Antenna parameters

Definition, general considerations and fundamental parameters

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Outline

- Radiation fundaments
- Fundamental parameters of antennas
  - Input impedance
  - Radiation patterns
  - Gain
  - Polarization
- Friis Formula
- Antenna noise temperature

Maxwell Equations

•The radiation phenomena of an antenna and of wave propagation are electromagnetic phenomena. So they are described by Maxwell equations that give the relation of electric and magnetic field with the sources (currents and charges).

•Constitute the mathematical basis for the resolution of electromagnetic problems of radiation and propagation waves.

FIELDS
E: Electric field intensity
H: Magnetic field intensity
D: Electric flux density
B: Magnetic flux density

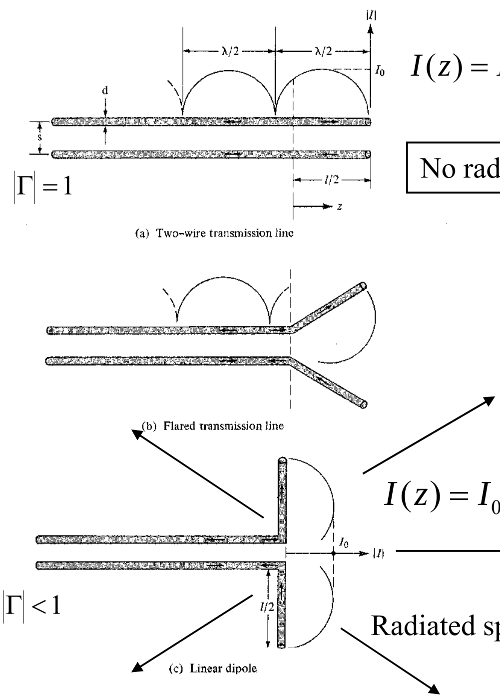
SOURCES
ρ: Electric charge density
J: Electric current density
J_c: Conduction electric current density

MEDIUM
ε: Electric permittivity
μ: Magnetic permeability
σ: Conductivity

$\nabla \times \vec{E} = -j\omega\vec{B}$	Faraday Law
$\nabla \times \vec{H} = j\omega\vec{D} + \vec{J}$	Generalized Ampere Law
$\nabla \cdot \vec{D} = \rho$	Gauss Law
$\nabla \cdot \vec{B} = 0$	Magnetic Flux Continuity
$\nabla \cdot \vec{J} + j\omega\rho = 0$	Continuity Equation
$\vec{D} = \varepsilon\vec{E}$, $\vec{B} = \mu\vec{H}$, $\vec{J}_c = \sigma\vec{E}$	Constitutive Material Equations

Current Distribution

- The current distribution is the function that define the way that takes the current on the antenna
- It is fixed by the boundary condition from Maxwell equations.
  - In permanent sinusoidal regime we apply:
    - » E_{tang} (on conductors)=0
- In some case, the distribution is modelled using very easy reasoning: as for example, the figure justify the approximated distribution in typical stationary wave of a dipole.



$$I(z) = I_0 \sin k_0\left(\frac{L}{2} - z\right)$$

No radiation if s<<λ

$|\Gamma| = 1$

$$I(z) = I_0 \sin\left(\frac{L}{2} - |z|\right)$$

$|\Gamma| < 1$

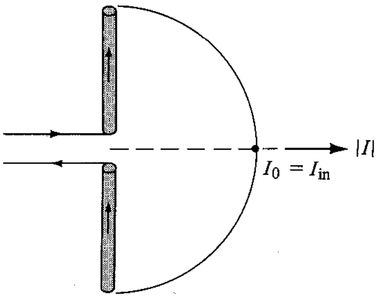
Radiated spherical wave

Current distribution: temporal variation

• For a dipole λ/2

Complex amplitude:

$I(z) = I_0 \cos(k_0 z)$



(b) $l = \lambda/2$

Instantaneous current:

$I(z,t) = \mathrm{Re}\left[I(z)e^{j\omega t}\right] = I_0 \cos(k_0 z)\cos(\omega t)$

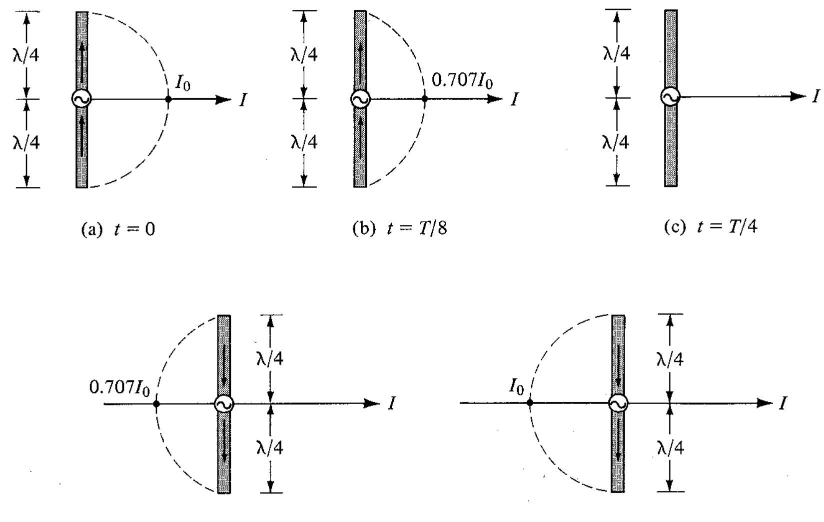


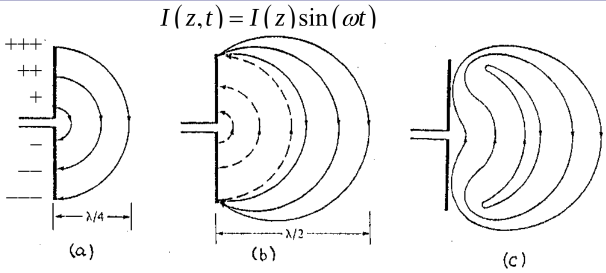
Figure 1.17 Current distribution on a λ/2 wire antenna for different times.

Radiation Mechanism

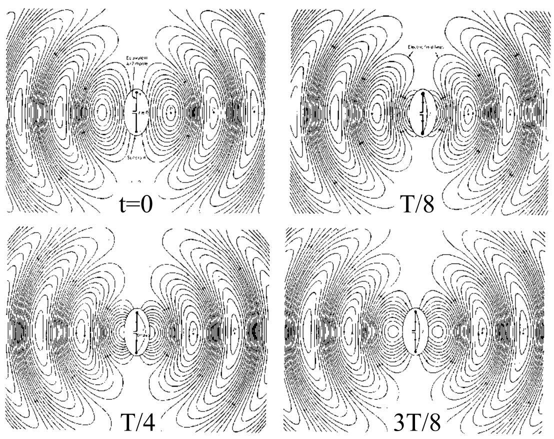
- Field line generation for a dipole:
  - (a) For the first quarter period, the current accumulate positive charge in the superior half-arm and negative in the inferior one, ending the circuit across the displacement currents that follow the field lines.
  - (b) In the following quarter of period, the current invert generating displacement current (field lines) in the contrary sense that push the previous toward outside.
  - (c) When the first half period finalize, the charge is null in all over the dipole and the field lines close on themselves.

- Radiated wave evolution in permanent sinusoidal regime:
  - The radiated electromagnetic waves behave like water waves in a pool.

$I(z,t) = I(z)\sin(\omega t)$



a) t=T/4 b) t=T/2 c) t>T/2



Potentials

- The electromagnetic problems of open geometry **as antennas** can be solve more easily if it is introduce auxiliary potentials derived from Maxwell equations.
  - $\vec{A}$ (potential magnetic vector)

$$\nabla \cdot \vec{B} = 0 \quad \Rightarrow \quad \vec{B} = \nabla \times \vec{A} \qquad \text{as} \qquad \nabla \cdot \left(\nabla \times \vec{A}\right) \equiv 0$$

  - Φ (scalar potential)

$$\nabla \times \vec{E} = -j\omega\vec{B}$$

$$\nabla \times \vec{E} = -j\omega\nabla \times \vec{A}$$

$$\nabla \times \left(\vec{E} + j\omega\vec{A}\right) = 0 \quad \Rightarrow \quad \vec{E} + j\omega\vec{A} = -\nabla\Phi \qquad \text{as} \qquad \nabla \times \left(\nabla\Phi\right) \equiv 0$$

$$k_o^{\ 2} \equiv \omega^2\mu_0\varepsilon_0$$

- Wave equation or Helmholtz equation:

$$\Delta\vec{A} + \omega^2\mu_0\varepsilon_0\vec{A} = -\mu\vec{J} \qquad \Delta\Phi + \omega^2\mu_0\varepsilon_0\Phi = -\frac{\rho}{\varepsilon_0}$$

The wave equations give a direct relation between the sources J and ρ

Electric and magnetic field:

$$\vec{E} = -\nabla\Phi - j\omega\vec{A} \qquad \vec{H} = \frac{1}{\mu_0}\nabla \times \vec{A} \qquad \vec{E} = \frac{1}{j\omega\varepsilon_0}\nabla \times \vec{H}$$

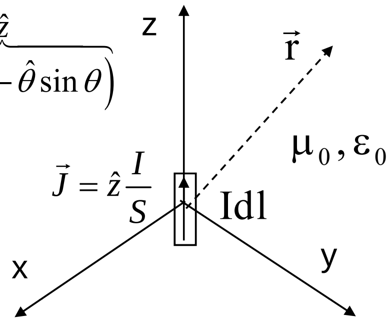
Radiated field for an element current

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{e^{-jk_0 r}}{r} Idl \overbrace{\left(\hat{r}\cos\theta - \hat{\theta}\sin\theta\right)}^{\hat{z}}$$

$$\vec{H} = \frac{1}{\mu_0}\nabla \times \vec{A}$$

$$\vec{E} = \frac{1}{j\omega\varepsilon_0}\nabla \times \vec{H}$$

$$\vec{J} = \hat{z}\frac{I}{S}$$



Radiated fields using as example the case of the more easiest antenna:

➢**An infinitesimal source with electric current going in the direction of z axis, situated in free space.**

- The fields that produce the element current in the origin, valid for any space point are:

$$\vec{H} = \hat{\phi}\frac{Idl\sin\theta}{4\pi r}\left(jk_0 + \frac{1}{r}\right)e^{-jk_0 r}$$

$$\vec{E} = \frac{j\eta Idl}{2\pi k_0}\left[\hat{r}\cos\theta\left(\frac{jk_0}{r^2} + \frac{1}{r^3}\right) + \hat{\theta}\frac{\sin\theta}{2}\left(-\frac{k_0^2}{r} + \frac{jk_0}{r^2} + \frac{1}{r^3}\right)\right]e^{-jk_0 r}$$

$$\eta = \sqrt{\mu_o/\varepsilon_o} = 120\pi = 377\Omega \qquad \text{(free space impedance)}$$

Farfield of an element current

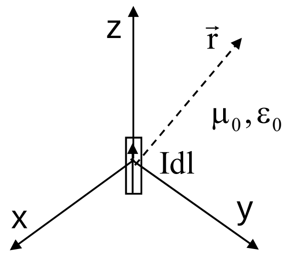
- If $k_0 r >> 1$ ($r >> \lambda$) predominate the terms in $1/r$ than $1/r^2$ o $1/r^3$, obtaining the following expressions valid for farfield:

$$\vec{H} = jk_0 I\, dl \sin\theta \frac{e^{-jk_0 r}}{4\pi r}\hat{\phi}$$

$$\vec{E} = j\eta k_0 I\, dl \sin\theta \frac{e^{-jk_0 r}}{4\pi r}\hat{\theta}$$

Radiated fields:

E ⊥ r, H⊥ r, E⊥ H



Radiated fields using as example the case of the more easiest antenna:

➢**An infinitesimal source with electric current going in the direction of z axis , situated in free space**

- **The radiated power density** (given by the Poynting vector) is radially outside pointing and decrease as $1/r^2$ for a lossless medium (progressive spherical wave)

$$<\vec{S}> = \frac{1}{2}\mathrm{Re}\left[\vec{E}\times\vec{H}^*\right] = \left[\frac{|I|^2 dl^2 k_o{}^2 \eta\, \mathrm{sen}^2(\theta)}{32\pi^2 r^2}\right]\hat{r} = \frac{1}{2\eta}|E|^2\hat{r}$$

- The fields terms in $1/r^2$ y $1/r^3$ represent reactive energy stored in these fields, with appreciate values only near the antenna.

Wavelength

- To visualize the radiates wave, it is convenient to compare the instantaneous expressions of the current source and the generated potential (for field is similar):

$$I(t) = \mathrm{Re}\left[I \exp(j\omega t)\right] = I\cos(\omega t)$$

$$\vec{A}(\vec{r},t) = \mathrm{Re}\left[\vec{A}e^{j\omega t}\right] = \mathrm{Re}\left[\hat{z}C_1\frac{e^{-jk_0 r}}{r}e^{j\omega t}\right] = \hat{z}\frac{C_1}{r}\cos(\omega t - k_0 r) = \hat{z}\frac{C_1}{r}\cos\left[\omega\left(t - \frac{r}{c}\right)\right]$$

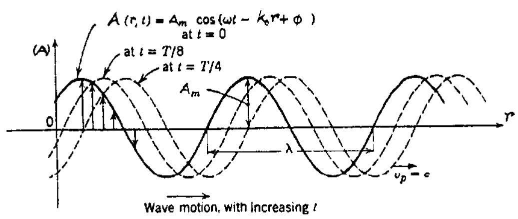
  - r/c=propagation time o delay that takes to the wave in travelling from the transmitter source until the observation point.
  - **At a large distance**, in an interval $\Delta r << r$, the spherical wave behave like a plane wave with a wavelength (distance between two consecutive and equal phase points)

Wave propagation and wavelength concept

$$\lambda = cT = c/f = \frac{2\pi}{\omega}\frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{2\pi}{k_0}$$

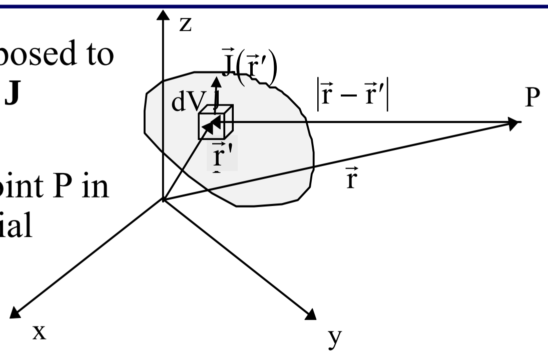
Propagation constant $= k_0 = \omega\sqrt{\mu_0\varepsilon_0} = 2\pi/\lambda$

Wavelength in cm: $\lambda(cm) = 30/f\,(GHz)$



Radiation of an antenna

- A real antenna have a current distribution supposed to be formed by infinites elements dV of current **J** situated in a point **r'**.
- Each infinites elements dV of current J in a point P in the space will generate a differential of potential magnetic vector dA



$$d\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{e^{-jk_0|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \vec{J}(\vec{r}')dV$$

•For the total radiated potential will be the superposition.

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint_{V'} \frac{\vec{J}(\vec{r}')e^{-jk_0|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} dV'$$

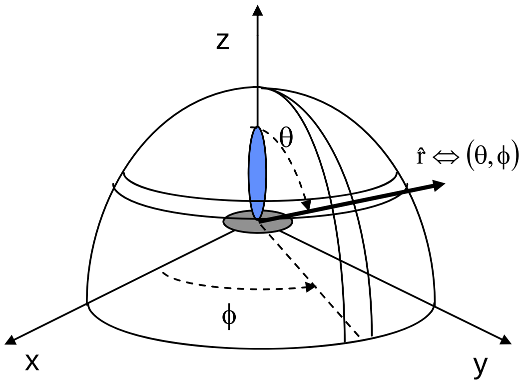
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_{S'} \frac{\vec{J}_s(\vec{r}')e^{-jk_0|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} dS'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_{L'} \frac{I(\vec{r}')e^{-jk_0|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} d\vec{l}'$$

Volume | Surface | Wire Antenna (diameter << λ)

Far field properties

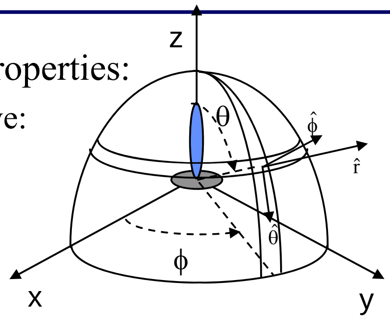
- An antenna radiates, in far field:
  - The radiated electromagnetic wave is propagated radially in all the space directions.
  - The dependence of **E** and **H** with *r* is <u>always</u> one of a spherical wave e^{-jk0r}/r. The field decrease with the distance as 1/r.
  - The field **E** y **H** depend with θ and ϕ because the spherical wave is not homogeneous. To analyze its variation, it is used the following spherical system.



$$0 \le \theta \le \pi$$
$$0 \le \phi < 2\pi$$

Far field properties

- The radiation fields of any antenna fulfil these general properties:
  - The radiated spherical wave behave locally as a plane wave: a fixed direction (θ,ϕ):



$0 \le \theta \le \pi$

$0 \le \phi < 2\pi$

$$\vec{E} \perp \hat{r} \qquad \vec{E} \perp \vec{H}$$
$$\vec{H} \perp \hat{r} \qquad |E| = \eta |H|$$

  - The fields E and H do not have radial components:

$$\left.\begin{aligned}\vec{A}(\vec{r}) &= A_r\hat{r} + A_\theta\hat{\theta} + A_\phi\hat{\phi} \\ \vec{E} &= -j\omega\left(\left(\hat{r}\times\vec{A}\right)\times\hat{r}\right)\end{aligned}\right\} \qquad \begin{aligned} E_r &= 0 & H_r &= 0 \\ E_\theta &= -j\omega A_\theta & E_\theta/H_\phi &= \eta \\ E_\phi &= -j\omega A_\phi & -E_\phi/H_\theta &= \eta \end{aligned}$$

  - **The power density** that transport the wave decrease as 1/r2. If the medium have no losses, it is defined:

$$<\vec{S}> = \frac{1}{2}\mathrm{Re}\left[\vec{E}\times\vec{H}^*\right] = \frac{1}{2\eta}\left[\left|E_\theta(r,\theta,\phi)\right|^2 + \left|E_\phi(r,\theta,\phi)\right|^2\right]\hat{r}$$

Fundamental parameters of radiation

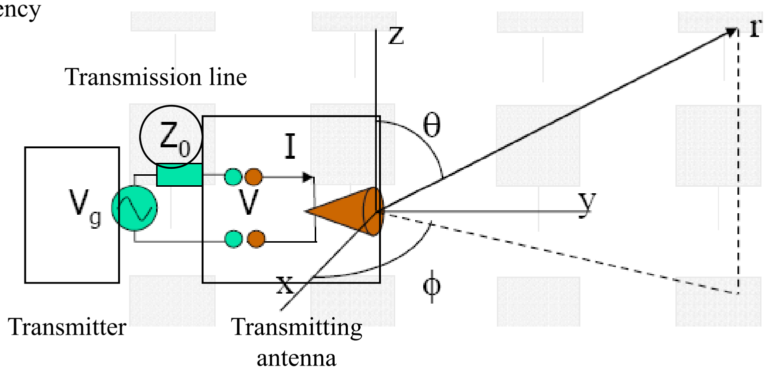
- Before, we saw how to determine, from the Maxwell equations, the radiated electric and magnetic fields of an antenna.
- As the field expressions are much complex, so **for the antenna characterization** we use parameters that can be measured and be easier to analyse.
- The measured parameters of an antenna follow the IEEE 145-1993 standard.
- Those parameters allow to consider the antenna as a black box and calculate the parameters values of a radio communication link.
- The most important parameters are:
  - **Input impedance**
  - **Radiation pattern**
  - **Gain**
  - **Polarization**

Parameters of an antenna: Impedance

- Feeding a linear antenna with a sinusoidal generator of frequency f, we can define a circuit model:
- From the input terminals
  - Input impedance is generally a complex number that varies with the frequency
  - Input reflection coefficient

$$Z_{antenna} = \frac{V}{I}$$

$$Z_{antenna} = R_{antenna} + jX_{antenna}$$



Generally, $X_{antenna}(f)=0$, resonant antenna

$$\Gamma_T = \frac{Z_{ant} - Z_g^{*}}{Z_{ant} + Z_g}$$

Impedance parameters

- The real part of the input impedance antenna is the sum of the loss resistance (associate to the energy that is dissipated) and the radiated resistance (associate to the radiated energy).

$$Radiation\ efficiency = \eta_{rad} = \frac{P_{radiated}}{P_{input}} = \frac{R_{radiation}}{R_{losses} + R_{radation}}$$

- Others alternative parameters to the input impedance, more easy to measure in the high frequency range are:
  - Return losses (dB):

$$\mathrm{RL(dB)} = 10\log\frac{P_{ref}}{P_{inc}} = 20\log|\Gamma_T|$$

VSWR:

$$VSWR = \frac{1+|\Gamma_T|}{1-|\Gamma_T|}$$

- Available power of the transmitter and transmitted to the antenna:

$$P_{inc} = \frac{1}{8}\frac{|V_g|^2}{R_g}$$

$$P_{trans} = P_{inc} - P_{ref} = P_{inc}\left(1-|\Gamma_T|^2\right)$$

- $VSWR_{MAX}$ should be 2 $\Rightarrow RL = -9.5\ dB \Rightarrow 12\%$ of power loss

Parameters of an antenna: Radiation pattern

- In the radiation far field
  - Radiation pattern of electric field F
  - Polarization pattern ê

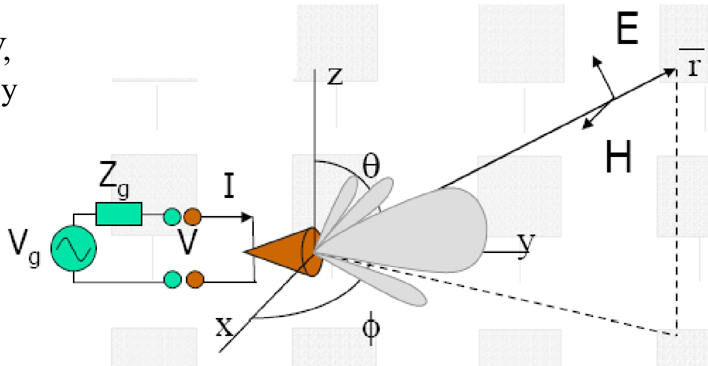
$$\vec{H} = jk_0 Idl sen\,\theta \frac{e^{-jk_0 r}}{4\pi r}\hat{\phi}$$

$$\vec{E} = j\eta k_0 Idl sen\,\theta \frac{e^{-jk_0 r}}{4\pi r}\hat{\theta}$$

$\hat{e}(\theta,\phi)$= polarization pattern

$\eta_0=120\pi$=free space impedance

When we feed the antenna with a voltage V, it is generated a current distribution (give by the Maxwell equations) that produce an electromagnetic radiation characterized by the fields **E** and **H**.



Radiation parameters: radiation pattern

- It is define as a *graphic representation of the radiation properties of an antenna in function of space angular coordinates*.
- Plot patterns of:
  - field : $|\underline{E}|$, E_θ,E_ϕ, arg(E_θ), arg(E_ϕ), E_{CP}, E_{XP}, etc
  - power : <S> = power density, Gain, Directivity.
- The formats that have the patterns are :
  - Absolute patterns: it plots the fields or power density for a delivered power to the antenna and a constant distance.
  - Relative patterns: like the anterior ones but normalized in relation to the maximum value of the plotted function. In this case the plot are in logarithmic scale (dB). So, the power and fields plots coincide because:

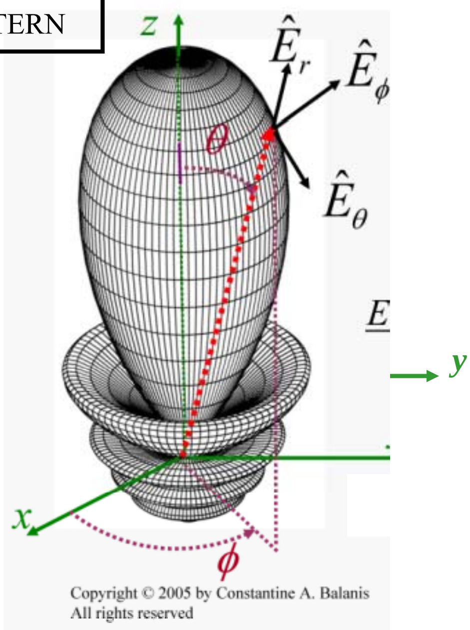
$$10\log\frac{<S>}{<S>_{max}} = 20\log\frac{|E|}{|E|_{max}}$$

Patterns type

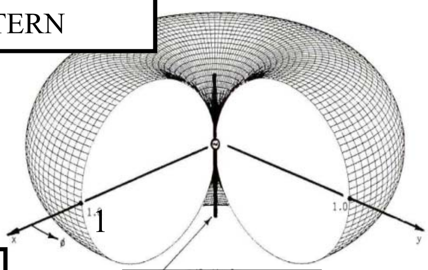
- Depending the application of the antenna, we classify:
  - Isotropic (quasi-isotropic)
  - Omnidirectional: Directional in one plane and isotropic in the other (symmetry revolution pattern).
  - Directional: concentrate fundamentally the radiation in a small angular cone:
    - » Pencil beam: conic beam (f.e. for point to point communication)
    - » Fan beam (f.e. base station mobile communication sectorial antennas)
    - » Contour beam, typical for adjusted coverage in DBS systems
    - » Beamforming, typical for security radar
    - » Multibeam (several main lobes)
  - Multi-pattern: several simultaneous patterns depending of the feeding input
  - Reconfigurable beamwidth antennas: when we can control the radiation pattern by control remote depending of the communication system. Interesting for antennas in satellites.
  - Adaptative antennas: when the radiation pattern is instantaneous adapted to the radio electric environment

3D patterns

DIRECTIVE PATTERN

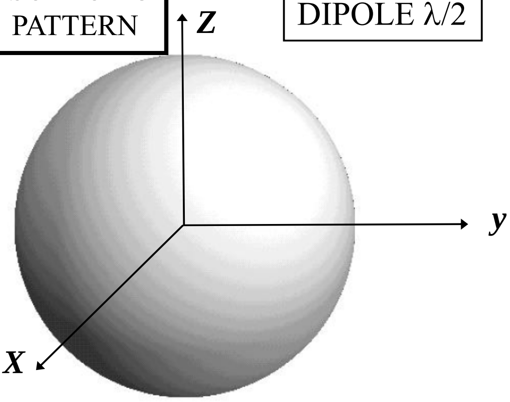


OMNIDIRECTIONAL PATTERN



DIPOLE λ/2

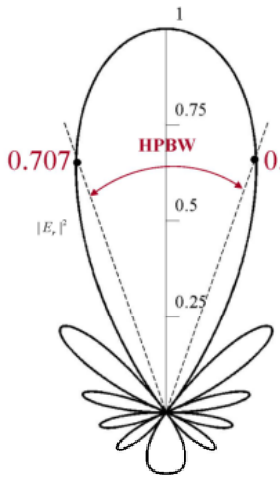
ISOTROPIC PATTERN

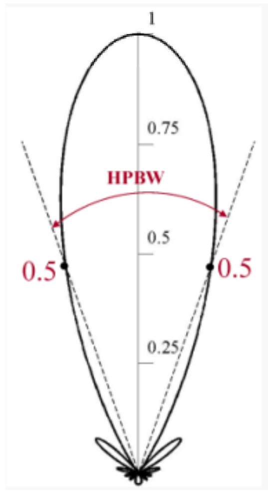


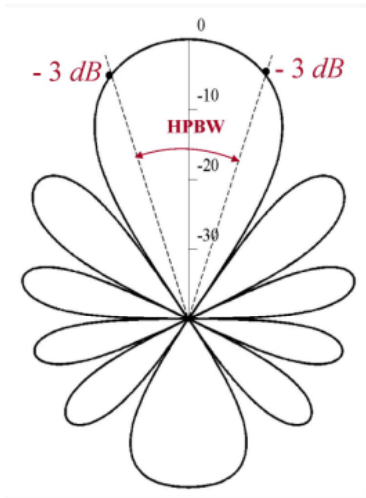
2D patterns

POLAR PLOTS AND BEAMWIDTH DEFINITION between half power points (at –3 dB)

HPBW= Half-Power Beamwidth



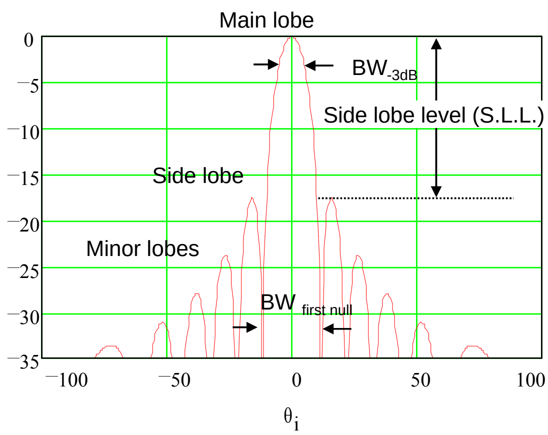




Normalized field pattern *Normalized power pattern* *Normalized pattern in [dB]*

Radiation pattern parameters

- Lobe: **portion of the radiation pattern bounded by regions of relatively weak radiation intensity (nulls)**
  - Major or main lobe: radiation lobe containing the direction of maximum radiation.
  - Minor lobes: any lobe except a major lobe.
  - Side lobes: a radiation lobe in any direction other than the intended lobe. Usually is adjacent to the main lobe.
  - Back lobe: a radiation lobe whose axis makes an angle of aprox. 180º with respect to the beam of the antenna.
- Side lobe level: **ratio of the power density in the lobe in question to that of the major lobe**
- Front to back ratio
- Half power beamwidth (HPBW), first null beamwidth (FNBW)



Radiation pattern 2D in dB. Cartesian plot

Radiation pattern: principal planes

- **For linear polarization directive antennas** it uses to be sufficient with the characterization of radiation pattern in main planes:
  - E-plane: the plane containing the electrical field vector and the direction of maximum radiation (YZ)
  - H-plane: the plane containing the magnetic field vector and the direction of maximum radiation (XZ)

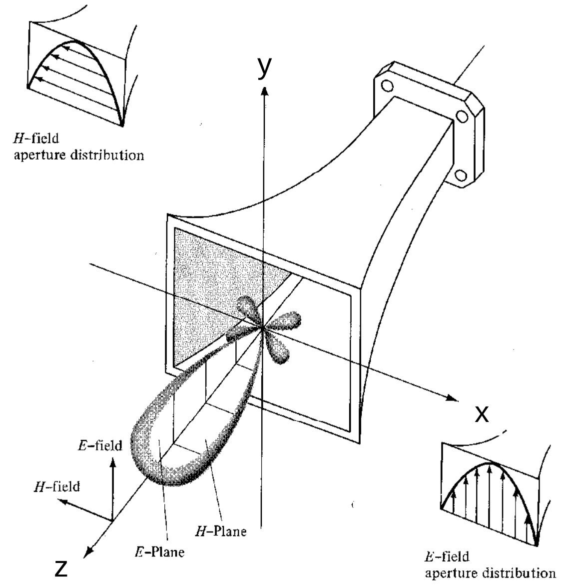
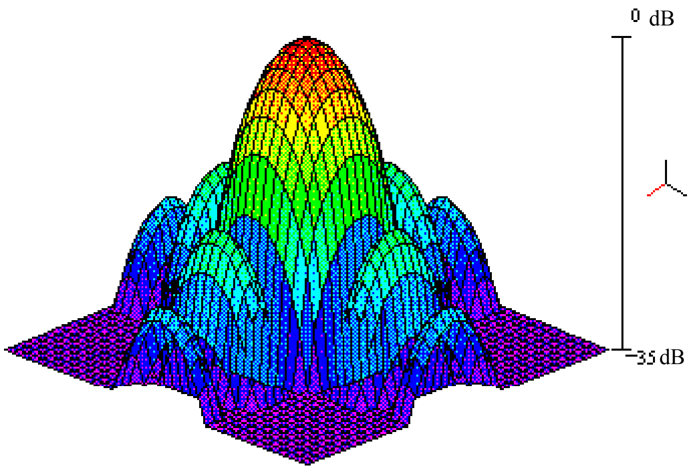


Figure 2.3 Principal E- and H-plane patterns for a pyramidal horn antenna.

Tridimensional representation

3D patterns in (u,v) coordinates

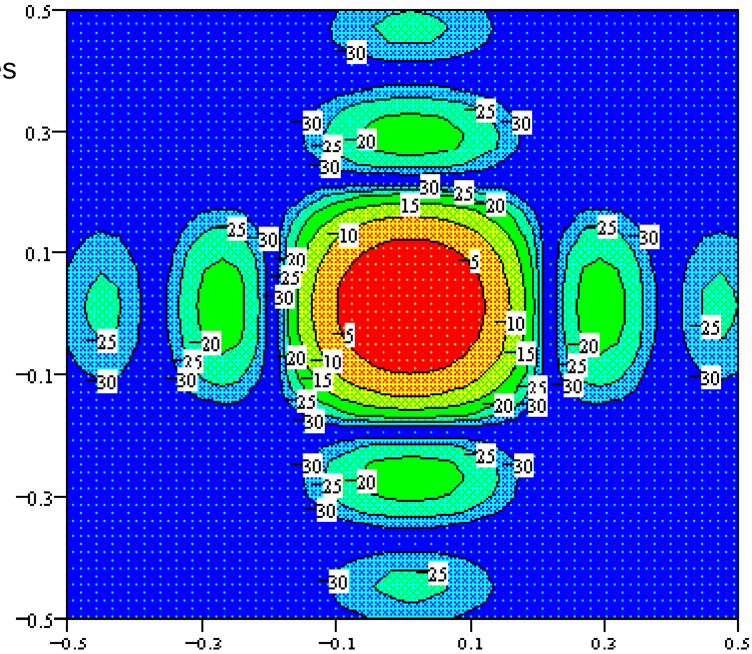
$$u = \sin\theta\cos\phi$$
$$v = \sin\theta\sin\phi$$



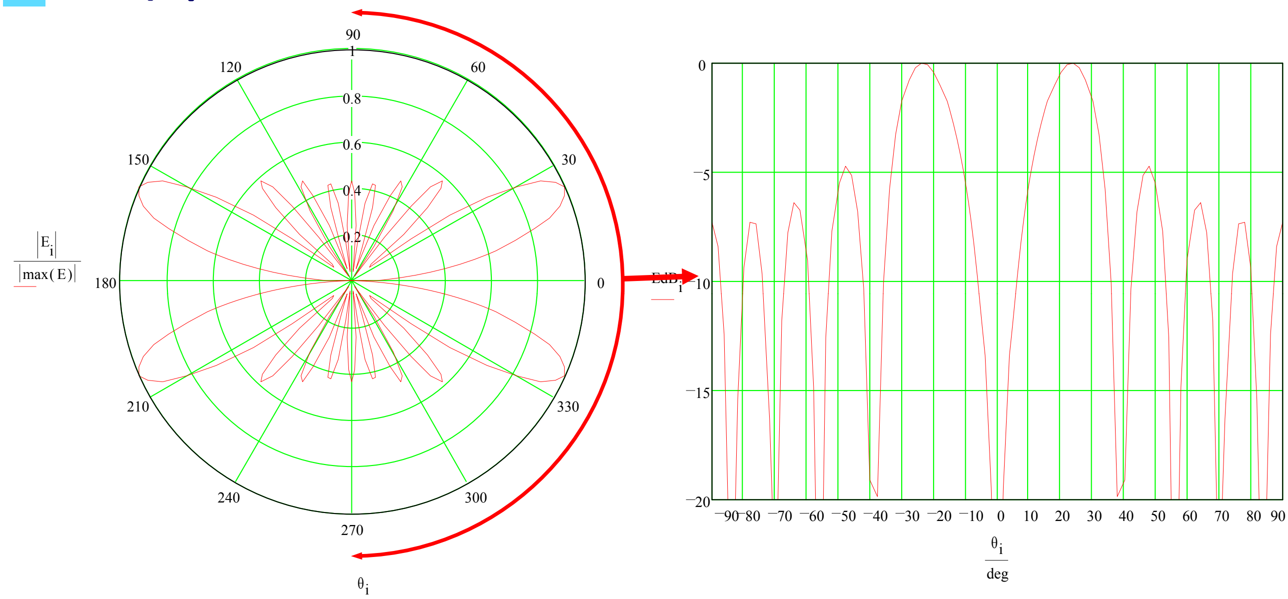
Bidimensional representation

2D patterns in (u,v) coordinates

$$u = \sin\theta\cos\phi$$
$$v = \sin\theta\sin\phi$$



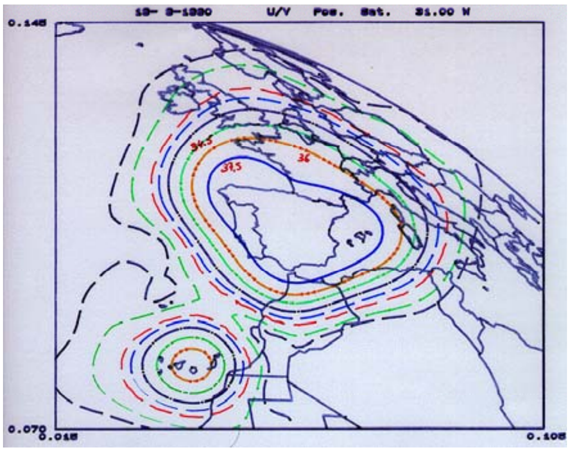
Radiation pattern planes: Polar and Cartesian representation



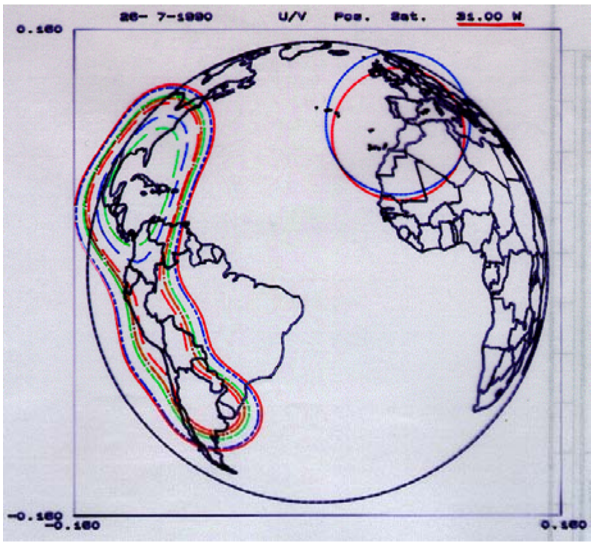
Polar (Linear) Cartesian (dB)

Examples of contour patterns

SSR



Multibeam pattern of contour beam DBS antenna from HISPASAT satellite.

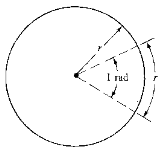


antenna TVA-GOV pattern (multipattern antenna) form HISPASAT satellite.

Radiation intensity

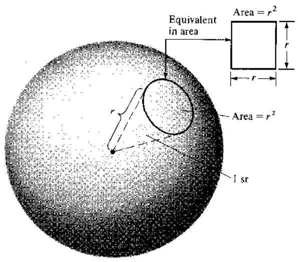
SSR

- Is the radiated power by solid angle in a determined direction.
- Represents the capacity that have an antenna to radiate the energy in this direction.



(a) Radian

- Solid angle:
  - Zone of the space included by a succession of radial lines with vertex in centre of a sphere.
  - Its unit is steradian (solid angle that includes a spherical surface r^2 with a radius r).

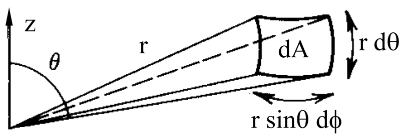


(b) Steradian

$$d\Omega = \frac{dA}{r^2} = \frac{r^2 \sin\theta d\theta d\phi}{r^2} = \sin\theta d\theta d\phi$$

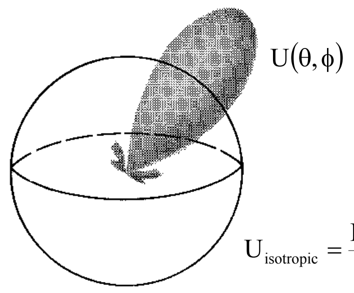
- Radiation intensity:
  - is the radiated power by solid angle unit.

$$U(\theta,\phi) = \frac{< S(r,\theta,\phi) > dA}{d\Omega} = r^2 < S(r,\theta,\phi) >$$



Directivity

- Directivity: D(θ,φ)
  - Represent the capacity that have the antenna to concentrate the radiation intensity in a determined direction.
  - The ratio of the radiation intensity in a given direction from the antenna to the radiation intensity of an isotropic antenna that radiated the equivalent total power averaged over all the directions.

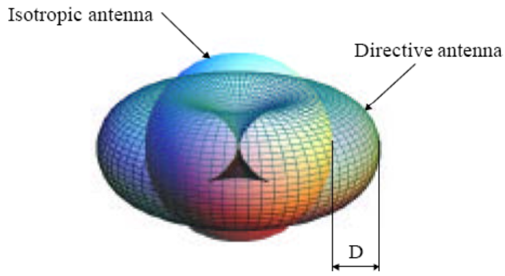


$$U_{isotropic} = \frac{P_{radiated}}{4\pi}$$

$$D(\theta,\phi) \stackrel{\Delta}{=} \frac{U(\theta,\phi)}{U_{Isotropic}} = 4\pi\frac{U(\theta,\phi)}{P_{radiated}} = 4\pi r^2 \frac{\langle S(r,\theta,\phi)\rangle}{P_{radiated}}$$

The total radiated power of an antenna:

$$P_{rad} = \int_{4\pi} U(\theta,\phi)d\Omega = \int_0^{\pi}\int_0^{2\pi} r^2 \langle S(r,\theta,\phi)\rangle \sin\theta d\theta d\phi$$



- Maximum directivity: D_0.
  - Directivity in maximum radiation direction.
  - It is always greater than 1 (0 dBi).
  - In dBi: 10 log D_0.

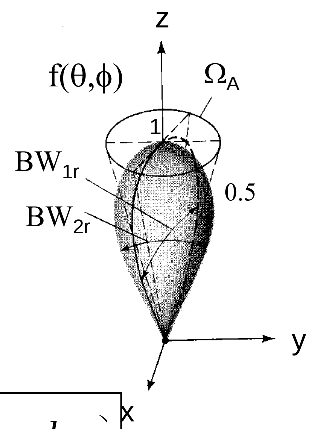
Directivity versus beamwidth

- From the normalized power plot: $f(\theta,\phi) = \frac{U(\theta,\phi)}{U_{max}} \le 1$

$$D(\theta,\phi) = 4\pi\frac{U(\theta,\phi)}{\int_{4\pi}U(\theta,\phi)d\Omega} = 4\pi\frac{f(\theta,\phi)}{\int_{4\pi}f(\theta,\phi)d\Omega} = D_0 f(\theta,\phi) \stackrel{\Delta}{=} 4\pi\frac{f(\theta,\phi)}{\Omega_A}$$

$$D_0 = \frac{4\pi}{\Omega_A}$$

- where Ω_A is the beam solid angle. $\Omega_A = \int_{4\pi} f(\theta,\phi)d\Omega$



- <u>For directive antennas</u>, pencil or fan beam type plot

$$\Omega_A \cong BW_{1r} \cdot BW_{2r} \quad (-3dB\ beamwidth)$$

$$D_0 \cong \frac{4\pi}{BW_{1r}\cdot BW_{2r}} = \frac{41253}{BW_{1d}\cdot BW_{2d}} \quad \begin{pmatrix} r: rad \\ d: degrees \end{pmatrix}$$

- <u>For omnidirectional antennas:</u>

$$D_0 \cong \frac{4\pi}{2\pi\cdot BW_{\theta r}} = \frac{114.6}{BW_{\theta d}} \quad \begin{pmatrix} r: rad \\ d: degrees \end{pmatrix}$$

Gain and Efficiency

- **Absolute gain: G(θ,ϕ),** is the ratio of the intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically.

$$G(\theta,\phi) \stackrel{\Delta}{=} 4\pi \frac{U(\theta,\phi)}{P_{in}} = 4\pi r^2 \frac{\langle S(r,\theta,\phi)\rangle}{P_{in}}$$

- **Maximum gain: G_0,** gain in maximum radiation direction
  - It can be lower than 1 (0 dBi)
  - In dBi: 10 log G_0.

- **Antenna Efficiency:** it is the relation between gain and directivity

$$\eta_R = \frac{P_{radiated}}{P_{in}} = \frac{G_0}{D_0} \qquad G(\theta,\phi) = \eta_R \cdot D(\theta,\phi)$$

Gain and Efficiency

- **E.I.R.P.: Equivalent Isotropic Radiated Power**

The EIRP is a figure of merit of the combined transmitter – antenna. If we divide it by $4\pi r^2$ (sphere area), we obtain the power density at a distance r. The EIRP curves are plotted in dBW.

$$EIRP(\theta,\phi) = G(\theta,\phi)\cdot P_{in} = D(\theta,\phi)\cdot P_{rad}$$

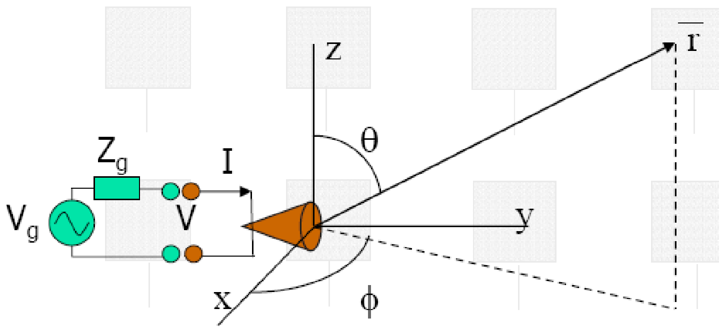
$$\langle S(r,\theta,\phi)\rangle = \frac{G(\theta,\phi)\cdot P_{in}}{4\pi r^2} \equiv \frac{EIRP(\theta,\phi)}{4\pi r^2} \qquad [W/m^2]$$

Parameters of an individual antenna: polarization pattern

- Another important element in the radiation pattern of the antenna is **the polarization**, that comes from its unitary vector.

$$\hat{e}(\theta,\phi) = \hat{\theta}\cdot\cos(\alpha(\theta,\phi)) + \hat{\phi}\sin(\alpha(\theta,\phi))e^{j\beta(\theta,\phi)}$$

- The shape and the orientation of the polarization ellipse depends on the amplitude relation α and the phases β between the electric field components.



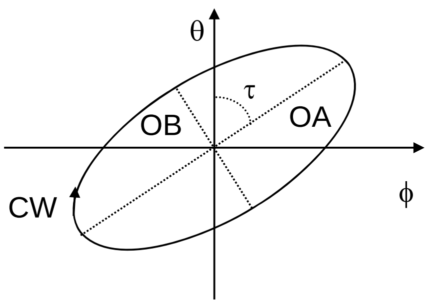
Polarization

- The polarization of an antenna in a given direction is defined as "the polarization of the wave transmitted (radiated) by the antenna.
- Polarization of a radiated wave is defined as "**the figure traced as a function of time, for a determined direction, by the extremity of the radiated field vector at a fixed location in space, and the sense in which it is traces, as observed situated on the antenna along the direction of propagation**".
- **The polarization concept** is important **in radio communication systems**, because the receiving antenna is only able to get the power contained in the field polarization that coincide with its own.

$$\vec{E} = E_\theta\hat{\theta} + E_\phi\hat{\phi} \qquad \begin{cases} E_\theta = |E_\theta|e^{j\delta_\theta} \\ E_\phi = |E_\phi|e^{j\delta_\phi} \end{cases}$$

$$E_{\theta i} = |E_\theta|\cos(\omega t + \delta_\theta)$$
$$E_{\phi i} = |E_\phi|\cos(\omega t + \delta_\phi)$$

$$\left(\frac{E_{\theta i}}{|E_\theta|}\right)^2 - 2\frac{E_{\theta i}}{|E_\theta|}\frac{E_{\phi i}}{|E_\phi|}\cos\delta + \left(\frac{E_{\phi i}}{|E_\phi|}\right)^2 = \sin^2\delta$$



Polarization ellipse

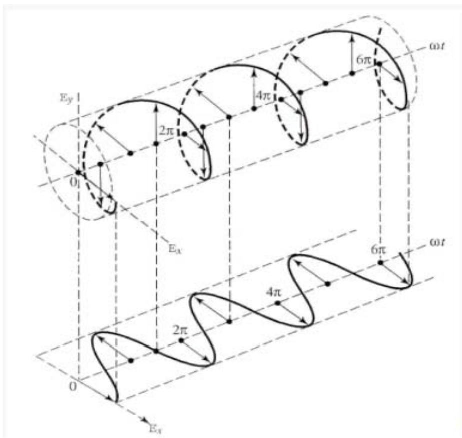
$\delta = \delta_\phi - \delta_\theta$ is the shift phase between the E_ϕ component with the E_θ component.

Polarization type

➢The polarization, that we generally achieved, is never perfectly circular of perfectly linear, but elliptic.

➢So this implies that an antenna radiated with a desired polarization, which have an undesired orthogonal polarization.

➢This is why we say “Copolar component (CP)” (for the desired field polarization) and “Cross polar component (XP)” (for the field polarization orthogonal to CP)



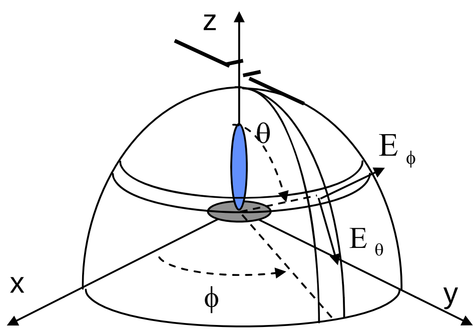
Right hand circular polarization

Horizontal linear polarization

Polarization: Co-polar and Cross-polar plots

$$\vec{E}(\theta,\phi) = E_\theta(\theta,\phi)\hat{\theta} + E_\phi(\theta,\phi)\hat{\phi}$$

$$\vec{E}(\theta,\phi) = E_{CP}(\theta,\phi)\hat{u}_{cp} + E_{XP}(\theta,\phi)\hat{u}_{xp}$$



CP and XP components:

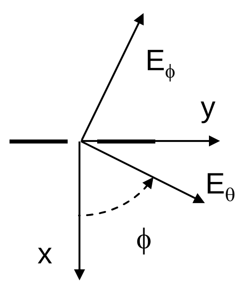
- Linear polarization:
  Ludwig 3rd definition for linear components (co polar on y-axes)

$$E_{CP}(\theta,\phi) = E_\theta(\theta,\phi)\sin\phi + E_\phi(\theta,\phi)\cos\phi$$
$$E_{XP}(\theta,\phi) = E_\theta(\theta,\phi)\cos\phi - E_\phi(\theta,\phi)\sin\phi$$

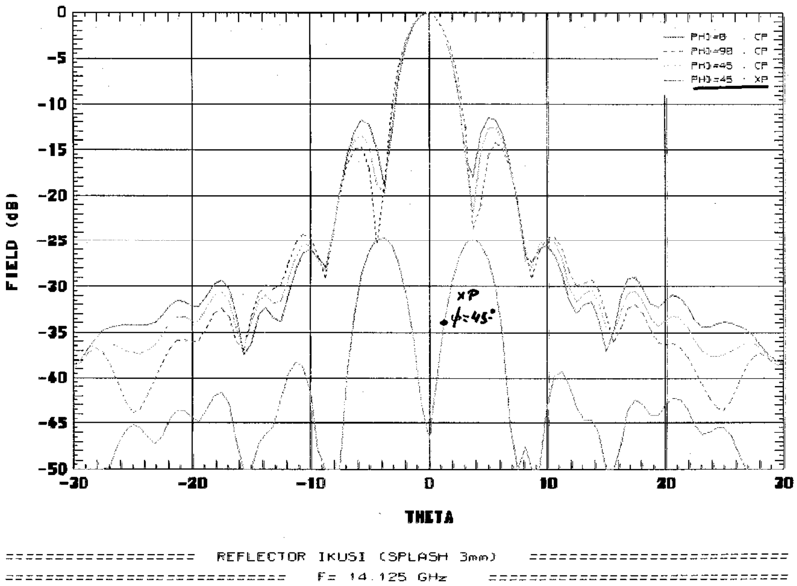
- Circular polarization:

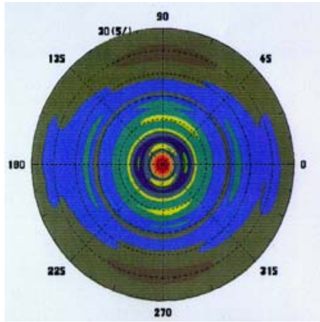
$$E_{RHC}(\theta,\phi) = \frac{1}{\sqrt{2}}\left(E_\theta(\theta,\phi) - jE_\phi(\theta,\phi)\right)e^{-j\phi}$$

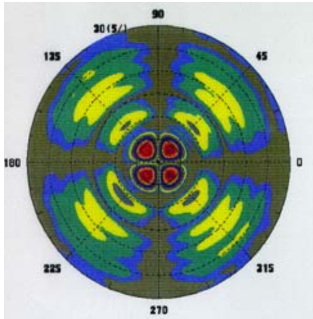
$$E_{LHC}(\theta,\phi) = \frac{1}{\sqrt{2}}\left(E_\theta(\theta,\phi) + jE_\phi(\theta,\phi)\right)e^{j\phi}$$



Typical CP-XP plots of a terrestrial station



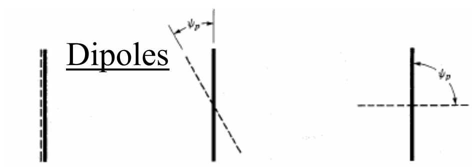




Polarization Loss Factor (PLF)

- Any field can be discompose in sum of two orthogonal components and to the direction of propagation to each other.
- **When a radio communication is settled down, the receiving antenna only coupled the component of coincident incident field with its polarization**. The polarization loss factor (PLF) is defined like the fraction of power that transports the incident wave in the polarization of the receiving antenna. This factor is calculated as the scalar product of the unitary vectors of polarization from the transmitting and receiving antenna in the link direction.

$$PLF = \left|\hat{e}_T(\theta,\phi)\cdot\hat{e}_R(\theta,\phi)\right|^2$$



PLF = 1 **PLF = $\cos^2\varphi_p$** **PLF = 0**

Polarization Loss Factor (PLF)

Examples:

•Linear polarization: a change of 1° in the polarization orientation, cause small losses in the copolar coupling. ⇒ PLF=$\cos^2(\varphi_p)$

•Circular polarization: perfect coupling (PLF=1) if the turn sense polarization of the transmitting and receiving antenna are the same. Complete uncoupling (PLF=0) if they are in contrary sense.

•For linear(transmitter) and circular polarization(receiver): PLF=1/2 (-3dB)

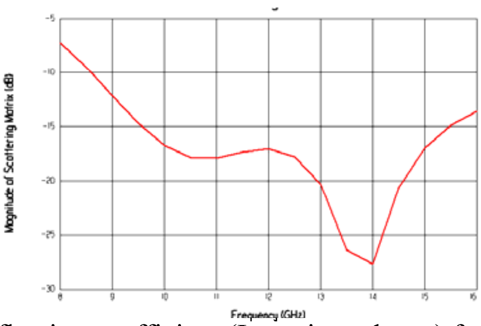
Dual polarization

- Nowadays because the saturation of the radio bands, the use of antennas of high polarization purity **allow to duplicate the capacity of a band using both polarization, transmitting and receiving channels that occupy the same band on two orthogonal polarizations**.
  - This is done for example in the fixed service by satellite, transmitting and receiving simultaneously orthogonal linear polarizations.
  - In order to avoid interferences between orthogonal channels, the radiation level crosspolar of the antennas do not have to be more than -35 dB.
- We notice that the anterior requirement also condition the position (adjustment) of the polarization axes of the terrestrial station.
  - A misalignment of 1° in the axis direction of polarization reference (maximum admitted variation in terrestrial stations) cause small losses in the copolar coupling but coupled -35 dB of crosspolar component.

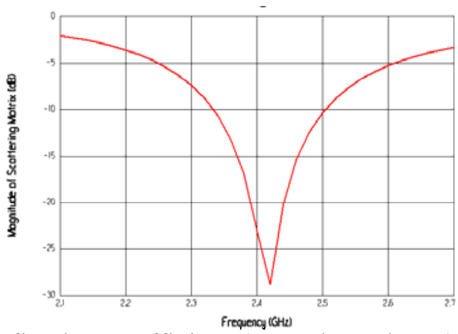
$$10\log\left(\cos^2 1^o\right) = 0.001\,\text{dB}$$
$$10\log\left(\cos^2 89^o\right) = -35.2\,\text{dB}$$

Bandwidth

- It is the frequency range where the characteristic parameters (input impedance (reflection coefficient), radiation pattern, gain,…) fulfil the prefixed specifications.
  - For the narrow band antennas (resonant antennas) it is usually defined in % of the resonance frequency.
  - For the broadband antennas, it is defined as the relation between the upper frequency of the band to the lower one, for example 2:1, 10:1 etc.



Reflection coefficient (Input impedance) for a wide bandwidth antenna

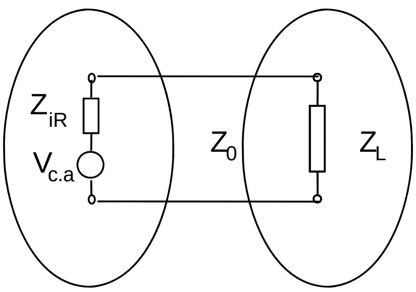


Reflection coefficient (Input impedance) for a narrow bandwidth antenna

- The antennas that are over a 2:1 relation for a certain specification (impedance…) they are designed based on angles and they receive the name of antennas independent of the frequency.

Circuital model of an antenna in reception

In an antenna, reciprocity $Z_{iR} = Z_{iT}$



Receiving Antenna Receiver

•Available power of the receiving antenna:

$$P_{available} = \frac{1}{8}\frac{|V_{ca}|^2}{R_{iR}}$$

• Input power at receiver:

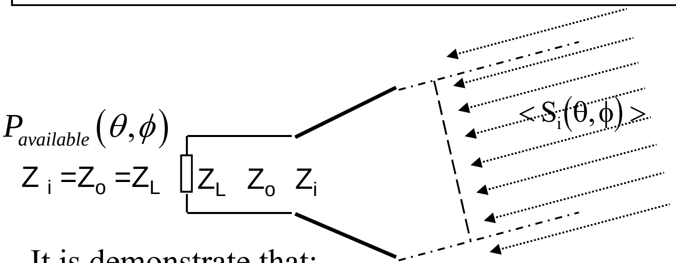
$$P_{received} = \frac{1}{2}|I_L|^2 R_L = P_{available}\left(1-|\Gamma_R|^2\right)$$

•Reflection coefficient (Z_o=Z_{iR}):

$$\Gamma_R = \frac{Z_L - Z_{iR}^*}{Z_{iR} + Z_L}$$

Absorption equivalent area

- If we consider the antenna as an aperture that get energy from the incident electromagnetic wave, we can define an equivalent antenna area (or effective area) as the "relation between the available power at the antenna and the power density of the incident wave".



$$A_e(\theta,\phi) = \frac{P_{available}(\theta,\phi)}{< S_i(\theta,\phi) >}$$

* This definition consider perfect coupling (PLF=1) in polarization between the incident wave and the antenna.

- It is demonstrate that:

$$A_e(\theta,\phi) = \frac{\lambda^2}{4\pi} G(\theta,\phi) \Rightarrow A_{emax} = \frac{\lambda^2}{4\pi} G_0$$

Reception pattern is identical to the transmission one

- Relation between the gain and the physical area for aperture antennas:

$$A_{emax} = \eta_r \cdot \varepsilon_a \cdot A_{aper} \qquad G_0 = \eta_r \varepsilon_a \frac{4\pi}{\lambda^2} A_{aperture}$$

ε_a: Aperture efficiency (≤ 1) $(0.5, 0.8)$

Friis Formulas: Propagation in free space

- **In all radiocomunication systems, we need to establish a power balance between the transmitter and the receiver to calculate the needed power in the transmitter that allow to reach a minimum level of signal in the receiver, that is over the noise.**

- Friis fomulas allow to calculate the insertion losses of a radiolink in function of the transmission parameters of each antennas, associated with the directions that which one see the other (Polarization Loss Factor (PLF)).

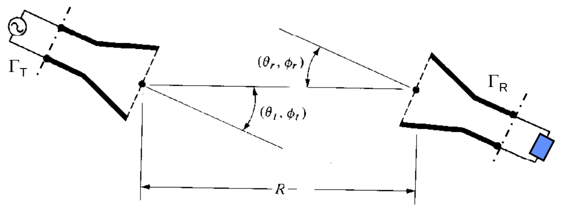
- These insertion losses are define as the ratio between the delivered power at the receiver P_{DR} and the available power at the transmitter P_{AT}.

Transmission equation: Friis Formulas

Using the definitions of power gain and the mismatches of the impedance in T_x and R_x, **a balance link can be made in conditions of free space**. This equation is what it is defined as **Friis Formula**:

$$\frac{P_{DR_x}}{P_{AT_x}} = \left|\hat{e}_T(\theta_t,\phi_t)\cdot\hat{e}_R(\theta_r,\phi_r)\right|^2 \cdot \left[1-\left|\Gamma_T\right|^2\right.$$

$$\left.1-\left|\Gamma_R\right|^2\right]\cdot\left(\frac{\lambda}{4\pi R}\right)^2 \cdot G_T(\theta_t,\phi_t)G_R(\theta_r,\phi_r)$$



Geometrical orientation of transmitting and receiving antennas

Alternative Friis Formula:

$$P_{DR} = \left\langle S_i(\theta,\phi)\right\rangle A_e(\theta,\phi)\cdot\left|\hat{e}_T\cdot\hat{e}_R\right|^2\left(1-\left|\Gamma_R\right|^2\right)$$

Antenna noise temperature

- **When a power balance is studied in a radiolink, not only the signal level is important also the noise that reach the receiver.**
- **All the bodies with a temperature different from 0K give incoherent radiation (noise).**
- **The antenna catches the radiation of all the bodies that surround it through their radiation pattern** and put it as an available noise power N_{AR} at the receiving antenna input.
- Being N_{AR}, the power of noise available in the antenna considering no losses, its noise temperature is defined as:
  - k, Boltzman cste. =1.38 10^{-23} (J/K)
  - B_f, noise bandwidth (Hz)
  - T_a, antenna noise temperature (K)

Nyquist Formula

$$T_a = \frac{N_{AR}}{k\cdot B_f}$$

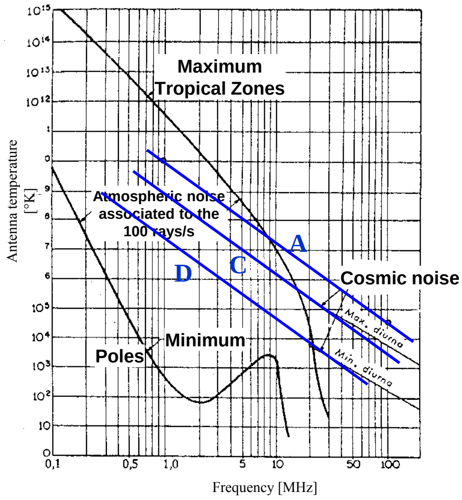
Antenna noise temperature

- Based on brightness temperature $T_B\,(\theta,\phi)$ associated to the noise radiation that impinges on the antenna for the (θ,ϕ) direction, the antenna temperature T_a is obtained as:

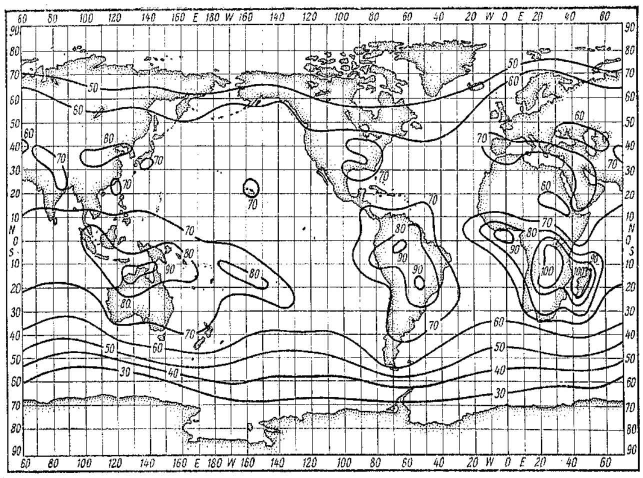
$$T_a = \frac{\int_{4\pi} T_B(\theta,\phi)\cdot f(\theta,\phi)d\Omega}{\int_{4\pi} f(\theta,\phi)d\Omega} = \frac{1}{\Omega_a}\int_{4\pi} T_B(\theta,\phi)\cdot f(\theta,\phi)d\Omega$$

The antenna noise temperature T_a depends on the antenna orientation in the atmosphere and of the frequency work.

Typical values of T_a (MF, HF y VHF)



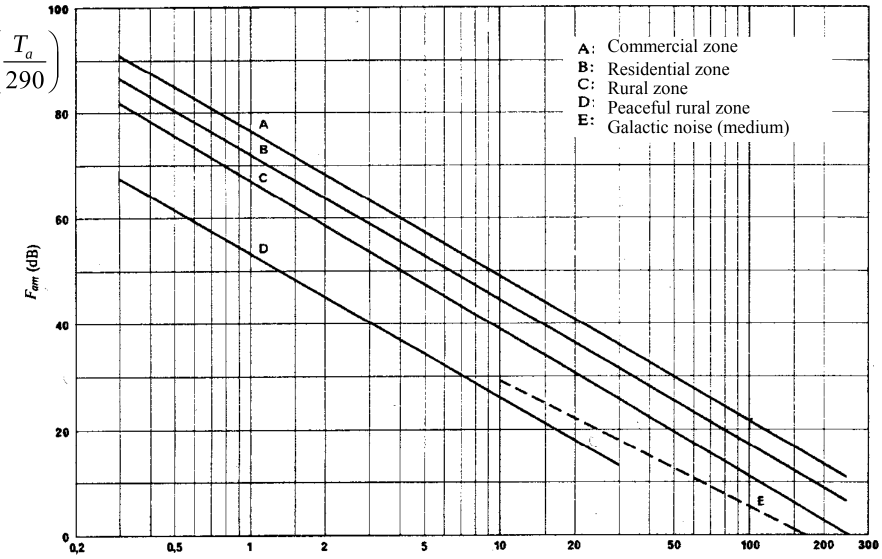
MF and HF: Noise temperature


Isolines of atmospheric noise at 1 MHz in dB refers to KT_aB

Typical values of T_a (MF, HF y VHF)

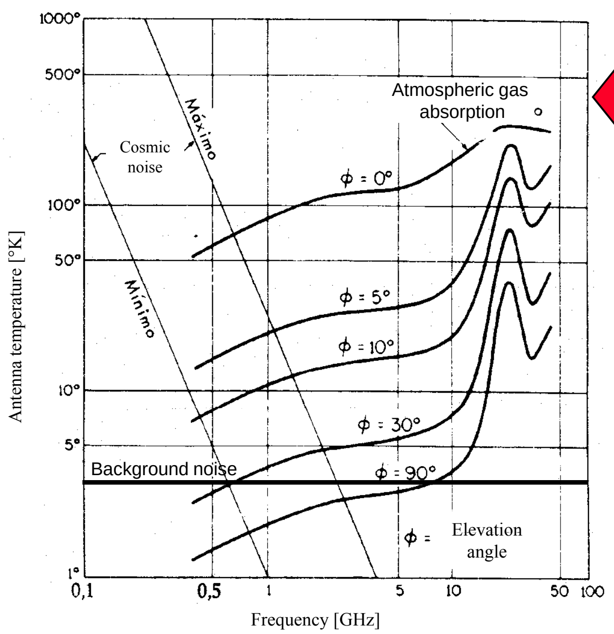
Noise from industrial type:

$$10\log\left(\frac{T_a}{290}\right)$$



Industrial type noise

Typical values of T_a (Microwave band)



Noise temperature in microwave frequency

Narrow beam antennas pointing with the main lobe at an elevation ϕ over the horizon with clear atmosphere (without considering ground contribution)

The atmospheric attenuation produce by the rain, the fog, etc. increase the antenna temperature as:

$$\Delta T_a = T_m\left(1-10^{-L/10}\right)$$

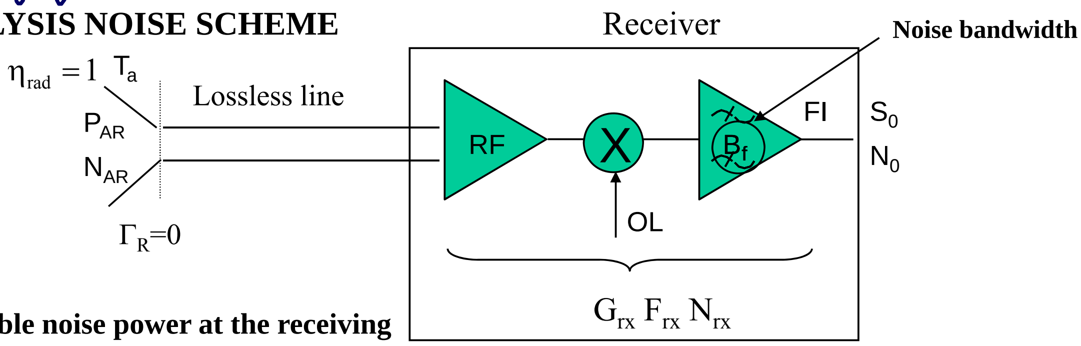
(T_m, medium value of the atmosphere temperature and L additional atmosphere attenuation.

Atmosphere attenuation L	Additional noise temperature of the antenna (ΔT_A)
0,5 dB	27 K
1 dB	51 K
2 dB	92 K
3 dB	124 K
5 dB	170 K

Typical increase in the microwave range

Noise analysis

ANALYSIS NOISE SCHEME



Available noise power at the receiving antenna:

$$N_{received} = kB_f T_A$$

$$N_{rx} = kT_{rx}B_f G_{rx} = k(f_{rx} - 1)T_o B_f G_{rx}$$

Noise power in the receiver

Signal/ noise ratio at the output:

$$\frac{S_o}{N_o} = \frac{G_{rx} \cdot P_{received}}{G_{rx} N_{received} + N_{rx}}$$

$$\frac{S_o}{N_o} = \frac{P_{received}}{kB_f (T_A + T_{rx})} = \frac{P_{received}}{kB_f T}$$

G/T parameter

Known the **total noise temperature of the system**:

$$T = T_r + T_a$$

$$\frac{S_0}{N_0} = \frac{G_A P_{DR}}{G_A N_{DR} + N_S} = \frac{P_{DR}}{kB_f (T_A + T_r)} = \frac{P_{DR}}{kB_f T}$$

SENSITIVITY= $P_{ARminimum}$

Friis Formula

$$\frac{P_{DRMinima}}{P_{DT}} = |\hat{e}_T \cdot \hat{e}_R|^2 \cdot \left[1 - |\Gamma_T|^2\right] \cdot \left(\frac{\lambda}{4\pi R}\right)^2 \cdot G_T \cdot G_R$$

Calculation of link parameters:
- Transmitter power
- Antennas gain, etc.

$$P_{DR} = \langle S_i \rangle A_e$$

$$A_e = \frac{\lambda^2}{4\pi} G_R$$

$$\frac{S_0}{N_0} = \frac{\langle S_i \rangle}{kB_f} \frac{\lambda^2}{4\pi} \left(\frac{G_R}{T}\right)$$

G/T (dB(1/K)) = 10 log (G/T)
is a global merit factor of the receiving system that are fixed by the antenna gain (G_R) and for the receiver quality (F_{rx}).

Exercice 1

1. We want to measure a parabolic antenna of 1 m diameter at 10 GHz. Calculate the minimum distance that has to be the probe of measurement to obtain the radiation pattern of the parabolic antenna in far field?

2. An antenna radiated in the z axis direction a field $\vec{E} = \frac{E_0}{z} e^{-j30z} \hat{\theta}$ What is the working frequency of the antenna?

Exercice 2

- Directivity calculation
  - Estimate the directivity of an omnidirectional antenna that has a symmetric radiation pattern in ϕ with a width of the main beam in elevation at -3dB of 10°.

Exercice 3

- Friis Fomulas
  - A linear polarized antenna that works at 3 GHz has a radiation efficiency η_r of 75%. The graph shows the radiation pattern that has a revolution symmetry respect to $\theta=0° \Rightarrow BW_{1d} = BW_{2d}$.
  a) Estimate the gain for a direction situated at 5° respect to the maximum radiation. G(5°)?
  b) Calculate the available power at the receiver input P_{DR} when a circular polarized wave of 10 mW/m^2 impinges on the antenna in the direction of a).

Exercice 4

- G/T parameter
  - A receiver system in S band has a G/T parameter of 30 [dB(1/K)]. If the antenna has a -3dB beamwidth of 1° (the radiation pattern has a revolution symmetry respect to $\theta=0° \Rightarrow BW_{1d} = BW_{2d}$) and a noise temperature T_a = 20K. Consider a radiation efficiency η_r = 1.

    Estimate the receiver noise figure F_{rx} in dB?