Fundamentals of Panels Theory

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Introduction

- Allows solving the potential incompressible problem (or linearized compressible) for complex geometries.
- It's based on distributing singularities over the body's surface calculating their intensities so they comply with the contour conditions on the body.
- **It lowers the problem's dimension**
  - 3D to 2D (the variables are on the wing's surface)
  - 2D to 1D (the variables are on the profile's curvature line)

Green's Integral

$\iiint_D \nabla\cdot\vec{A} = -\iint_\Sigma \vec{A}\cdot\vec{N}ds$, donde $\vec{A}$ es de clase C^1 en D y

$\vec{N}$ es el vector normal interior a D.

Tomando:

$\vec{A} = F\nabla G - G\nabla F$, con F y G funciones de clase C^2 en D

que cumplen $\Delta F = 0$, $\Delta G = 0$, se tiene:

$$\iint_\Sigma (F\nabla G - G\nabla F)\cdot\vec{N}ds = 0$$

Basic Formulation

- Apply Green's integral, with:
  - $G = \Phi_m(\vec{x},\vec{x}_p)$: Speed potential in the point $\vec{x}$ of a source with strength one located on the point P

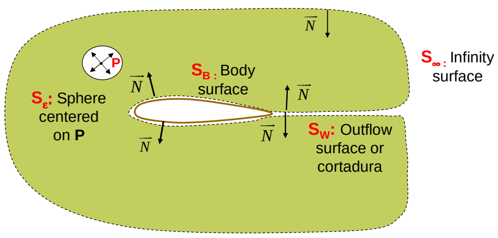
    2D: $\Phi_m(\vec{x};\vec{x}_P) = \frac{1}{2\pi}\log|\vec{x}-\vec{x}_P| = f(|\vec{x}-\vec{x}_P|) = f(|\vec{x}_P-\vec{x}|)$

    3D: $\Phi_m(\vec{x};\vec{x}_P) = -\frac{1}{4\pi|\vec{x}-\vec{x}_P|} = f(|\vec{x}-\vec{x}_P|) = f(|\vec{x}_P-\vec{x}|)$
  - $F = \Phi(\vec{x})$: Speed potential in the point $\vec{x}$

$$\iint_\Sigma (F\nabla G - G\nabla F)\cdot\vec{N}ds = \iint_\Sigma (\Phi\nabla\Phi_m - \Phi_m\nabla\Phi)\cdot\vec{N}ds = 0$$

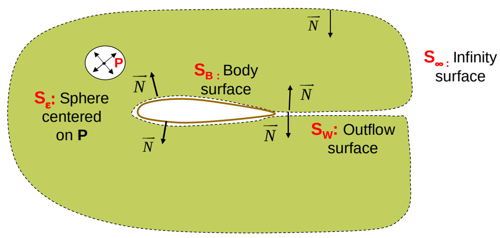
Basic Formulation

- **Definition for the contour surfaces:** $\sum = S_B + S_\varepsilon + S_\infty + S_W$



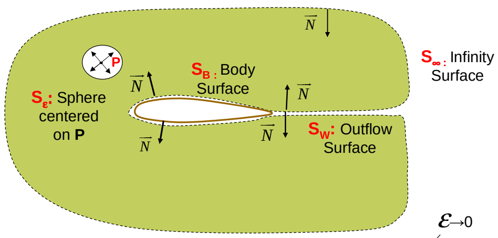
$$\iint_\Sigma (F\nabla G - G\nabla F)\cdot \vec{N}ds = \iint_\Sigma (\Phi\nabla\Phi_m - \Phi_m\nabla\Phi)\cdot \vec{N}ds = 0$$

Basic Formulation



$$\iint_\Sigma (\Phi\nabla\Phi_m - \Phi_m\nabla\Phi)\cdot \vec{N}ds = \iint_{S_B+S_W+S_\infty+S_\varepsilon} (\Phi\nabla\Phi_m - \Phi_m\nabla\Phi)\cdot \vec{N}ds = 0$$

Basic Formulation

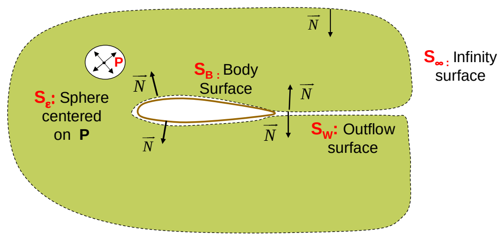


$\mathcal{E}\to 0$

$$\iint_{S_B+S_W+S_\infty} (\Phi\nabla\Phi_m - \Phi_m\nabla\Phi)\cdot \vec{N}ds = -\iint_{S_\varepsilon} \Phi\nabla\Phi_m \cdot \vec{N}ds + \iint_{S_\varepsilon} \Phi_m\nabla\Phi\cdot \vec{N}ds$$

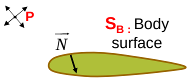
$$\to -\Phi(\vec{x}_p)$$

Basic Formulation



$$\Phi(\vec{x}_p) = \iint_{S_B+S_W+S_\infty} (\Phi_m\nabla\Phi - \Phi\nabla\Phi_m)\cdot \vec{N}ds$$

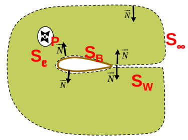
Basic Formulation



- $F = \Phi_i(\vec{x})$, con $\Delta\Phi_i = 0$ en D_i: Inner speed potential in the point $\vec{x}$. The contour condition is not defined

$$\iint_{\Sigma} (F\nabla G - G\nabla F)\cdot \vec{N}ds = \iint_{S_B} (\Phi_i \nabla\Phi_m - \Phi_m \nabla\Phi_i)\cdot \vec{N}ds = 0$$

Basic Formulation



$$\Phi(\vec{x}_P) = \iint_{S_B+S_W+S_\infty} (\Phi_m \nabla\Phi - \Phi\nabla\Phi_m)\cdot \vec{N}dS$$

$$0 = \iint_{S_B} (\Phi_m \nabla\Phi_i - \Phi_i \nabla\Phi_m)\cdot \vec{N}dS$$

- **Subtracting both equations we get:**

$$\Phi(\vec{x}_P) = \iint_{S_B} \left(\Phi_m(\nabla\Phi - \nabla\Phi_i)\cdot\vec{N} - (\Phi - \Phi_i)\nabla\Phi_m\cdot\vec{N}\right)dS \; +$$

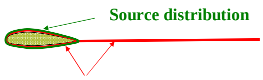
$$\iint_{S_W} \left(\underbrace{\Phi_m \nabla\Phi\cdot\vec{N}}_{=0} - \Phi\nabla\Phi_m\cdot\vec{N}\right)dS \; + \underbrace{\iint_{S_\infty} \left(\Phi_m\nabla\Phi\cdot\vec{N} - \Phi\nabla\Phi_m\cdot\vec{N}\right)dS}_{\to \Phi_\infty = U_\infty(x\cos\alpha + y\sin\alpha)}$$

Basic Formulation

- **Green formula**:

$$\Phi(\vec{x}_P) = \underbrace{\iint_{S_B}\left(\frac{\partial\Phi}{\partial n} - \frac{\partial\Phi_i}{\partial n}\right)\Phi_m dS}_{\text{Potencial producido en } \vec{x}_p \text{ por una distribucion de manantiales sobre el cuerpo}} - \underbrace{\iint_{S_B}(\Phi - \Phi_i)\nabla\Phi_m\cdot\vec{N}dS}_{\text{Potencial producido en } \vec{x}_p \text{ por una distribucion de dobletes sobre el cuerpo}}$$

$$- \underbrace{\iint_{S_W}\left(\Phi^+ - \Phi^-\right)\nabla\Phi_m\cdot\vec{N}dS}_{\text{Potencial producido en } \vec{x}_p \text{ por una distribucion de dobletes sobre la estela}} + \Phi_\infty$$



Basic Formulation

- Φ and Φ_i must comply with the following equations:

$\Delta\Phi = 0$ en D
$\frac{\partial\Phi}{\partial n} = 0$ en S_B
$\Phi \to U_\infty(x\cos\alpha + y\sin\alpha)$ en S_∞
Condición de Kutta

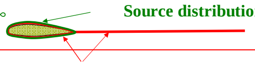
$\Delta\Phi_i = 0$ en D_i
$\frac{\partial\Phi_i}{\partial n} = ?$ o $\Phi_i = ?$ en S_B

- The contour condition for the inner potential can be set freely, it's the degree of freedom that allows to choose different singularities.

Diritchlet's Formulation

$$\Phi(\vec{x}_P)=\iint_{S_B}\left(\cancel{\frac{\partial\Phi}{\partial n}}-\frac{\partial\Phi_i}{\partial n}\right)\Phi_m dS-\iint_{S_B}(\Phi-\Phi_i)\underbrace{\nabla\Phi_m\cdot\vec{N}}_{\text{Potential of a doublet: }\Phi_d}dS$$
$$-\iint_{S_W}\left(\Phi^+-\Phi^-\right)\underbrace{\nabla\Phi_m\cdot\vec{N}}_{\text{Potential of a doublet: }\Phi_d}dS+\Phi_\infty$$

$$\Phi(\vec{x}_P)=-\iint_{S_B}\frac{\partial\Phi_i}{\partial n}(\vec{x})\Phi_m\left(|\vec{x}_P-\vec{x}|\right)dS-\iint_{S_B}\left(\Phi(\vec{x})-\Phi_i(\vec{x})\right)\Phi_d\left(|\vec{x}_P-\vec{x}|\right)dS$$
$$-\iint_{S_W}\left(\Phi^+-\Phi^-\right)\Phi_d\left(|\vec{x}_P-\vec{x}|\right)dS+\Phi_\infty$$



- The integral equation must be solved making the point *P* tend towards the surface of the body

Diritchlet's Formulation

- Choosing the inner potential as zero:

$$\Phi(\vec{x}_P)=-\iint_{S_B}\cancel{\frac{\partial\Phi_i}{\partial n}}(\vec{x})\Phi_m\left(|\vec{x}_P-\vec{x}|\right)dS-\iint_{S_B}\left(\Phi(\vec{x})-\cancel{\Phi_i(\vec{x})}\right)\Phi_d\left(|\vec{x}_P-\vec{x}|\right)dS$$
$$-\iint_{S_W}\underbrace{\left(\Phi^+-\Phi^-\right)}_{=\Gamma}\Phi_d\left(|\vec{x}_P-\vec{x}|\right)dS+\Phi_\infty$$

$$\Phi(\vec{x}_P)=-\iint_{S_B}\Phi(\vec{x})\Phi_d\left(|\vec{x}_P-\vec{x}|\right)dS-\Gamma\iint_{S_W}\Phi_d\left(|\vec{x}_P-\vec{x}|\right)dS+\Phi_\infty$$

Neumann Formulation

$$\Phi(\vec{x}_P)=\iint_{S_B}\left(\cancel{\frac{\partial\Phi}{\partial n}}-\frac{\partial\Phi_i}{\partial n}\right)\Phi_m dS-\iint_{S_B}(\Phi-\Phi_i)\nabla\Phi_m\cdot\vec{N}dS$$
$$-\iint_{S_W}\left(\Phi^+-\Phi^-\right)\nabla\Phi_m\cdot\vec{N}dS+\Phi_\infty$$

$$\Phi(\vec{x}_P)=\iint_{S_B}\sigma(\vec{x})\Phi_m\left(|\vec{x}_P-\vec{x}|\right)dS-\iint_{S_B+S_W}\mu(\vec{x})\Phi_d\left(|\vec{x}_P-\vec{x}|\right)dS+\Phi_\infty$$

- The equation's derivative is found to calculate the perpendicular speed on the body ($\vec{V}(\vec{x}_P)=\nabla\Phi$) and then it's made zero. The variables are: $\sigma(\vec{x})$ and $\mu(\vec{x})$. If the inner potential is zero, the only variable is $\mu(\vec{x})$ (doublets distribution).
- Some information is lost