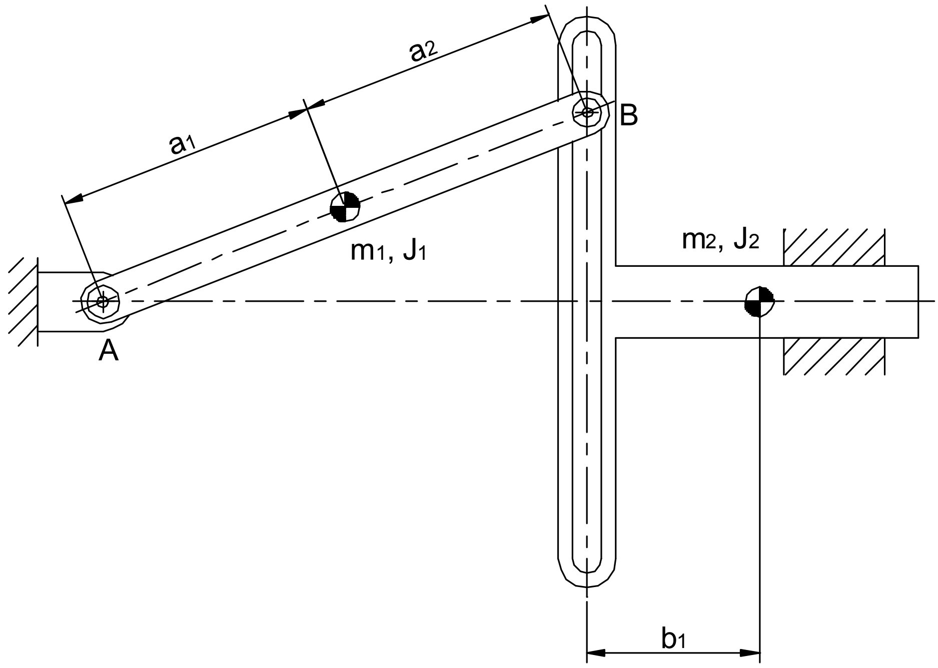
Ejercicio 1:

Determinar a velocidad horizontal del cuerpo 2 sabiendo que $j_1 = w_0 t$



Determinación de los grados de libertad:

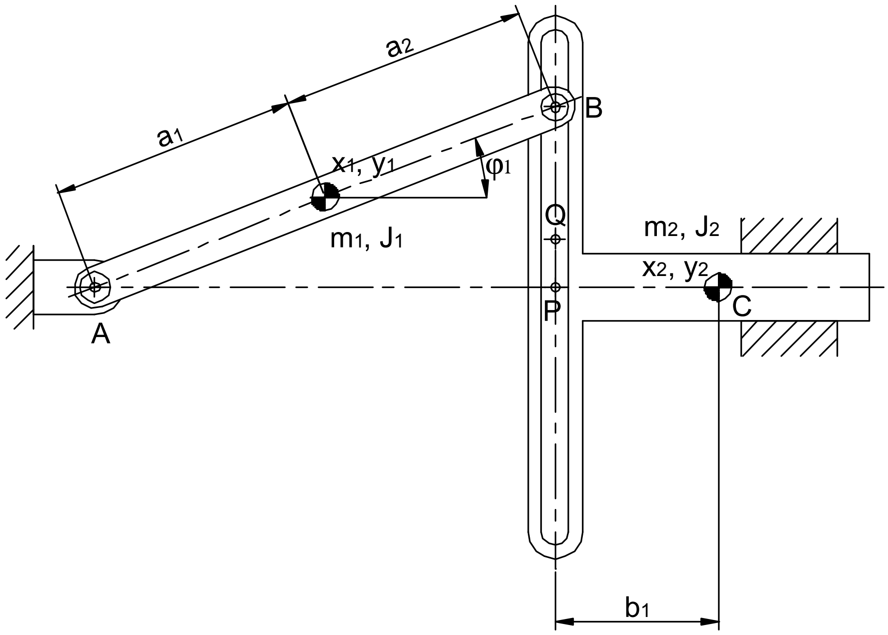
3 Eslabones :1, 2 y 0	9 coordenadas
Un cuerpo fijo (0). 3 restricciones simples	3 ec. rest,
Un par de revolución (A)	2 ec. rest.
Un par de traslación-rotación (B)	1 ec. rest.
Un par de traslación (C)	2 ec. rest.
TOTAL	1 g.d.l.

Coordenadas de los cuerpos:

Cuerpo 1 : $\begin{bmatrix} x_1 & y_1 & j_1 \end{bmatrix}^T$

Cuerpo 2 : $\begin{bmatrix} x_2 & y_2 & j_2 \end{bmatrix}^T$

Cuerpo 0 : $\begin{bmatrix} x_0 & y_0 & j_0 \end{bmatrix}^T$



Vectores de posición de los puntos implicados:

Punto A

$$\vec{r}_1^{\,A} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} \cos\varphi_1 & -sen\varphi_1 \\ sen\varphi_1 & \cos\varphi_1 \end{bmatrix} \begin{bmatrix} -a_1 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1 - a_1\cos\varphi_1 \\ y_1 - a_1 sen\varphi_1 \end{bmatrix}$$

Punto B

$$\vec{r}_1^{\,B} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} \cos\varphi_1 & -sen\varphi_1 \\ sen\varphi_1 & \cos\varphi_1 \end{bmatrix} \begin{bmatrix} a_2 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1 + a_2\cos\varphi_1 \\ y_1 + a_2 sen\varphi_1 \end{bmatrix}$$

Punto C

$$\vec{r}_2^{\,C} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + \begin{bmatrix} \cos\varphi_1 & -sen\varphi_1 \\ sen\varphi_1 & \cos\varphi_1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

Punto P

$$\vec{r}_2^{\,P} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + \begin{bmatrix} \cos\varphi_2 & -sen\varphi_2 \\ sen\varphi_2 & \cos\varphi_2 \end{bmatrix} \begin{bmatrix} -b_1 \\ 0 \end{bmatrix} = \begin{bmatrix} x_2 - b_1\cos\varphi_2 \\ y_2 - b_1 sen\varphi_2 \end{bmatrix}$$

Punto Q

$$\vec{r}_2^{\,Q} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + \begin{bmatrix} \cos\varphi_2 & -sen\varphi_2 \\ sen\varphi_2 & \cos\varphi_2 \end{bmatrix} \begin{bmatrix} -b_1 \\ 1 \end{bmatrix} = \begin{bmatrix} x_2 - b_1\cos\varphi_2 - sen\varphi_2 \\ y_2 - b_1 sen\varphi_2 + \cos\varphi_2 \end{bmatrix}$$

Restricciones:

Restricciones simples:

$$x_0 = 0$$
$$y_0 = 0$$
$$j_0 = 0$$

Par de revolución A:

$$\vec{r}_1^{\,A} = 0$$

$$\begin{cases} x_1 - a_1 \cos j_1 = 0 \\ y_1 - a_1 sen j_1 = 0 \end{cases}$$

Par traslación-revolución B:

$$\overrightarrow{BP} // \overrightarrow{QP} = 0$$

$$\overrightarrow{BP} = \begin{bmatrix} x_2 - b_1 \cos j_2 - x_1 - a_2 \cos j_1 \\ y_2 - b_1 sen j_2 - y_1 - a_2 sen j_1 \end{bmatrix}$$

$$\overrightarrow{QP} = \begin{bmatrix} - sen j_2 \\ \cos j_2 \end{bmatrix}$$

$$\overrightarrow{BP} \times \overrightarrow{QP} = 0$$

$$(x_2 - b_1 \cos j_2 - x_1 - a_2 \cos j_1) \times (\cos j_2) - (y_2 - b_1 sen j_2 - y_1 - a_2 sen j_1) \times (- sen j_2) = 0$$

Par de traslación C:

$$\vec{u} // \overrightarrow{OC} = 0$$

$$\vec{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \overrightarrow{OC} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

$$\vec{u} \times \overrightarrow{OC} = 0 \qquad 1 \times y_2 - 0 \times x_2 = 0 \qquad y_2 = 0$$

$$j_2 - j_3 = 0 \qquad j_3 = 0 \quad \Rightarrow \quad j_2 = 0$$

$$\begin{cases} y_2 = 0 \\ j_2 = 0 \end{cases}$$

Ecuaciones de restricción:

Operando con las ecuaciones anteriores y eliminando las que resultan valores nulos resulta:

$$x_1 - a_1 \cos j_1 = 0$$
$$y_1 - a_1 sen j_1 = 0$$
$$x_2 - b_1 - x_1 - a_2 \cos j_1 = 0$$

Jacobiana

$$\begin{array}{c|cccc} & \dfrac{\partial}{\partial x_1} & \dfrac{\partial}{\partial y_1} & \dfrac{\partial}{\partial j_1} & \dfrac{\partial}{\partial x_2} \\ \hline f_1 & 1 & 0 & a_1 sen j_1 & 0 \\ f_2 & 0 & 1 & -a_1 \cos j_1 & 0 \\ f_3 & -1 & 0 & a_2 sen j_1 & 1 \end{array}$$

$$j_1 = w_0 t$$
$$x_1 = a_1 \cos j_1 = a_1 \cos w_0 t$$
$$y_1 = a_1 \sin j_1 = a_1 \sin w_0 t$$
$$x_2 = b_1 + x_1 + a_2 \cos j_1 = b_1 + a_1 \cos w_0 t + a_2 \cos w_0 t$$

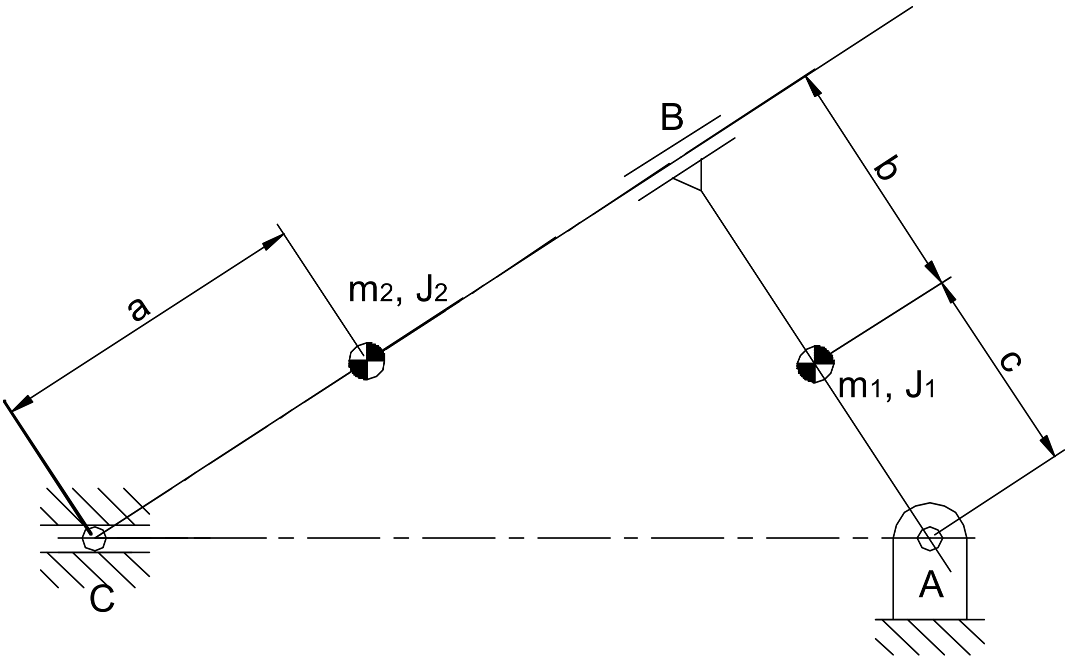
$$\begin{bmatrix} 1 & 0 & 0 & a_1 \sin j_1 \\ 0 & 1 & 0 & -a_1 \cos j_1 \\ -1 & 0 & 1 & a_2 \sin j_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{x}_2 \\ \dot{j}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ w_0 \end{bmatrix}$$

$$\dot{j}_1 = w_0$$
$$\dot{x}_1 = -a_1 \dot{j}_1 \sin j_1 = -a_1 w_0 \sin j_1$$
$$\dot{y}_1 = a_1 \dot{j}_1 \cos j_1 = a_1 w_0 \cos j_1$$
$$\dot{x}_2 = \dot{x}_1 - a_2 \dot{j}_1 \sin j_1 = -a_1 w_0 \sin j_1 - a_2 w_0 \sin j_1$$

Ejercicio 2 :

Determinar La velocidad del punto C sabiendo que el brazo AB gira según la expresión:

$j_1 = w_0 t + p / 2$



Determinación de los grados de libertad:

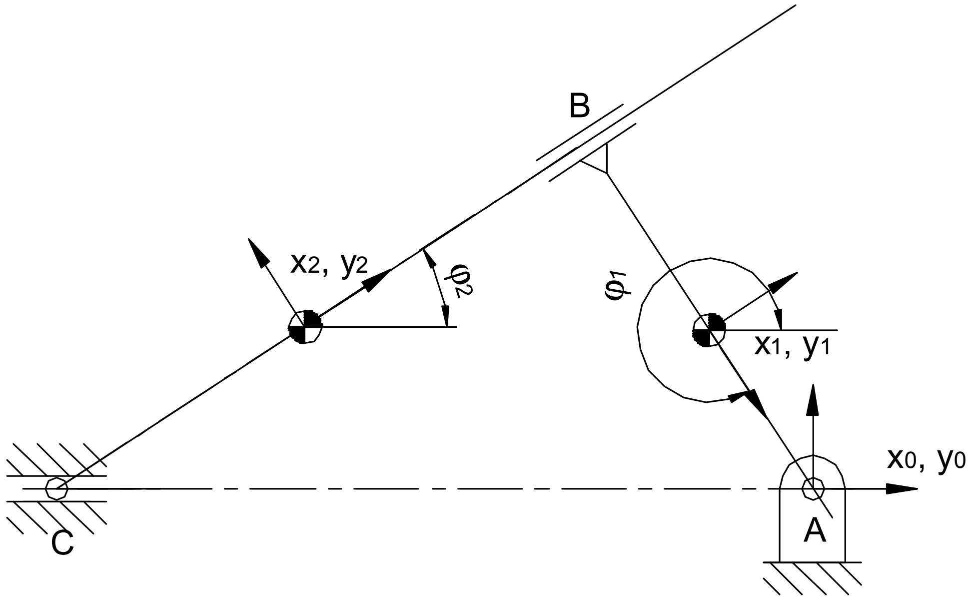
3 Eslabones :1, 2 y 0	9 coordenadas
Un cuerpo fijo (0). 3 restricciones simples	3 ec. rest,
Un par de revolución (A)	2 ec. rest.
Un par de traslación (B)	2 ec. rest.
Un par de revolución-traslación (C)	1 ec. rest.
TOTAL	1 g.d.l.

Coordenadas de los cuerpos:

Cuerpo 1 : $[x_1 \quad y_1 \quad j_1]^T$

Cuerpo 2 : $[x_2 \quad y_2 \quad j_2]^T$

Cuerpo 0 : $[x_0 \quad y_0 \quad j_0]^T$



Vectores de posición de los puntos implicados:

Cuerpo 0:

Punto A: $\vec{r}_0^{\,A} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Cuerpo 1:

Punto A: $\vec{r}_1^{\,A} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} \cos\varphi_1 & -\sin\varphi_1 \\ \sin\varphi_1 & \cos\varphi_1 \end{bmatrix} \begin{bmatrix} c \\ 0 \end{bmatrix} = \begin{bmatrix} x_1 + c\cos\varphi_1 \\ y_1 + c\sin\varphi_1 \end{bmatrix}$

Punto B: $\vec{r}_1^{\,B} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} \cos\varphi_1 & -\sin\varphi_1 \\ \sin\varphi_1 & \cos\varphi_1 \end{bmatrix} \begin{bmatrix} -b \\ 0 \end{bmatrix} = \begin{bmatrix} x_1 - b\cos\varphi_1 \\ y_1 - b\sin\varphi_1 \end{bmatrix}$

Cuerpo 2:

Punto C: $\vec{r}_2^{\,C} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + \begin{bmatrix} \cos\varphi_2 & -\sin\varphi_2 \\ \sin\varphi_2 & \cos\varphi_2 \end{bmatrix} \begin{bmatrix} -a \\ 0 \end{bmatrix} = \begin{bmatrix} x_2 - a\cos\varphi_2 \\ y_2 - a\sin\varphi_2 \end{bmatrix}$

Restricciones:

Restricciones simples:

$$\begin{cases} x_0 = 0 \\ y_0 = 0 \\ \varphi_0 = 0 \end{cases}$$

Par de revolución A:

$$\vec{r}_1^{\,A} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{cases} x_1 + c\cos j_1 = 0 \\ y_1 + c\sin j_1 = 0 \end{cases}$$

Par de traslación B:

$$\begin{cases} j_1 - j_2 - 3p\ /2 = 0 \\ \overrightarrow{u_2} // \overrightarrow{X_2B_1} \Rightarrow \overrightarrow{CB} \times \overrightarrow{X_2B_1} = \vec{0} \end{cases}$$

$$\overrightarrow{u_2} = \begin{bmatrix} \cos j_2 \\ \sin j_2 \end{bmatrix}$$

$$\overrightarrow{X_2B_1} = \begin{bmatrix} x_1 - b\cos j_1 - x_2 \\ y_1 - b\sin j_1 - y_2 \end{bmatrix}$$

$$\begin{cases} j_1 - j_2 - 3p\ /2 = 0 \\ \cos j_2 (y_1 - b\sin j_1 - y_2) - \sin j_2 (x_1 - b\cos j_1 - x_2) = 0 \end{cases}$$

Par de revolución - traslación C:

$$\overrightarrow{u_0} // \overrightarrow{CA} \Rightarrow \overrightarrow{u_0} \times \overrightarrow{CA} = \vec{0}$$

$$\overrightarrow{u_0} = \begin{bmatrix} \cos j_0 \\ \sin j_0 \end{bmatrix}$$

$$\overrightarrow{CA} = \vec{r}_2^{\,C} - \vec{r}_0^{\,A} = \begin{bmatrix} x_2 - a\cos j_2 \\ y_2 - a\sin j_2 \end{bmatrix} - \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

$$\cos j_0 (y_2 - a\sin j_2 - y_0) - \sin j_0 (x_2 - a\cos j_2 - x_0) = 0$$

Ecuaciones de restricción:

Operando con las ecuaciones anteriores y simplificando resulta:

$$x_0 = 0$$
$$y_0 = 0$$
$$j_0 = 0$$
$$x_1 + c\cos j_1 = 0$$
$$y_1 + c\sin j_1 = 0$$
$$j_1 - j_2 - 3p\ /2 = 0$$
$$\cos j_2 (y_1 - b\sin j_1 - y_2) - \sin j_2 (x_1 - b\cos j_1 - x_2) = 0$$
$$\cos j_0 (y_2 - a\sin j_2 - y_0) - \sin j_0 (x_2 - a\cos j_2 - x_0) = 0$$

$$x_1 + c\cos j_1 = 0$$
$$y_1 + c\sin j_1 = 0$$
$$j_1 - j_2 - 3p/2 = 0$$
$$\cos j_2(y_1 - b\sin j_1 - y_2) - \sin j_2(x_1 - b\cos j_1 - x_2) = 0$$
$$y_2 - a\sin j_2 = 0$$

$$x_1 = -c\cos j_1$$
$$y_1 = -c\sin j_1$$
$$j_2 = j_1 - 3p/2$$
$$y_2 = a\sin j_2 = a\sin(j_1 - 3p/2)$$
$$x_2 = x_1 - b\cos j_1 - \frac{\cos j_2}{\sin j_2}(y_1 - b\sin j_1 - y_2) =$$
$$= -c\cos j_1 - b\cos j_1 - \frac{\cos j_2}{\sin j_2}(-c\sin j_1 - b\sin j_1 - a\sin(j_1 - 3p/2))$$

Velocidades:

Matriz Jacobiana:

$$f_1 = x_1 + c\cos j_1$$
$$f_2 = y_1 + c\sin j_1$$
$$f_3 = j_1 - j_2 - 3p/2$$
$$f_4 = \cos j_2(y_1 - b\sin j_1 - y_2) - \sin j_2(x_1 - b\cos j_1 - x_2)$$
$$f_5 = y_2 - a\sin j_2$$
$$f_6 = j_1 - w_0 t - p/2$$

	$\partial/\partial x_1$	$\partial/\partial y_1$	$\partial/\partial j_1$	$\partial/\partial x_2$	$\partial/\partial y_2$	$\partial/\partial j_2$
f_1	1	0	$-c\sin j_1$	0	0	0
f_2	0	1	$c\cos j_1$	0	0	0
f_3	0	0	1	0	0	-1
f_4	$-\sin j_2$	$\cos j_2$	$-b\cos j_1\cos j_2 - b\sin j_1\sin j_2$	$\sin j_2$	$-\cos j_2$	$J_{4,6}$
f_5	0	0	0	0	1	$-a\cos j_2$
f_6	0	0	1	0	0	0

$$
\begin{aligned}
J_{4,6} &= -\sin\varphi_2(y_1 - b\sin\varphi_1 - y_2) - \cos\varphi_2(x_1 - b\cos\varphi_1 - x_2) = \\
&= -y_1\sin\varphi_2 + b\sin\varphi_1\sin\varphi_2 + y_2\sin\varphi_2 - x_1\cos\varphi_2 + b\cos\varphi_1\cos\varphi_2 + x_2\cos\varphi_2 = \\
&= -y_1\sin\varphi_2 + y_2\sin\varphi_2 - x_1\cos\varphi_2 + x_2\cos\varphi_2 + b\cos(\varphi_1 - \varphi_2) = \\
&= -y_1\sin\varphi_2 + y_2\sin\varphi_2 - x_1\cos\varphi_2 + x_2\cos\varphi_2 + b\cos(3\pi/2) = \\
&= -y_1\sin\varphi_2 + y_2\sin\varphi_2 - x_1\cos\varphi_2 + x_2\cos\varphi_2 = \\
&= a\sin\varphi_2\sin\varphi_2 + (-c\cos\varphi_1 - b\cos\varphi_1 - \frac{\cos\varphi_2}{\sin\varphi_2}(-c\sin\varphi_1 - b\sin\varphi_1 - a\sin\varphi_2))\cos\varphi_2 = \\
&= a + (b + c)\frac{\cos\varphi_2}{\sin\varphi_2}
\end{aligned}
$$

$$\cos(\varphi_1 - \varphi_2) = \cos(3\pi/2) = 0$$

	$\partial/\partial x_1$	$\partial/\partial y_1$	$\partial/\partial \varphi_1$	$\partial/\partial x_2$	$\partial/\partial y_2$	$\partial/\partial \varphi_2$
f_1	1	0	$-c\sin\varphi_1$	0	0	0
f_2	0	1	$c\cos\varphi_1$	0	0	0
f_3	0	0	1	0	0	-1
f_4	$-\sin\varphi_2$	$\cos\varphi_2$	0	$\sin\varphi_2$	$-\cos\varphi_2$	$a + (b+c)\frac{\cos\varphi_2}{\sin\varphi_2}$
f_5	0	0	0	0	1	$-a\cos\varphi_2$
f_6	0	0	1	0	0	0

$$
\begin{bmatrix}
1 & 0 & -c\sin\varphi_1 & 0 & 0 & 0 \\
0 & 1 & c\cos\varphi_1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 \\
-\sin\varphi_2 & \cos\varphi_2 & 0 & \sin\varphi_2 & -\cos\varphi_2 & a + (b+c)\frac{\cos\varphi_2}{\sin\varphi_2} \\
0 & 0 & 0 & 0 & 1 & -a\cos\varphi_2 \\
0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
\cdot
\begin{bmatrix}
\dot{x}_1 \\ \dot{y}_1 \\ \dot{\varphi}_1 \\ \dot{x}_2 \\ \dot{y}_2 \\ \dot{\varphi}_2
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \omega_0
\end{bmatrix}
$$

$$\dot{j}_1 = w_0$$
$$\dot{j}_2 = \dot{j}_1 = w_0$$
$$\dot{x}_1 = c\dot{j}_1 \sin j_1 = cw_0 \sin j_1$$
$$\dot{y}_1 = -c\dot{j}_1 \cos j_1 = -cw_0 \cos j_1$$
$$\dot{y}_2 = a\dot{j}_2 \cos j_2 = aw_0 \cos j_2$$
$$\dot{x}_2 = \frac{1}{\sin j_2}(\sin j_2 \dot{x}_1 - \cos j_2 \dot{y}_1 + \cos j_2 \dot{y}_2 - J_{4,6}\dot{j}_2) =$$
$$= \frac{1}{\sin j_2}(\sin j_2 cw_0 \sin j_1 + \cos j_2 cw_0 \cos j_1 + \cos j_2 aw_0 \cos j_2 - (a + (b+c)\frac{\cos j_2}{\sin j_2})w_0) =$$
$$= \frac{1}{\sin j_2}(a\cos^2 j_2 - (a + (b+c)\frac{\cos j_2}{\sin j_2}))w_0 = \frac{1}{\sin j_2}(a\cos^2 j_2 - a - (b+c)\frac{\cos j_2}{\sin j_2})w_0 =$$
$$= \frac{1}{\sin j_2}(a\cos^2 j_2 - a - (b+c)\frac{\cos j_2}{\sin j_2})w_0 = \frac{1}{\sin j_2}(-a\sin^2 j_2 - (b+c)\frac{\cos j_2}{\sin j_2})w_0 =$$
$$= (-a\sin j_2 - (b+c)\frac{\cos j_2}{\sin^2 j_2})w_0$$

$$= -c\cos j_1 - b\cos j_1 - \frac{\cos j_2}{\sin j_2}(-c\sin j_1 - b\sin j_1 - a\sin(j_1 - 3p\ /2)) =$$
$$+ c\sin j_1 + b\sin j_1 - \frac{\cos j_2}{\sin j_2}(-c\cos j_1 - b\cos j_1 - a\cos(j_1 - 3p\ /2)) +$$
$$+ \frac{1}{\sin^2 j_2}(-c\sin j_1 - b\sin j_1 - a\sin(j_1 - 3p\ /2))$$