EXERCICE 1

Design of the steering system of an automobile, as it is shown in the figure.

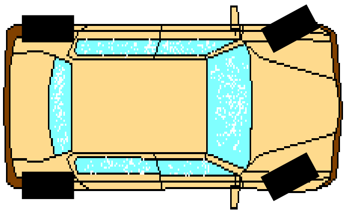


Figure E1.1.- Design of the steering system of an automobile.

Solution:

a) First requirements:

- Frontal steering system.
- Null yaw tyre angles .

b) Kinematic requirements:

- As figure E.2. shows, the correct motion of the car forces to $\delta i \neq \delta e$.

The ratio between δi and δe is the known "Proportion of Akerman":

$$Cot(\delta_i) - Cot(\delta_e) = \frac{b}{L}$$

(ec.E1.1)

Where "b" is the wheel track of the car, and "L" is the wheelbase.

For an average car:

b = 1,74 m. ; L = 3,67 m.

Therefore:

$$Cot(\delta i) - Cot(\delta e) = 0,47411$$

(ec.E1.2)

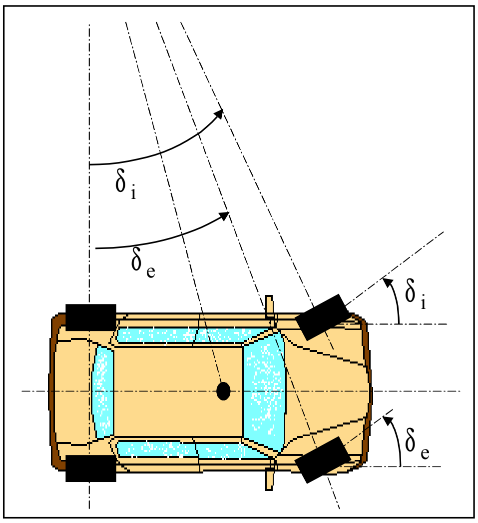


Figure E1.2.- Kinematic requirement for the right operation of the steering system.

c) Precision points.

We consider a car, where usually the steering angle takes values into the interval of ±20º. Because steering system has a symmetrical motion, in order to start the synthesis of the mechanism it is considered an interval of values between 0º to 20º for the internal angle δi.

Now the *Spaced of Chesbyshev* is applied, (see fig. E1.3 and E1.4):

$$\delta_{ij} = \frac{1}{2}(\delta_{if} + \delta_{ii}) - \frac{1}{2}(\delta_{if} - \delta_{ii}) \cos \frac{\pi(2j-1)}{2n}$$

With three precision points, the results for the internal steering angle are:

δ_{i1} = 1.339745 º = 0.023382961 rad.
δ_{i2} = 9.999993 º = 0.1745328 rad.
δ_{i3} = 18.66024 º = 0.325682639 rad.

For the external steering angle, applying the Proportion of Akerman, (ec.E1.2), the results are:

δ_{e1} = 1.325058 º = 0.023126621 rad.
δ_{e2} = 9.242352 º = 0.161309478 rad.
δ_{e3} = 16.23025 º = 0.283271304 rad.

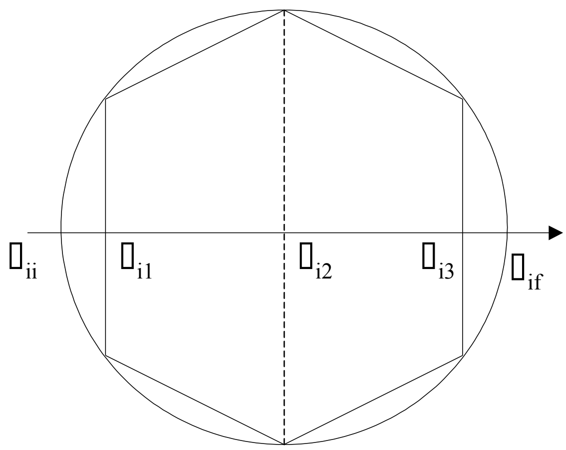


Figure E1.3.- Spaced of *Chesbyshev* with three precision points.

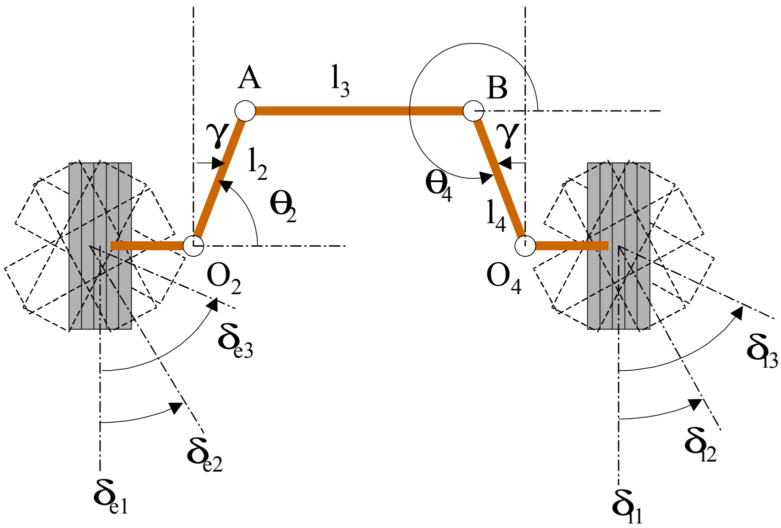


Figure E1.4.- Steering system scheme.

d) Synthesis de Bloch and Equation of Freudenstein.

In order to do the synthesis of the mechanism for the steering system, the angles θ_2 , θ_4 are used, (see fig. E1.4 and E1.5):

$\theta_2 = \pi/4 - \gamma + \delta_e$	$\theta_4 = 3\pi/4 + \gamma + \delta_i$
$\theta_{21} = \pi/4 - \gamma + \delta_{e1} = 1.245$	$\theta_{41} = 3\pi/4 + \gamma + \delta_{i1} = 5.085$
$\theta_{22} = \pi/4 - \gamma + \delta_{e2} = 1.383$	$\theta_{42} = 3\pi/4 + \gamma + \delta_{i2} = 5.236$
$\theta_{23} = \pi/4 - \gamma + \delta_{e3} = 1.505$	$\theta_{43} = 3\pi/4 + \gamma + \delta_{i3} = 5.387$

Where angle γ takes the value, $\gamma = 0.34906585$ rad. = 20 º.

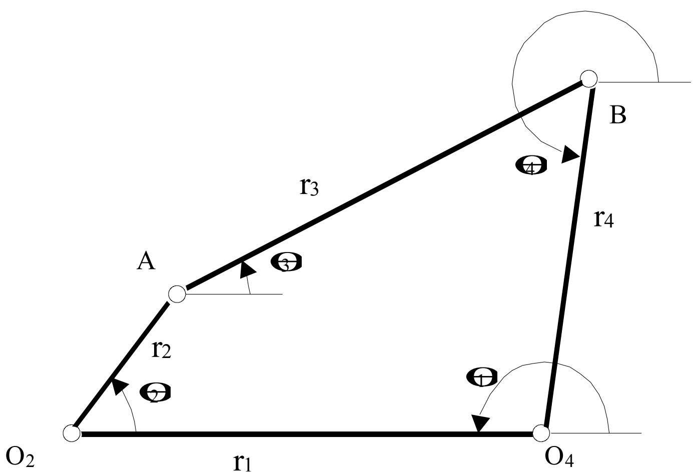


Figure E1.5.- Scheme of the mechanism.

Now is able to apply the Synthesis of Bolch, and Equation of Freudenstein.

$$K_1 \cos\theta_2 + K_2 \cos\theta_4 + K_3 = \cos(\theta_2 - \theta_4)$$

$$K_1 = \frac{r_1}{r_4}\ ;\ K_2 = \frac{r_1}{r_2}\ ;\ K_3 = \frac{r_3^2 - r_1^2 - r_2^2 - r_4^2}{2r_2r_4}$$

Using the before values of θ_{2j}, θ_{4j}, a set of three equations with three unknowns K_1, K_2 and K_3 is obtained:

$$K_1 \cos\theta_{21} + K_2 \cos\theta_{41} + K_3 = \cos(\theta_{21} - \theta_{41})$$
$$K_1 \cos\theta_{22} + K_2 \cos\theta_{42} + K_3 = \cos(\theta_{22} - \theta_{42})$$
$$K_1 \cos\theta_{23} + K_2 \cos\theta_{43} + K_3 = \cos(\theta_{23} - \theta_{43})$$

Calculating the values of K_1, K_2 and K_3:

$$K_1 = 8.062$$
$$K_2 = 7.973$$
$$K_3 = -6.249$$

The length of a bar 1 is chosen:

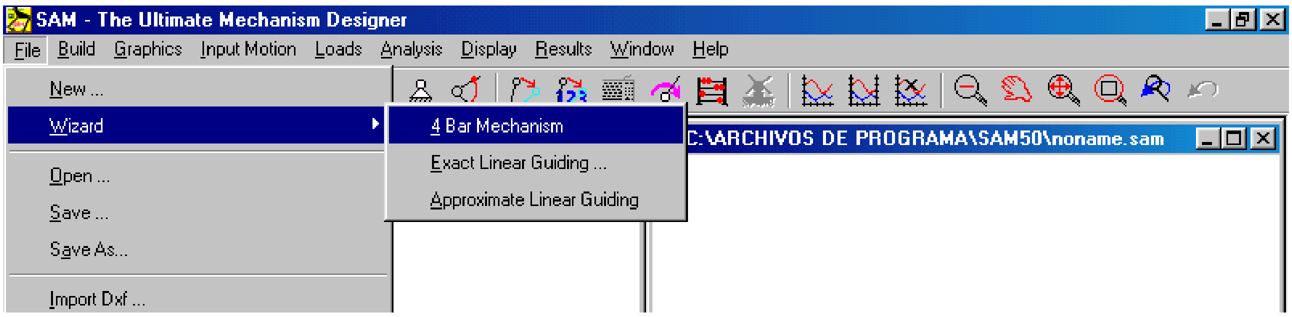
$$\mathbf{r_1 = 1\ m.}$$

Then the length of the three remaining bars of a four bars mechanism are obtained:

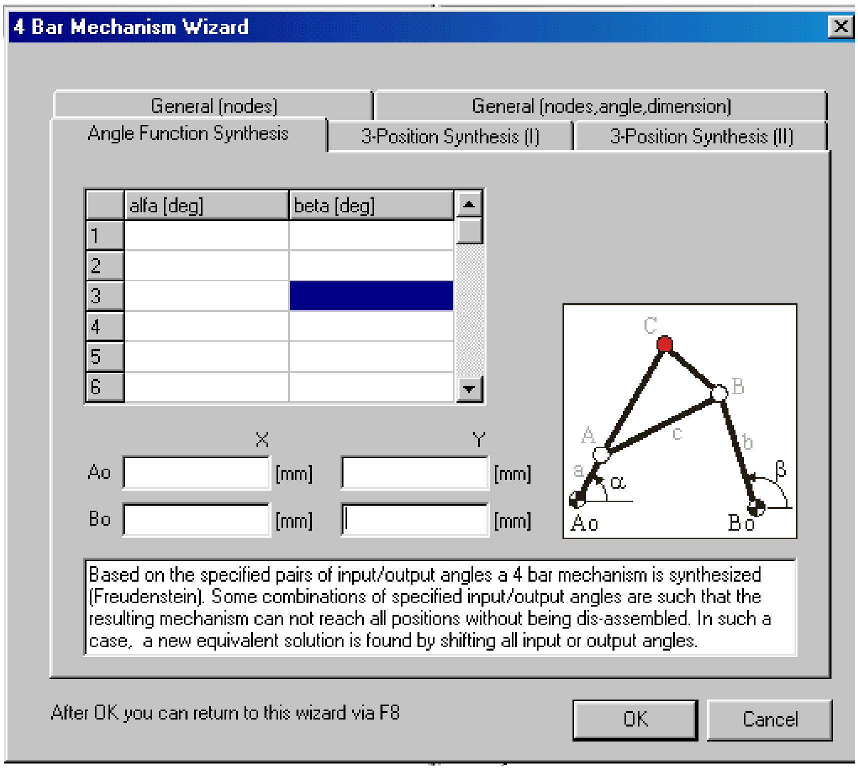
$$\mathbf{r_2 = 0.125\ m.}$$
$$\mathbf{r_3 = 0.125\ m.}$$
$$\mathbf{r_4 = 0.837\ m.}$$

e) Synthesis of the mechanism using the computer program SAM PC v5.0:

We can test the before solution using computer programs, as SAM PC v5.0. This computer program provides a *"Wizard"* menu that can be used in order to apply synthesis of functions:



We choose the “4 Bar Mechanism” option. Then the following form appears:



Next step is to introduce the pairs of angles (α, β), and the coordinates of A_0 and B_0,

As it is considered in the section f) of this exercise, for the synthesis we have used the angles θ_2 , θ_4, (see figure E1.5.) This implies:

$$\alpha = \theta_2$$

$$\beta = \pi - (2\pi - \theta_4) = \theta_4 - \pi$$

$$\alpha_1 = 71.3332603\ ^\circ\ ; \quad \beta_1 = 111.349099\ ^\circ$$

$$\alpha_2 = 79.2400796\ ^\circ\ ; \quad \beta_2 = 120.000764\ ^\circ$$

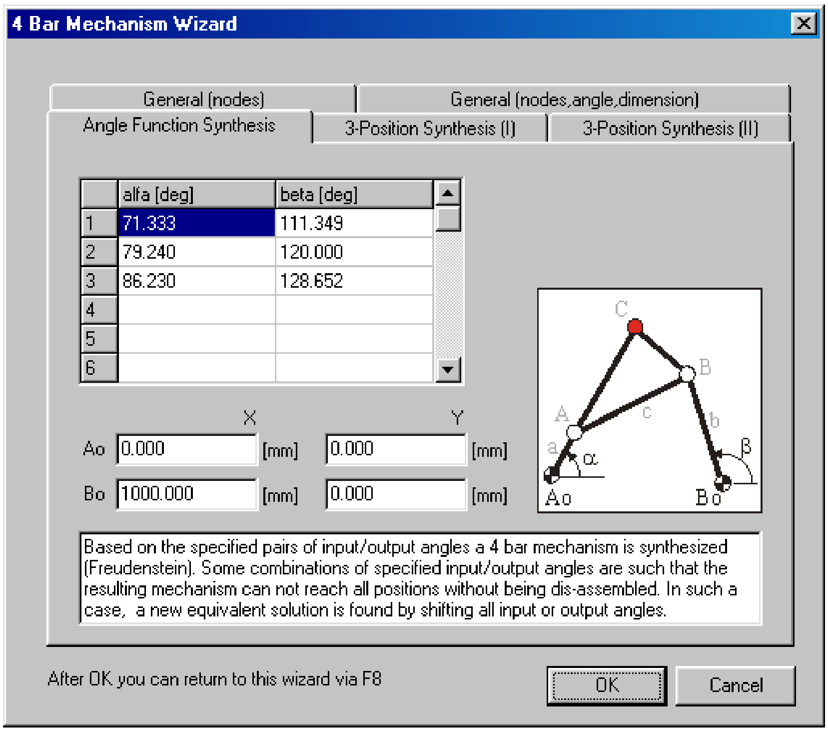
$$\alpha_3 = 86.2301661\ ^\circ\ ; \quad \beta_3 = 128.652428\ ^\circ$$

Also, in section f), we have chosen, r_1 = 1 m. Consistently, we can adjust the following coordinates of A_0 and B_0:

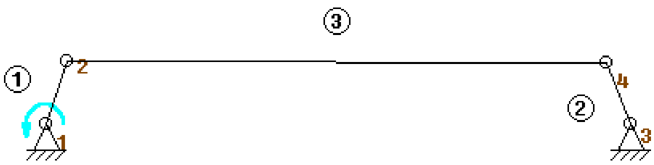
A_0 (0, 0) mm.

B_0 (1000, 0) mm.

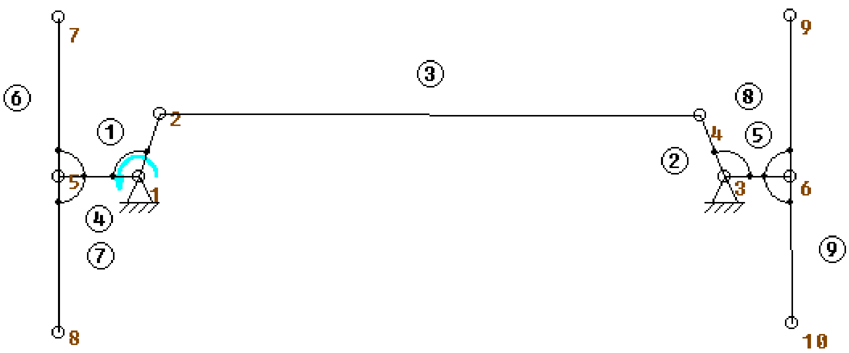
Now, we introduce all the before information in the form, as follows:



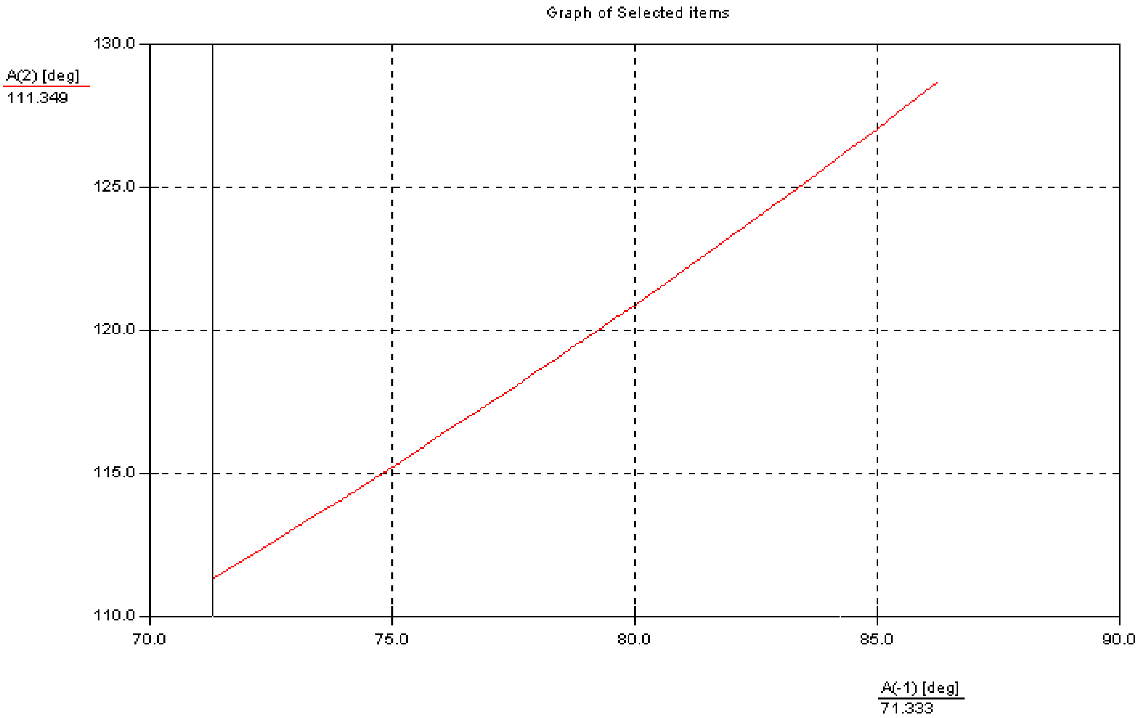
Then "O.K." is stroke, and it appears the solution:



We can complete the mechanism in order to design it more realistic than before:



The next figure shows the relation between both angles α and β



Graph of Selected items
A(2) [deg]
111.349
130.0
125.0
120.0
115.0
110.0
70.0
75.0
80.0
85.0
90.0
A(-1) [deg]
71.333