Discreet Vortex Method (2D)

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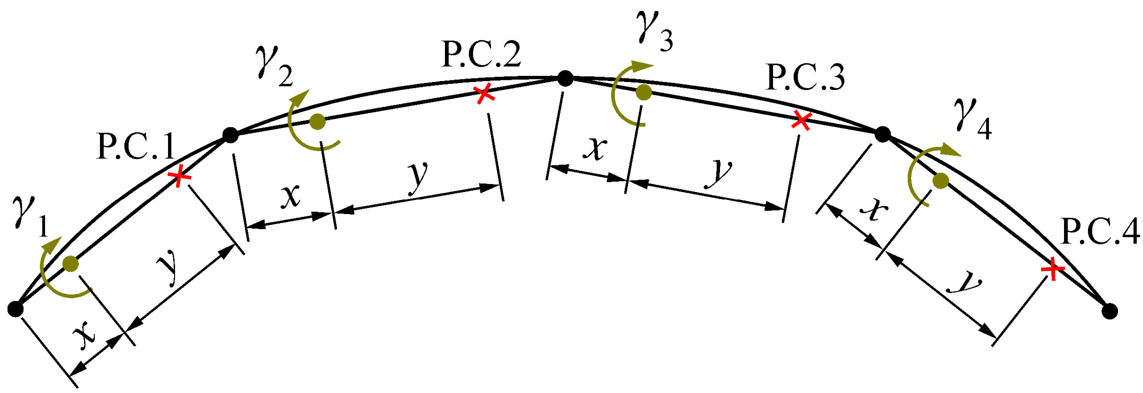


Figure 1

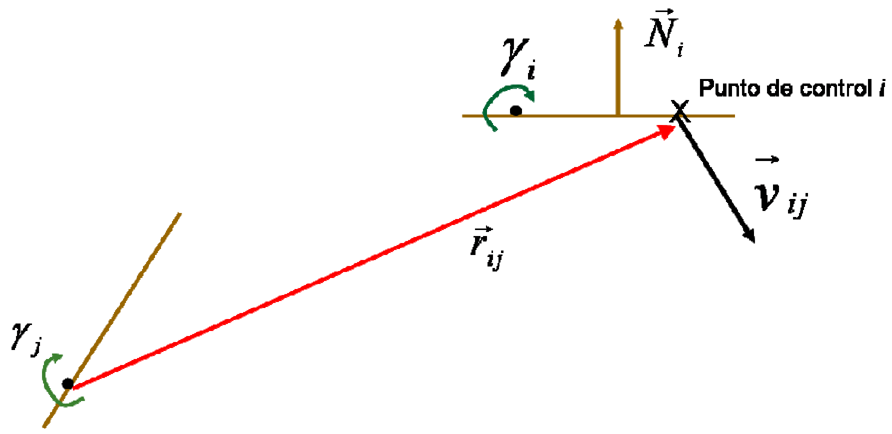
This method emerges from the panel method (that will be seen in following classes) and is used only for curvature lines (camberlines). The profile is discretized with *N* panels and then each panel is replaced with a vortex, placed at a distance *x* from the initial point of each panel. Since the solution thus obtained is a superposition of elemental solutions, it will comply with the differential equation $\Delta\Phi=0$.

<u>Contour conditions for the problem:</u>

i. Orthogonal flow is null on the profile's boundary.
ii. Uniform flow at infinity.
iii. Kutta Condition.

The problem is now reduced to determining the strength of the vortex that comply with the problem's contour conditions (moving them to the discretized problem appropriately). For this, the contour conditions will be imposed in certain points (collocation points). There is a collocation point for each panel, located at a distance *y* from the vortex.

Equation System



The speed on the collocation point *i* will be the speed induced by the *N* vortices plus the speed of the incoming flow.

Calling $\vec{r}_{ij} = (\xi_{ij}, \eta_{ij})$ the vector starting on the vortex *j* and ending on the collocation point *i*, $\vec{r}_{Tj} = (\xi_{Tj}, \eta_{Tj})$ the position of the vortex *j* and $\vec{r}_{PCi} = (\xi_{PCi}, \eta_{PCi})$ the position of the collocation point *i*, we have:

$$\vec{r}_{ij} = \vec{r}_{PCi} - \vec{r}_{Tj}$$

The speed induced in the collocation point *i* by the vortex *j*, $\vec{v}_{ij}$, is perpendicular to $\vec{r}_{ij}$ and can be calculated as:

$$\vec{v}_{ij} = \frac{\gamma_j}{2\pi\left|\vec{r}_{ij}\right|}\vec{n}_{ij}, \text{ con } \vec{n}_{ij} = \frac{(\eta_{ij}, -\xi_{ij})}{\left|\vec{r}_{ij}\right|}$$

This way, the speed on the collocation point *i* is:

$$\vec{v}_i = \sum_{j=1}^{N} \frac{\gamma_j (\eta_{ij}, -\xi_{ij})}{2\pi\left|\vec{r}_{ij}\right|^2} + \vec{v}_\infty \qquad (1)$$

We have *N* unknown variables (γ_j) and N equations, obtained when imposing that the perpendicular speed to each panel in the collocation point be zero:

$$\vec{v}_i \cdot \vec{N}_i = 0,\ i = 1, ..., N \qquad (2)$$

where $\vec{N}_i$ is the vector perpendicular to the panel *i.* Calling (ξ_i, η_i) the coordinates of the node *i*, a vector in the panel's tangent direction would be:

$$\vec{T}_i = (\xi_{i+1} - \xi_i, \eta_{i+1} - \eta_i) = (\Delta\xi_i, \Delta\eta_i),\ \text{luego}\ \vec{N}_i = (-\Delta\eta_i, \Delta\xi_i) \qquad (3)$$

Taking the Equations (1) and (3) to the equation (2) we obtain:

$$\sum_{j=1}^{N} \frac{\gamma_j\ (\eta_{ij}, -\xi_{ij})(-\Delta\eta_i, \Delta\xi_i)}{2\pi\left|\vec{r}_{ij}\right|^2} = -\vec{v}_\infty \cdot (-\Delta\eta_i, \Delta\xi_i), \quad i = 1, ..., N$$

That can be written in matrix form as:

$$A\gamma = B$$

where:

$$A_{ij} = \frac{(\eta_{ij}, -\xi_{ij})(-\Delta\eta_i, \Delta\xi_i)}{2\pi\left|\vec{r}_{ij}\right|^2},\ B_i = \vec{v}_\infty \cdot (\Delta\eta_i, -\Delta\xi_i)$$