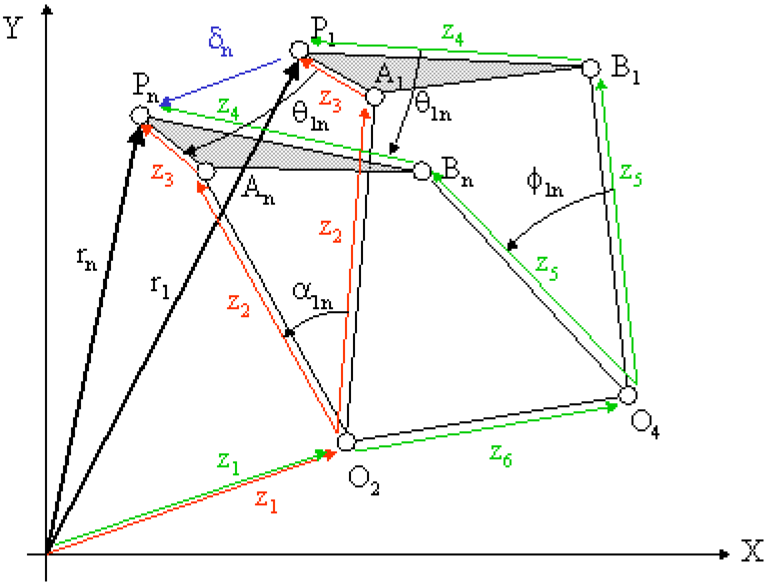
Circumference of focus



2. Generation of trajectories with three precision points by means of the complex numbers method

Considering two positions 1 and n of the mechanism, the position vectors of a coupler point by two ways can be defined:

$r_1 = z_1 + z_2 + z_3$
$r_1 = z_1 + z_6 + z_5 + z_4$
$r_n = z_1 + z_2 eia1n + z_3 eiq1n$
$r_n = z_1 + z_6 + z_5 eif1n + z_4 eiq1n$



The position of point n respect to point 1, can be defined by the vectors:

$\delta_n = r_n - r_1$
$\delta_n = z_2(e^{i\alpha 1n} - 1) + z_3(e^{i\theta 1n} - 1)$
$\delta_n = z_5(e^{i\Phi 1n} - 1) + z_4(e^{i\theta 1n} - 1)$

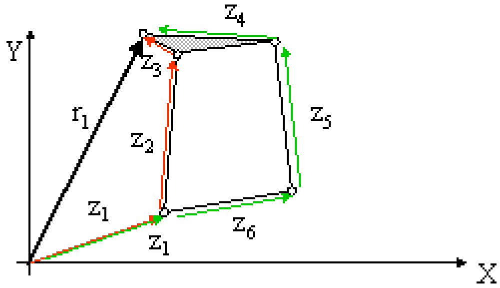
Having three points 1, 2 and 3 by where it is desired that the coupler point of the mechanism passes, from point 1 to 2 and from point 1 to 3, the following vectors can be defined:

$\delta_2 = z_2(e^{i\alpha 12} - 1) + z_3(e^{i\theta 12} - 1)$
$\delta_3 = z_2(e^{i\alpha 13} - 1) + z_3(e^{i\theta 13} - 1)$
$\delta_2 = z_5(e^{i\Phi 12} - 1) + z_4(e^{i\theta 12} - 1)$
$\delta_3 = z_5(e^{i\Phi 13} - 1) + z_4(e^{i\theta 13} - 1)$

The previous system of 4 equations with 4 unknowns allows to calculate the dimensions of the four bars mechanism and the coupler point which passes through the three specified points 1, 2 and 3:

$$\left.\begin{array}{l}\delta_2 = z_2(e^{i\alpha 12} - 1) + z_3(e^{i\theta 12} - 1)\\ \delta_3 = z_2(e^{i\alpha 13} - 1) + z_3(e^{i\theta 13} - 1)\end{array}\right\} \Longrightarrow z_2, z_3 \Downarrow z_1 = r_1 - z_2 - z_3$$

$$\left.\begin{array}{l}\delta_2 = z_5(e^{i\phi 12} - 1) + z_4(e^{i\theta 12} - 1)\\ \delta_3 = z_5(e^{i\phi 13} - 1) + z_4(e^{i\theta 13} - 1)\end{array}\right\} \Longrightarrow z_4, z_5 \Downarrow z_6 = r_1 + z_1 - z_5 - z_4$$



Go to Exercise 2