#### **Mathematics**





#### **Initial Value ODE Problem Solvers**

These are the MATLAB initial value problem solvers. The table lists the kind of problem you can solve with each solver, and the method each solver uses.

Solver	<b>Solves These Kinds of Problems</b>	Method
ode45	Nonstiff differential equations	Runge-Kutta
ode23	Nonstiff differential equations	Runge-Kutta
<u>ode113</u>	Nonstiff differential equations	Adams
ode15s	Stiff differential equations and DAEs	NDFs (BDFs)
ode23s	Stiff differential equations	Rosenbrock
ode23t	Moderately stiff differential equations and DAEs	Trapezoidal rule
ode23tb	Stiff differential equations	TR-BDF2

#### **ODE Solution Evaluation**

If you call an ODE solver with one output argument, it returns a structure that you can use to evaluate the solution at any point on the interval of integration.

Function	Description
<u>deval</u>	Evaluate the numerical solution using output of ODE solvers.

### **ODE Option Handling**

An options structure contains named integration properties whose values are passed to the solver, and which affect problem solution. Use these functions to create, alter, or access an options structure.

Function	Description
odeset	Create or alter options structure for input to ODE solvers.
<u>odeget</u>	Extract properties from options structure created with odeset.

### **ODE Solver Output Functions**

If an output function is specified, the solver calls the specified function after every successful integration step. You can use <u>odeset</u> to specify one of these sample functions as the <u>OutputFcn property</u>, or you can modify them to create your own functions.

Function	Description
<u>odeplot</u>	Time-series plot

odephas2	Two-dimensional phase plane plot
odephas3	Three-dimensional phase plane plot
<u>odeprint</u>	Print to command window

### **ODE Initial Value Problem Examples**

These examples illustrate the kinds of problems you can solve in MATLAB. From the MATLAB Help browser, click the example name to see the code in an editor. Type the example name at the command line to run it.

The Differential Equations Examples browser enables you to view the code for the ODE examples and DAE examples. You can also run the examples from the browser. Click on these links to invoke the browser, or type odeexamples('ode') or odeexamples('dae') at the command line.

Example	Description			
amp1dae	Stiff DAE - electrical circuit			
<u>ballode</u>	Simple event location - bouncing ball			
batonode	ODE with time- and state-dependent mass matrix - motion of a baton			
brussode	Stiff large problem - diffusion in a chemical reaction (the Brusselator)			
burgersode	ODE with strongly state-dependent mass matrix - Burger's equation solved using a moving mesh technique			
<u>femlode</u>	Stiff problem with a time-dependent mass matrix - finite element method			
<u>fem2ode</u>	Stiff problem with a constant mass matrix - finite element method			
<u>hb1dae</u>	Stiff DAE from a conservation law			
<u>hblode</u>	Stiff problem solved on a very long interval - Robertson chemical reaction			
<u>orbitode</u>	Advanced event location - restricted three body problem			
<u>rigidode</u>	Nonstiff problem - Euler equations of a rigid body without external forces			
vdpode	Parameterizable van der Pol equation (stiff for large $\mu$ )			

**★** Initial Value Problems for ODEs and DAEs

Introduction to Initial Value ODE Problems





# ode45, ode23, ode113, ode15s, ode23s, ode23t, ode23tb

Solve initial value problems for ordinary differential equations (ODEs)

# **Syntax**

```
[T,Y] = solver(odefun,tspan,y0)
[T,Y] = solver(odefun,tspan,y0,options)
[T,Y] = solver(odefun,tspan,y0,options,p1,p2...)
[T,Y,TE,YE,IE] = solver(odefun,tspan,y0,options)
sol = solver(odefun,[t0 tf],y0...)
```

where solver is one of ode45, ode23, ode113, ode15s, ode23s, ode23t, or ode23tb.

### **Arguments**

odefun

A function that evaluates the right-hand side of the differential equations. All solvers solve systems of equations in the form y' = f(t, y) or problems that involve a mass matrix,

M(t, y)y' = f(t, y). The ode23s solver can solve only equations with constant mass matrices.

ode15s and ode23t can solve problems with a mass matrix that is singular, i.e., differential-algebraic equations (DAEs).

tspan

A vector specifying the interval of integration, [t0,tf]. To obtain solutions at specific times (all increasing or all decreasing), use tspan = [t0,t1,...,tf].

Y<sup>0</sup> A vector of initial conditions.

options Optional integration argument created using the odeset function. See odeset for details.

p1,p2... Optional parameters to be passed to odefun.

# **Description**

[T,Y] = solver(odefun, tspan, y0) with tspan = [t0 tf] integrates the system of differential equations y' = f(t, y) from time to to tf with initial conditions y0. Function f = odefun(t, y), for a scalar t and a column vector f, must return a column vector f corresponding to f(t, y). Each row in the solution array f corresponds to a time returned in column vector f. To obtain solutions at the specific times f (all increasing or all decreasing), use f (all f (all increasing) are f (all f (al

[T,Y] = solver(odefun,tspan,y0,options) solves as above with default integration parameters replaced by property values specified in options, an argument created with the odeset function. Commonly used properties include a scalar relative error tolerance RelTol (le-3 by default) and a vector of absolute error tolerances AbsTol (all components are le-6 by default). See odeset for details.

[T,Y] = solver(odefun,tspan,y0,options,p1,p2...) solves as above, passing the additional parameters p1,p2... to the function odefun, whenever it is called. Use options = [] as a place holder if no options are set.

[T,Y,TE,YE,IE] = solver(odefun,tspan,y0,options) solves as above while also finding where functions of (t,y), called event functions, are zero. For each event function, you specify whether the integration is to

terminate at a zero and whether the direction of the zero crossing matters. This is done by setting the Events property to, say, @EVENTS, and creating a function [value,isterminal,direction] = EVENTS(t,y). For the ith event function:

- value(i) is the value of the function.
- isterminal(i) = 1 if the integration is to terminate at a zero of this event function and 0 otherwise.
- direction(i) = 0 if all zeros are to be computed (the default), +1 if only the zeros where the event function increases, and -1 if only the zeros where the event function decreases.

Corresponding entries in  $_{\text{TE}}$ ,  $_{\text{YE}}$ , and  $_{\text{IE}}$  return, respectively, the time at which an event occurs, the solution at the time of the event, and the index  $_{\text{I}}$  of the event function that vanishes.

sol = solver(odefun,[t0 tf],y0...) returns a structure that you can use with deval to evaluate the solution at any point on the interval [t0,tf]. You must pass odefun as a function handle. The structure sol always includes these fields:

```
Steps chosen by the solver.

Sol.y

Each column sol.y(:,i) contains the solution at sol.x(i).

Sol.solver

Solver name.
```

If you specify the Events option and events are detected, sol also includes these fields:

- Points at which events, if any, occurred. sol.xe(end) contains the exact point of a terminal event, if any.
- sol.ye Solutions that correspond to events in sol.xe.
- Indices into the vector returned by the function specified in the Event option. The values indicate which event has been detected.

If you specify an output function as the value of the <code>OutputFon</code> property, the solver calls it with the computed solution after each time step. Four output functions are provided: <code>odeplot</code>, <code>odephas2</code>, <code>odephas3</code>, <code>odeprint</code>. When you call the solver with no output arguments, it calls the default <code>odeplot</code> to plot the solution as it is computed. <code>odephas2</code> and <code>odephas3</code> produce two- and three-dimnesional phase plane plots, respectively. <code>odeprint</code> displays the solution components on the screen. By default, the ODE solver passes all components of the solution to the output function. You can pass only specific components by providing a vector of indices as the value of the <code>OutputSel</code> property. For example, if you call the solver with no output arguments and set the value of <code>OutputSel</code> to <code>[1,3]</code>, the solver plots solution components 1 and 3 as they are computed.

For the stiff solvers ode15s, ode23s, ode23t, and ode23tb, the Jacobian matrix  $\partial f/\partial y$  is critical to reliability and efficiency. Use odeset to set Jacobian to @FJAC if FJAC(T,Y) returns the Jacobian  $\partial f/\partial y$  or to the matrix  $\partial f/\partial y$  if the Jacobian is constant. If the Jacobian property is not set (the default),  $\partial f/\partial y$  is approximated by finite differences. Set the Vectorized property 'on' if the ODE function is coded so that odefun(T,[Y1,Y2...]) returns [odefun(T,Y1), odefun(T,Y2)...]. If  $\partial f/\partial y$  is a sparse matrix, set the JPattern property to the sparsity pattern of  $\partial f/\partial y$ , i.e., a sparse matrix s with S(i,j) = 1 if the ith component of f(t,y) depends on the jth component of y, and 0 otherwise.

The solvers of the ODE suite can solve problems of the form M(t, y)y' = f(t, y), with time- and state-

dependent mass matrix M. (The ode23s solver can solve only equations with constant mass matrices.) If a problem has a mass matrix, create a function M = MASS(t,y) that returns the value of the mass matrix, and use odeset to set the Mass property to @MASS. If the mass matrix is constant, the matrix should be used as the value of the Mass property. Problems with state-dependent mass matrices are more difficult:

- If the mass matrix does not depend on the state variable y and the function MASS is to be called with one input argument, t, set the MStateDependence property to 'none'.
- If the mass matrix depends weakly on y, set MStateDependence to 'weak' (the default) and otherwise, to 'strong'. In either case, the function MASS is called with the two arguments (t,y).

If there are many differential equations, it is important to exploit sparsity:

- Return a sparse M(t, y).
- Supply the sparsity pattern of  $\partial f/\partial y$  using the JPattern property or a sparse  $\partial f/\partial y$  using the Jacobian property.
- For strongly state-dependent M(t, y), set MvPattern to a sparse matrix s with S(i,j) = 1 if for any k, the (i,k) component of M(t,y) depends on component j of y, and 0 otherwise.

If the mass matrix M is singular, then M(t, y)y' = f(t, y) is a differential algebraic equation. DAEs have solutions only when  $y_0$  is consistent, that is, if there is a vector  $y_0$  such that

 $M(t_0, y_0)yp_0 = f(t_0, y_0)$ . The ode15s and ode23t solvers can solve DAEs of index 1 provided that  $y_0$  is sufficiently close to being consistent. If there is a mass matrix, you can use odeset to set the MassSingular property to 'yes', 'no', or 'maybe'. The default value of 'maybe' causes the solver to test whether the problem is a DAE. You can provide  $y_{0}$  as the value of the InitialSlope property. The default is the zero vector. If a problem is a DAE, and  $y_0$  and  $y_{0}$  are not consistent, the solver treats them as guesses, attempts to compute consistent values that are close to the guesses, and continues to solve the problem. When solving DAEs, it is very advantageous to formulate the problem so that M is a diagonal matrix (a semi-explicit DAE).

Solver	Problem Type	Order of Accuracy	When to Use
ode45	Nonstiff	Medium	Most of the time. This should be the first solver you try.
ode23	Nonstiff	Low	If using crude error tolerances or solving moderately stiff problems.
ode113	Nonstiff	Low to high	If using stringent error tolerances or solving a computationally intensive ODE file.
ode15s	Stiff	Low to medium	If ode45 is slow because the problem is stiff.
ode23s	Stiff	Low	If using crude error tolerances to solve stiff systems and the mass matrix is constant.
ode23t	Moderately Stiff	Low	If the problem is only moderately stiff and you need a solution without numerical damping.
ode23tb	Stiff	Low	If using crude error tolerances to solve stiff systems.

The algorithms used in the ODE solvers vary according to order of accuracy [6] and the type of systems (stiff or

nonstiff) they are designed to solve. See Algorithms for more details.

# **Options**

Different solvers accept different parameters in the options list. For more information, see <u>odeset</u> and <u>Improving ODE Solver Performance</u> in the "Mathematics" section of the MATLAB documentation.

Parameters	ode45	ode23	ode113	ode15s	ode23s	ode23t	ode23tb
RelTol, AbsTol, NormControl	1	1	√	1	1	1	4
OutputFcn, OutputSel, Refine, Stats	1	4	√	4	1	1	4
Events	1	4	√	1	1	1	4
MaxStep, InitialStep	4	4	√	4	1	4	4
Jacobian, JPattern, Vectorized				4	4	4	4
Mass MStateDependence MvPattern MassSingular	 	77:	77	444	<b>√</b>  	<b>~~~</b>	: 444
InitialSlope				4		4	
MaxOrder, BDF				4			

### **Examples**

**Example 1.** An example of a nonstiff system is the system of equations describing the motion of a rigid body without external forces.

$$y'_1 = y_2 y_3$$
  $y_1(0) = 0$   
 $y'_2 = -y_1 y_3$   $y_2(0) = 1$   
 $y'_3 = -0.51 y_1 y_2$   $y_3(0) = 1$ 

To simulate this system, create a function rigid containing the equations

```
function dy = rigid(t,y)

dy = zeros(3,1); % a column vector

dy(1) = y(2) * y(3);

dy(2) = -y(1) * y(3);

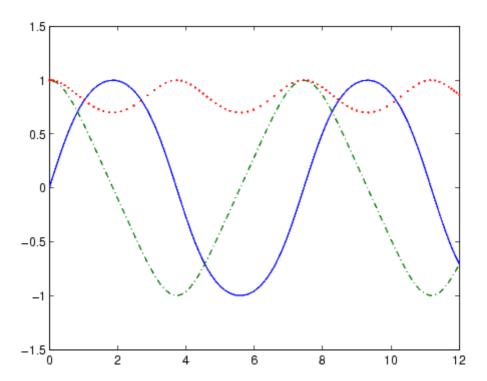
dy(3) = -0.51 * y(1) * y(2);
```

In this example we change the error tolerances using the  $\underline{\mathtt{odeset}}$  command and solve on a time interval [0 12] with an initial condition vector [0 1 1] at time 0.

```
options = odeset('RelTol',1e-4,'AbsTol',[1e-4 1e-4 1e-5]);
[T,Y] = ode45(@rigid,[0 12],[0 1 1],options);
```

Plotting the columns of the returned array y versus T shows the solution

```
plot(T,Y(:,1),'-',T,Y(:,2),'-.',T,Y(:,3),'.')
```



**Example 2.** An example of a stiff system is provided by the van der Pol equations in relaxation oscillation. The limit cycle has portions where the solution components change slowly and the problem is quite stiff, alternating with regions of very sharp change where it is not stiff.

$$y'_1 = y_2$$
  $y_1(0) = 0$   
 $y'_2 = 1000(1 - y_1^2)y_2 - y_1$   $y_2(0) = 1$ 

To simulate this system, create a function vdp1000 containing the equations

```
function dy = vdp1000(t,y)

dy = zeros(2,1); % a column vector

dy(1) = y(2);

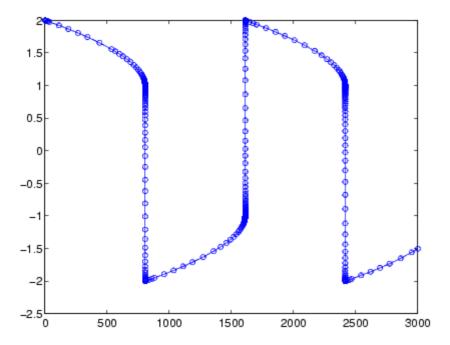
dy(2) = 1000*(1 - y(1)^2)*y(2) - y(1);
```

For this problem, we will use the default relative and absolute tolerances (1e-3 and 1e-6, respectively) and solve on a time interval of [0 3000] with initial condition vector [2 0] at time 0.

```
[T,Y] = ode15s(@vdp1000,[0 3000],[2 0]);
```

Plotting the first column of the returned matrix y versus T shows the solution

```
plot(T,Y(:,1),'-o')
```



### **Algorithms**

ode45 is based on an explicit Runge-Kutta (4,5) formula, the Dormand-Prince pair. It is a *one-step* solver - in computing  $y(t_n)$ , it needs only the solution at the immediately preceding time point,  $y(t_{n-1})$ . In general, ode45 is the best function to apply as a "first try" for most problems. [3]

ode23 is an implementation of an explicit Runge-Kutta (2,3) pair of Bogacki and Shampine. It may be more efficient than ode45 at crude tolerances and in the presence of moderate stiffness. Like ode45, ode23 is a one-step solver. [2]

ode113 is a variable order Adams-Bashforth-Moulton PECE solver. It may be more efficient than ode45 at stringent tolerances and when the ODE file function is particularly expensive to evaluate. ode113 is a *multistep* solver - it normally needs the solutions at several preceding time points to compute the current solution. [7]

The above algorithms are intended to solve nonstiff systems. If they appear to be unduly slow, try using one of the stiff solvers below.

ode15s is a variable order solver based on the numerical differentiation formulas (NDFs). Optionally, it uses the backward differentiation formulas (BDFs, also known as Gear's method) that are usually less efficient. Like ode113, ode15s is a multistep solver. Try ode15s when ode45 fails, or is very inefficient, and you suspect that the problem is stiff, or when solving a differential-algebraic problem. [9], [10]

ode23s is based on a modified Rosenbrock formula of order 2. Because it is a one-step solver, it may be more efficient than ode15s at crude tolerances. It can solve some kinds of stiff problems for which ode15s is not effective. [9]

ode23t is an implementation of the trapezoidal rule using a "free" interpolant. Use this solver if the problem is only moderately stiff and you need a solution without numerical damping. ode23t can solve DAEs. [10]

ode23tb is an implementation of TR-BDF2, an implicit Runge-Kutta formula with a first stage that is a trapezoidal rule step and a second stage that is a backward differentiation formula of order two. By construction, the same iteration matrix is used in evaluating both stages. Like ode23s, this solver may be more efficient than

ode15s at crude tolerances. [8], [1]

#### See Also

deval, odeset, odeget, @ (function handle)

### References

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- [4] Forsythe, G., M. Malcolm, and C. Moler, *Computer Methods for Mathematical Computations*, Prentice-Hall, New Jersey, 1977.
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- [8] Shampine, L. F. and M. E. Hosea, "Analysis and Implementation of TR-BDF2," *Applied Numerical Mathematics* 20, 1996.
- [9] Shampine, L. F. and M. W. Reichelt, "The MATLAB ODE Suite," *SIAM Journal on Scientific Computing*, Vol. 18, 1997, pp 1-22.
- [10] Shampine, L. F., M. W. Reichelt, and J.A. Kierzenka, "Solving Index-1 DAEs in MATLAB and Simulink," *SIAM Review*, Vol. 41, 1999, pp 538-552.

**♦** nzmax

odefile 🗪



# odeget

Extract properties from options structure created with odeset

# **Syntax**

```
o = odeget(options,'name')
o = odeget(options,'name',default)
```

# **Description**

o = odeget(options, 'name') extracts the value of the property specified by string 'name' from integrator options structure options, returning an empty matrix if the property value is not specified in options. It is only necessary to type the leading characters that uniquely identify the property name. Case is ignored for property names. The empty matrix [] is a valid options argument.

o = odeget(options, 'name', default) returns o = default if the named property is not specified in options.

### **Example**

Having constructed an ODE options structure,

```
options = odeset('RelTol',1e-4,'AbsTol',[1e-3 2e-3 3e-3]);
```

you can view these property settings with odeget.

#### See Also

odeset



odeset 🗭



# odeset

Create or alter options structure for input to ordinary differential equation (ODE) solvers

# **Syntax**

```
options = odeset('name1',value1,'name2',value2,...)
options = odeset(oldopts,'name1',value1,...)
options = odeset(oldopts,newopts)
odeset
```

### **Description**

The odeset function lets you adjust the integration parameters of the ODE solvers. The ODE solvers can integrate systems of differential equations of one of these forms

$$y' = f(t, y)$$

or

$$M(t, y)y' = f(t, y)$$

See below for information about the integration parameters.

options = odeset('name1', value1, 'name2', value2,...) creates an integrator options structure in which the named properties have the specified values. Any unspecified properties have default values. It is sufficient to type only the leading characters that uniquely identify a property name. Case is ignored for property names.

```
options = odeset(oldopts, 'name1', value1,...) alters an existing options structure oldopts.
```

options = odeset(oldopts, newopts) alters an existing options structure oldopts by combining it with a new options structure newopts. Any new options not equal to the empty matrix overwrite corresponding options in oldopts.

odeset with no input arguments displays all property names as well as their possible and default values.

### **ODE Properties**

The available properties depend on the ODE solver used. There are several categories of properties:

- Error tolerance
- Solver output
- Jacobian matrix
- Event location
- Mass matrix and differential-algebraic equations (DAEs)
- Step size
- ode15s

**Note** This reference page describes the ODE properties for MATLAB, Version 6. The Version 5 properties are supported only for backward compatibility. For information on the Version 5 properties,

type at the MATLAB command line: more on, type odeset, more off.

# **Error Tolerance Properties**

Property	Value	Description
RelTol	Positive scalar {1e-3}	A relative error tolerance that applies to all components of the solution vector. The estimated error in each integration step satisfies  e(i) <= max(RelTol*abs(y(i)),AbsTol(i))
AbsTol	Positive scalar or vector {1e-6}	The absolute error tolerance. If scalar, the tolerance applies to all components of the solution vector. Otherwise the tolerances apply to corresponding components.
NormControl	on   {off}	Control error relative to norm of solution. Set this property on to request that the solvers control the error in each integration step with norm(e) <= max (RelTol*norm(y), AbsTol). By default the solvers use a more stringent component-wise error control.

# **Solver Output Properties**

Property	Value	Description		
OutputFcn	Function	Installable output function. The ODE solvers provide sample functions that you can use or modify:		
		odeplot	Time series plotting (default)	
		odephas2	Two-dimensional phase plane plotting	
		odephas3 Three-dimensional phase plane plotting		
		odeprint	Print solution as it is computed	
		To create or modify an output function, see ODE Solver Output Properties in the "Differential Equations" section of the MATLAB documentation.		
OutputSel	Vector of indices	Specifies the components of the solution vector that the solver passes to the output function.		
Refine	Positive integer	Produces smoother output, increasing the number of output points by the specified factor. The default value is 1 in all solvers except ode45, where it is 4. Refine doesn't apply if length(tspan) > 2.		
Stats	on   {off}	Specifies whether the solver should display statistics about the computational cost of the integration.		

# Jacobian Matrix Properties (for ode15s, ode23s, ode23t, and ode23tb)

Property	Value	Description
Jacobian	Function   constant matrix	Jacobian function. Set this property to @FJac (if a function FJac(t,y) returns $\partial f/\partial y$ ) or to the constant value of $\partial f/\partial y$ .
JPattern	Sparse matrix of {0,1}	Sparsity pattern. Set this property to a sparse matrix $S$ with $S(i, j) = 1$ if component $i$ of $f(t, y)$ depends on component $j$ of $y$ , and $0$ otherwise.

Vectorized	Vectorized ODE function. Set this property on to inform the stiff solver that the ODE function $F$ is coded so that $F(t,[y1\ y2\ \dots])$ returns the vector $[F(t,y1)\ F(t,y2)\ \dots]$ . That is, your ODE function can pass to the solver a whole array of column vectors at once. A stiff function calls your ODE function a vectorized manner only if it is generating Jacobians numerically (the default
	1

# **Event Location Property**

Property	Value	Description
Events		Locate events. Set this property to @Events, where Events is the event function. See the <a href="ODE solvers">ODE solvers</a> for details.

# Mass Matrix and DAE-Related Properties

Property	Value	Description
Mass	Constant matrix   function	For problems $My' = f(t, y)$ set this property to the value of the constant mass matrix $m$ . For problems $M(t, y)y' = f(t, y)$ , set this property to <code>@Mfun</code> , where <code>Mfun</code> is a function that evaluates the mass matrix $M(t, y)$ .
MStateDependence	none   {weak}   strong	Dependence of the mass matrix on $y$ . Set this property to none for problems $M(t)y' = f(t, y)$ . Both weak and strong indicate $M(t, y)$ , but weak results in implicit solvers using approximations when solving algebraic equations. For use with all solvers except ode23s.
MvPattern	Sparse matrix	$\partial(M(t,y)v)/\partial y$ sparsity pattern. Set this property to a sparse matrix $S$ with $S(i,j)=1$ if for any $k$ , the $(i,k)$ component of $M(t,y)$ depends on component $j$ of $y$ , and $0$ otherwise. For use with the ode15s, ode23t, and ode23tb solvers when MStateDependence is strong.
MassSingular	yes   no   {maybe}	Indicates whether the mass matrix is singular. The default value of 'maybe' causes the solver to test whether the problem is a DAE. For use with the ode15s and ode23t solvers.
InitialSlope	Vector	Consistent initial slope $yp_0$ , where $yp_0$ satisfies $M(t_0, y_0)yp_0 = f(t_0, y_0) \text{ . For use with the } \frac{\text{ode15s}}{\text{solvers when solving DAEs}} \text{ and } \frac{\text{ode23t}}{\text{solvers when solving DAEs}}.$

# **Step Size Properties**

Property	Value	Description
MaxStep	Positive	An upper bound on the magnitude of the step size that the solver uses. The

	scalar	default is one-tenth of the tspan interval.
InitialStep	Positive scalar	Suggested initial step size. The solver tries this first, but if too large an error results, the solver uses a smaller step size.

In addition there are two options that apply only to the ode15s solver.

# ode15s Properties

Property	Value	Description
MaxOrder	1   2   3   4   {5}	The maximum order formula used.
BDF	on {off}	Set on to specify that ode15s should use the backward differentiation formulas (BDFs) instead of the default numerical differentiation formulas (NDFs).

# See Also

 $\underline{\texttt{deval}}, \underline{\texttt{odeget}}, \underline{\texttt{ode23}}, \underline{\texttt{ode23t}}, \underline{\texttt{ode23tb}}, \underline{\texttt{ode113}}, \underline{\texttt{ode15s}}, \underline{\texttt{ode23s}}, \underline{\texttt{@}} \ (function \ handle)$ 





# •

# deval

Evaluate the solution of a differential equation problem

# **Syntax**

```
sxint = deval(sol,xint)
```

# **Description**

sxint = deval(sol, xint) evaluates the solution of a differential equation problem at each element of the vector xint. For each i, sxint(:,i) is the solution corresponding to xint(i).

The input argument sol is a structure returned by an initial value problem solver (ode45, ode23, ode23, ode23, ode23t, ode23tb) or the boundary value problem solver (bvp4c). The ordered row vector sol.x contains the independent variable. For each i, the column sol.y(:,i) contains the solution at sol.x(i). The structure sol also contains data needed for interpolating the solution in (x(i), x(i+1)). The form of this data depends on the solver that creates sol. The field sol.solver contains the name of that solver.

Elements of xint must be in the interval [sol.x(1), sol.x(end)].

#### See Also

ODE solvers: ode45, ode23, ode113, ode15s, ode23s, ode23t, ode23tb

BVP solver: bvp4c



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