PANEL METHOD FOR CONSTANT POTENTIAL (2D)

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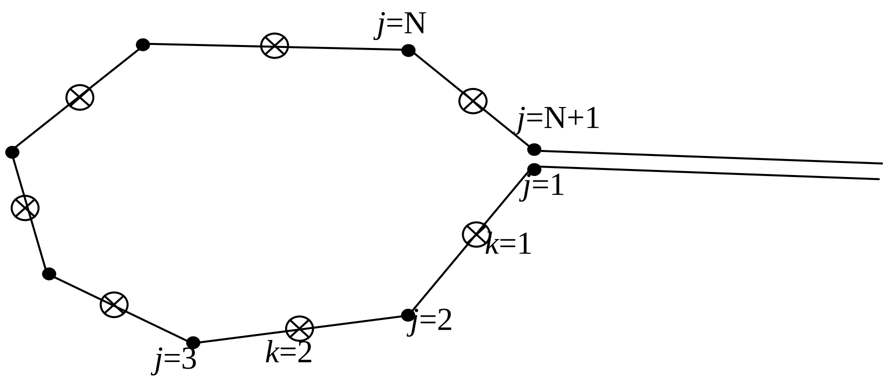
In this method the Dirichlet solving method is used, and for the inner potential we take $\Phi_i = 0$, so there is no source distribution, only doublets, thus the Green formula is rewritten:

$$\Phi_P = -\int_{S_B} \Phi(\nabla\Phi_m \cdot \bar{N})\,\mathrm{d}S - \Gamma \int_{S_W} (\nabla\Phi_m \cdot \bar{N})\,\mathrm{d}S + \Phi_\infty \qquad (1)$$

where it is known that for 2D it is $\Gamma = \Phi^+ - \Phi^-$. To discretize this equation the airfoil is replaced with N panels and in each panel we assume that the doublets strength (the potential Φ) is constant. The unknown variables of the problem are the potential value in each panel and the equations which have to be solved are obtained particularizing the Green formula, Eq. (1), on N collocation points.



The collocation points are in the middle of each panel. So, if the node position is (x_j, z_j), the collocation point position will be:

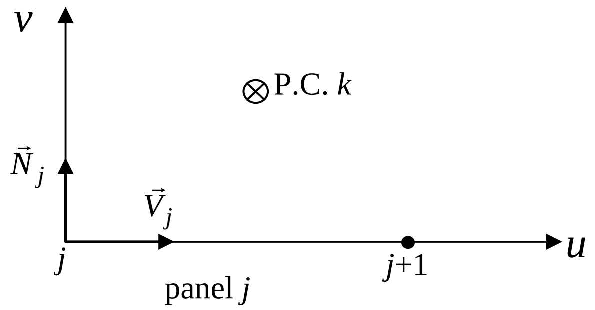
$$x_{PCj} = \frac{x_j + x_{j+1}}{2} \quad , \quad z_{PCj} = \frac{z_j + z_{j+1}}{2}$$

The Green formula in these collocation points is:

$$\Phi_k = U_\infty(x_{PCk}\cos\alpha + z_{PCk}\sin\alpha) - \sum_{j=1}^{N}\frac{\Phi_j}{2\pi}\int_{\text{panel j}}\frac{(\bar{x}-\bar{x}_{PCk})\cdot\bar{N}\,\mathrm{d}S}{\left|\bar{x}-\bar{x}_{PCk}\right|^2} - \frac{\Phi_N-\Phi_1}{2\pi}\int_{S_W}\frac{(\bar{x}-\bar{x}_{PCk})\cdot\bar{N}\,\mathrm{d}S}{\left|\bar{x}-\bar{x}_{PCk}\right|^2}$$

(2)

To solve these integrals use the following reference system for each panel:



$$\vec{V}_j = \frac{\bar{x}_{j+1}-\bar{x}_j}{l_j} \quad , \quad l_j = \text{panel length}$$

$$\vec{V}_j = (V_{jx}, V_{jz}) \quad , \quad \vec{N}_j = (-V_{jz}, V_{jx})$$

According to the numeration of the panels (clockwise), the $\vec{N}_j$ vector (perpendicular to $\vec{V}_j$ and forming a right-hand system) always points to the outside of the body, meaning $\vec{N}_j = \bar{N}$.

The coordinates of the collocation point *k* in the reference system of the panel *j* are (u^j_{PCk}, v^j_{PCk}), and the contribution of the panel *j* to the potential in the collocation point *k* will be (Eq. 2):

$$-\frac{\Phi_j}{2\pi}\int_{\text{panel j}}\frac{(\bar{x}-\bar{x}_{PCk})\cdot\bar{N}\,\mathrm{d}S}{\left|\bar{x}-\bar{x}_{PCk}\right|^2} = \frac{\Phi_j}{2\pi}\int_0^{l_j}\frac{v^j_{PCk}\,du}{(u-u^j_{PCk})^2+(v^j_{PCk})^2} = \frac{\Phi_j}{2\pi}\left\{\underbrace{\mathrm{arc\,tan}\left(\frac{l_j-u^j_{PCk}}{v^j_{PCk}}\right)}_{\beta^j_{kF}} + \underbrace{\mathrm{arc\,tan}\left(\frac{u^j_{PCk}}{v^j_{PCk}}\right)}_{\beta^j_{kI}}\right\}$$

$$\beta^j_k = \beta^j_{kI} + \beta^j_{kF}$$

For the panel $k = j$ is

$$u_k^j = \frac{l_j}{2} \quad , \quad v_k^j \to 0^+, \quad k = j$$

so

$$\beta_{kI}^j \to \frac{\pi}{2} \quad , \quad \beta_{kF}^j \to \frac{\pi}{2}, \qquad k = j$$

For the layer surface S_w a unique panel is used ($j = N+1$) with infinite length, so $l_{N+1} \to \infty$, and

$$\beta_{kF}^{N+1} = \frac{\pi}{2}\operatorname{sgn}(v_k^{N+1})$$

Finally, rewriting (1) we obtain:

$$\Phi_k = U_\infty(x_{PCk}\cos\alpha + z_{PCk}\sin\alpha) + \sum_{j=1}^{N}\frac{\Phi_j}{2\pi}\beta_k^j + \frac{\Phi_N - \Phi_1}{2\pi}\left(\frac{\pi}{2}\operatorname{sgn}(v_k^{N+1}) + \beta_{kI}^{N+1}\right)$$

and in matrix form

$$A_k^j \Phi_j = B_k, \quad k = 1,....N, \quad j = 1,....N$$

with

$$B_k = U_\infty(x_{PCk}\cos\alpha + z_{PCk}\sin\alpha)$$

and

$$A_k^j = \delta_k^j - \frac{\beta_k^j}{2\pi} \quad , \quad j \neq 1, N,$$

$$A_k^1 = \delta_k^1 - \frac{\beta_k^1}{2\pi} + \frac{\left(\frac{\pi}{2}\operatorname{sgn}(v_k^{N+1}) + \beta_{Ik}^{N+1}\right)}{2\pi}$$

$$A_k^N = \delta_k^N - \frac{\beta_k^N}{2\pi} - \frac{\left(\frac{\pi}{2}\operatorname{sgn}(v_k^{N+1}) + \beta_{Ik}^{N+1}\right)}{2\pi}$$

where

$$\beta_k^j = \left\{ \operatorname{arc\,tan}\left(\frac{l_j - u_k^j}{v_k^j} \right) + \operatorname{arc\,tan}\left(\frac{u_k^j}{v_k^j} \right) \right\} , \quad k \neq j ,$$

$$\beta_k^j = \pi \ , \quad k = j,$$

and

$$\beta_{kI}^{N+1} = \operatorname{arc\,tan}\left(\frac{u_k^{N+1}}{v_k^{N+1}} \right).$$