

# Neumann formulation review for non-zero thickness airfoils

M<sup>a</sup> Victoria Lapuerta González y Ana Laverón Simavilla

The equation for the velocity potential which is obtained with Dirichlet formulation for constant potential is:

$$\Phi_k = U_\infty (x_{PCk} \cos \alpha + z_{PCk} \sin \alpha) + \sum_{j=1}^N \frac{\Phi_j}{2\pi} \beta_k^j + \frac{\Phi_N - \Phi_1}{2\pi} \left( \frac{\pi}{2} \operatorname{sgn}(v_k^{N+1}) + \beta_{kl}^{N+1} \right)$$

and it can be rewritten also as

$$\Phi_k(\vec{x}_k) = \sum_{j=1}^N \frac{\Phi_j}{2\pi} (\theta_{kF}^j - \theta_{kI}^j) + \frac{\Gamma}{2\pi} \theta_k^\infty + \Phi_\infty(\vec{x}_k), \quad (1)$$

where  $\Gamma = \Phi_N - \Phi_1$ , as it is shown in Figure 1,  $\theta_{kI}^j$  is the angle between the line which join the collocation point  $\vec{x}_k$  and the initial node of  $j$  panel with the layer panel, and  $\theta_{kF}^j$  is the angle between the line which join  $\vec{x}_k$  point and the final node of the same panel with the layer panel, so:

$$\beta_k^j = \beta_{kI}^j + \beta_{kF}^j = \theta_{kF}^j - \theta_{kI}^j$$

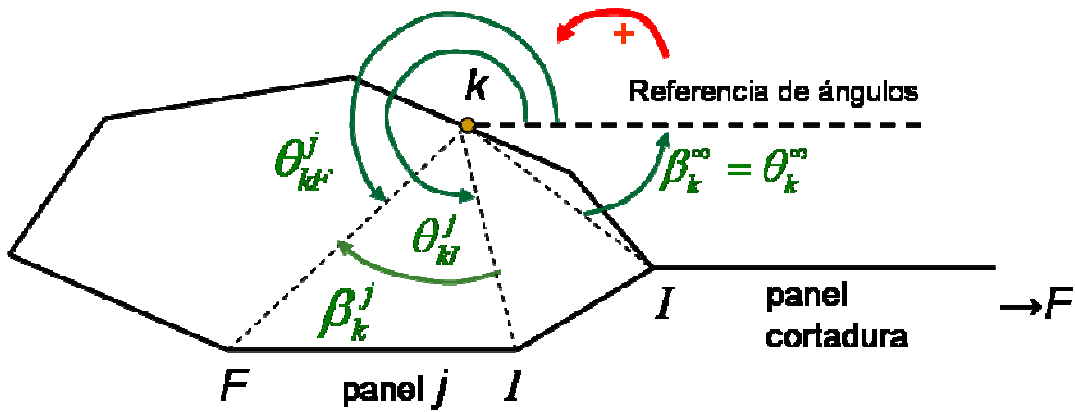


Figure 1

(1) equation shows that the velocity potential due to an uniform distribution of doublets with  $\Phi_j$  intensity through the panel is equivalent to the potential due to a couple of vortex located at the extremes of the panel with  $\Phi_j$  and  $-\Phi_j$  intensities, see Fig. 2 (remember: with that angle definition the potential that a vortex of strength one located at point  $j$  would produce would be  $-\theta_j/2\pi$ ).

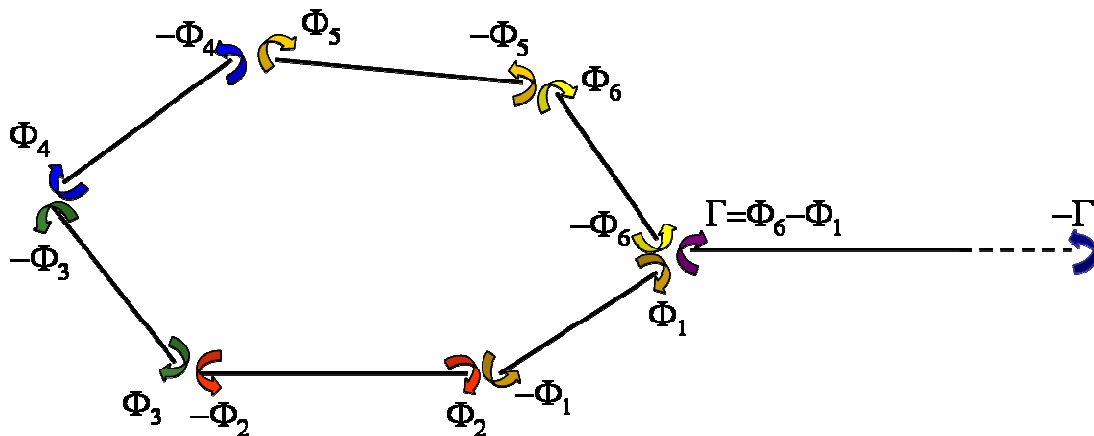


Figure 2

Derivate Eq. (1) and multiply by the normal vector to  $k$  panel to obtain the normal velocity to that panel and make it equal to zero,

$$\nabla_k \Phi \cdot \vec{n}_k = 0,$$

we obtain

$$0 = \frac{1}{2\pi} \sum_{i=1}^N \Phi_j \left( \theta'_{kF}{}^j - \theta'_{kI}{}^j \right) + \frac{\Gamma}{2\pi} \theta'_k{}^\infty + V_{nk\infty},$$

where

$$\theta'_k{}^j = \nabla_k \theta_k^j \cdot \vec{n}_k = \frac{(\vec{x}_k - \vec{x}_j) \cdot \vec{t}_k}{|\vec{x}_k - \vec{x}_j|^2}$$

$\vec{t}_k$  is an unit vector tangent to k panel  $k$  and  $V_{nj\infty}$  is the normal component to j panel of the infinite airfoil. Multiplying the previous equation by  $2\pi$ , ordering the different terms, knowing that  $\theta'_{kl}{}^j = \theta'_{kf}{}^{j-1}$  and calling  $\gamma_j$  the difference of numerical potential between two consecutive panels,  $\gamma_j = \Phi_j - \Phi_{j-1}$   $j = 2, \dots, N$ , we obtain

$$\begin{aligned} 0 &= \Phi_N \theta'_{kF}{}^N - \Phi_N \theta'_{kF}{}^{N-1} + \Phi_{N-1} \theta'_{kF}{}^{N-1} - \Phi_{N-1} \theta'_{kF}{}^{N-2} + \dots + \\ &+ \Phi_2 \theta'_{kF}{}^2 - \Phi_2 \theta'_{kF}{}^1 + \Phi_1 \theta'_{kF}{}^1 - \Phi_1 \theta'_{kI}{}^1 + (\Phi_N - \Phi_1) \theta'_k{}^\infty + 2\pi V_{nk\infty} = \\ &= \Phi_N \theta'_{kF}{}^N - \Phi_1 \theta'_{kI}{}^1 + (\Phi_N - \Phi_1) \theta'_k{}^\infty - \sum_{j=2}^N (\Phi_j - \Phi_{j-1}) \theta'_{kF}{}^{j-1} + 2\pi V_{nk\infty} = \\ &= \Phi_N \underbrace{(\theta'_{kF}{}^N + \theta'_k{}^\infty)}_0 - \Phi_1 \underbrace{(\theta'_{kI}{}^1 + \theta'_k{}^\infty)}_0 - \sum_{j=2}^N \gamma_j \theta'_{kF}{}^{j-1} + 2\pi V_{nk\infty} \end{aligned}$$

Finally the equation for the K point is

$$\sum_{j=2}^N \gamma_j \theta'_{kF}{}^j = 2\pi V_{nk\infty}, \quad (2)$$

thus what seemed to be a system with  $N$  unknown quantities:

$\Phi_1, \dots, \Phi_N$  becomes a system with  $N-1$  unknown quantities:  $\gamma_2, \dots, \gamma_N$ , see Fig. 3. That means that despite dividing the airfoil in  $N$  panels, **only N-1 equations can be imposed in N-1 collocation points.**

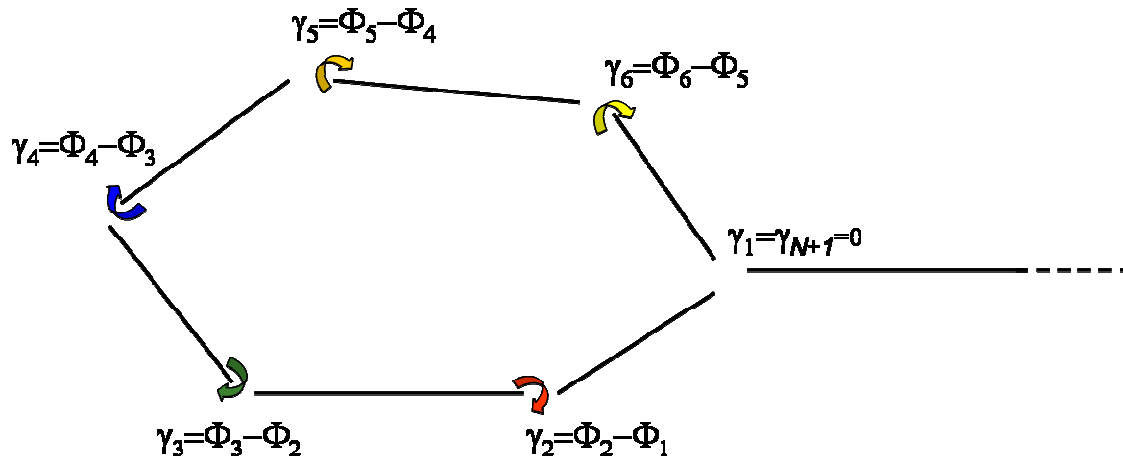


Figure 3

It is important to note that even though some publications assure that the Neumann formula for the non-zero thickness airfoil can be solved in the numerical velocity potential variables  $\Phi_j$ , it is not true since the coefficient matrix for the linear equation system in those variables is singular and renders it impossible to solve. However, it is possible to solve the system in the variables  $\gamma_j = \Phi_{j+1} - \Phi_j$  because the coefficient matrix is regular. The circulation around an airfoil is calculated as:

$$\Gamma = \Phi_N - \Phi_1 = \sum_{j=2}^N \gamma_j .$$

To apply the Neumann method we consider N-1 vortices located at the nodes (the vortices located on nodes 1 and N+1 have zero intensity  $\gamma_1 = \gamma_{N+1} = \Gamma + \Phi_1 - \Phi_N = 0$ ) **and we impose that the velocity normal to the panel is zero on the N-1 collocation points: 2,.....,N.**