Array antennas introduction

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Outline

- **Array antennas definition**
- **Arrays types**
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- **Arrays theory**
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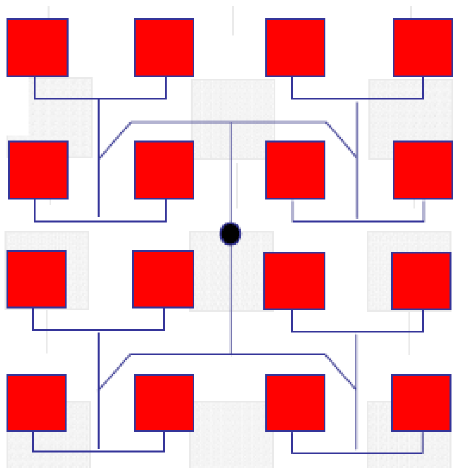
  - **Equispace linear arrays**
  - **Effects of the feeding elements**

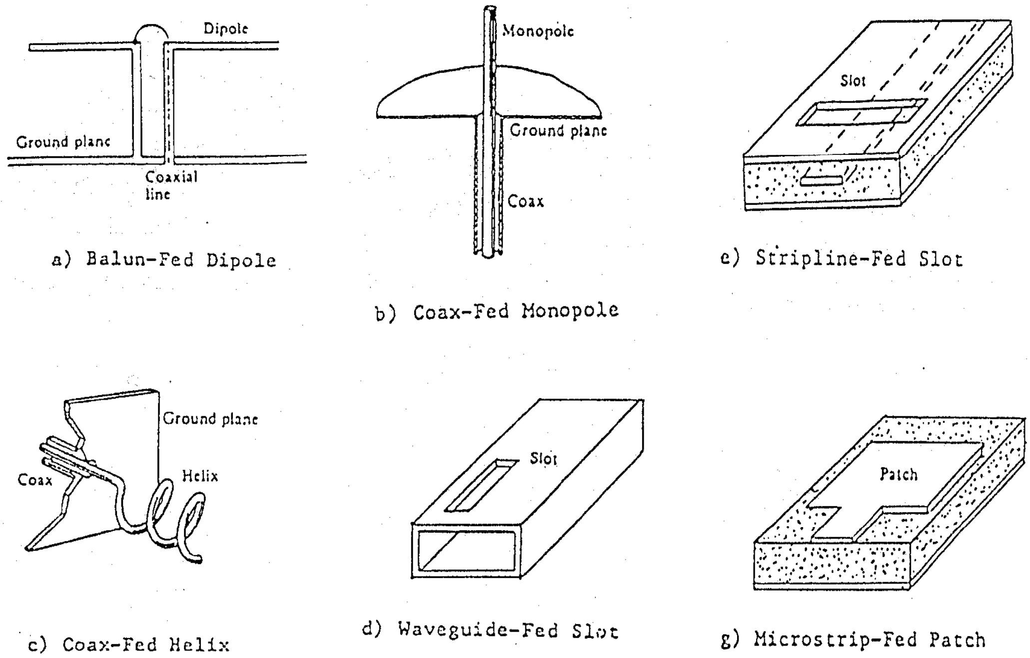
Array antenna definition

What is an array antenna?

- **Definition:**
  - **An array antenna is a spatially extended collection of N similar radiating elements, and the term "similar radiating elements" means that <u>all the elements have the same radiation patterns</u>, orientated in the same direction in 3D space.**
  - **The elements don't have to be necessary spaced on a regular grid, but it is assumed that <u>they are all fed with the same frequency</u>.**
  - **Group of individual radiating elements**
  - **Feed from a common terminal**
  - **By linear networks**



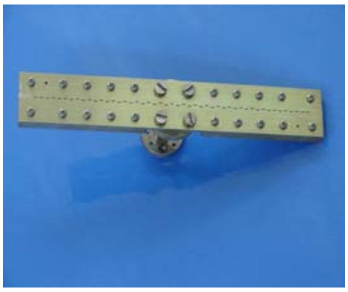
Radiating elements used to form arrays

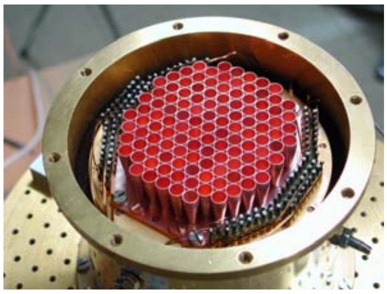


Array types

Arrays types: elements

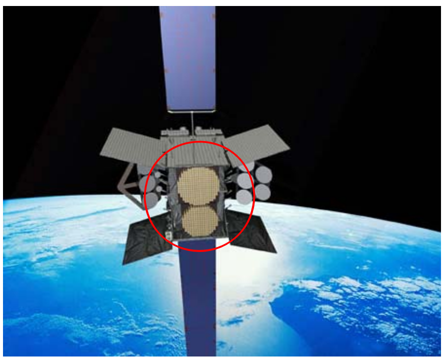
- **Depending on its elements**
  - Wires → wire array antennas
  - Printed elements → printed array antennas
  - Slot → slot array antennas
  - Horn → horn array antennas





Arrays types: application

- **Depending on it application**
  - Communications
    - mobile
    - Satellite
  - Radar
  - …



Arrays types: geometry (I)

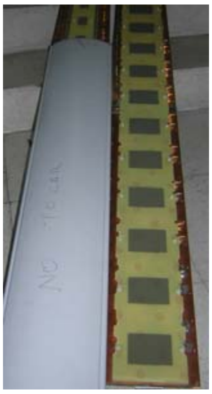
- **Depending on it geometry**
  - Linear
  - Planar
  - Conformal
    - » Cylindrical
    - » Spherical

This classification depends on the position where the different elements are placed:

- Linear (elements in a line)
- Planar (elements in a plane): rectangular (elements in a rectangular shape), triangular (elements in a triangle shape, circular (elements on concentric circumferences)
- Conformal (elements in a 3D-surface): cylinder, sphere...

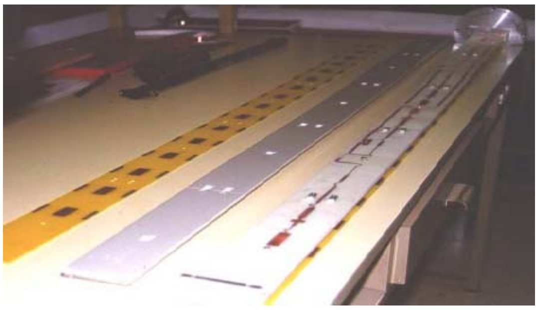
Arrays types: geometry (II)
Examples of linear arrays

Base station antennas for mobile systems application: DECT (3.5 GHz): Vertical 65°, 90° antennas



Base station antennas for mobile systems applications: GSM 1800 MHz: Vertical pol.° sectorial 65° & 90° antennas

Base station antennas for mobile systems applications: UMTS: crosspolar ± 45° sectorial 65° antennas

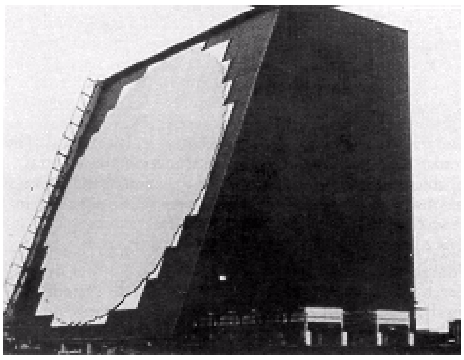


The printed antennas have the advantage to be easy to fabricate and low cost

Arrays types: geometry (III) Examples of planar arrays

- Satcom antenna

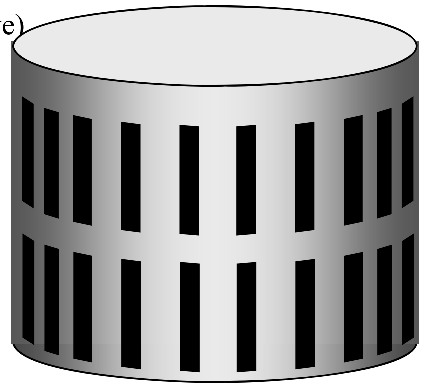
  -airborne radar technology for satellite communications placed on the F16



- Cobra Dane
  - A big antenna formed of 34769 radiating elements
  - works at 1200 MHz
  - part of the security radar in USA

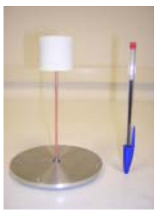
Arrays types: geometry (IV) Examples of conformal arrays

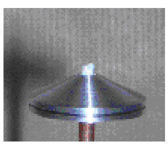
- Radiating elements placed on a no planar surface (for example curve)
  - Cylindrical (Elements placed over a cylinder)
  - Conical (Elements placed over a cone)
  - Spherical (Elements placed on a sphere)
  - Different surfaces (flight wings, vehicle, etc.)



Example: Cylindrical array of slots

Omnidirectional Circularly Polarized Slot Antenna Fed by a Cylindrical Waveguide for Identification Friend or Foe System in the 37GHz band





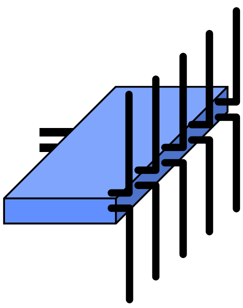
Circularly Polarized Omnidirectional Millimeter Wave Monopole with Parasitic Strip Elements for Identification Friend or Foe System

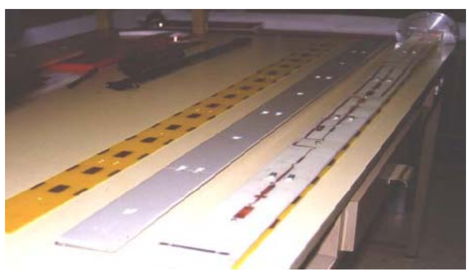
Arrays types: network (I)

- **Depending on the network**
  - Passive
    - » A single beam
    - » Multibeam
  - Active
  - Adaptative

Arrays types: network (II)
Pasive arrays

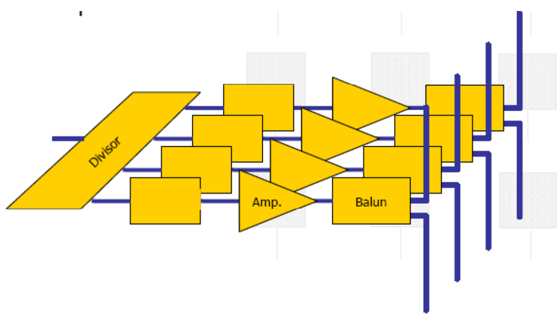
- Use a feeding network with passives elements (power divider, transmission lines, matching network etc.)
  - The radiation pattern and polarization are fixed.
  - Work as a unique antenna.
  - Depending on the network
    - » A single beam
    - » multibeam
  - Can have different input terminals in the network (multi-diagram or multibeam antenna).
  - Are reciprocal, works in transmission and reception.

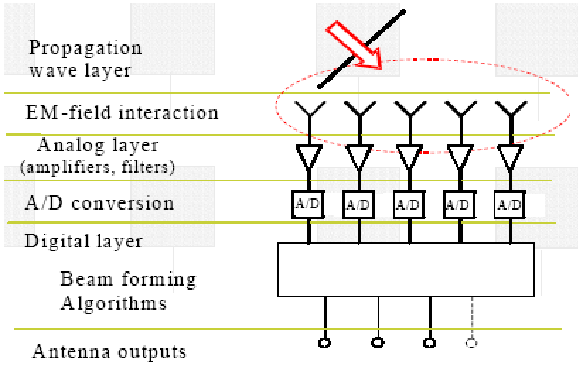




Arrays types: network (III)
Active arrays

- Linear active network to feed the arrays
  - Allow amplified distribution in the antenna
  - Allow active control of the excitations and of the radiation patterns.
  - Allow signal processing

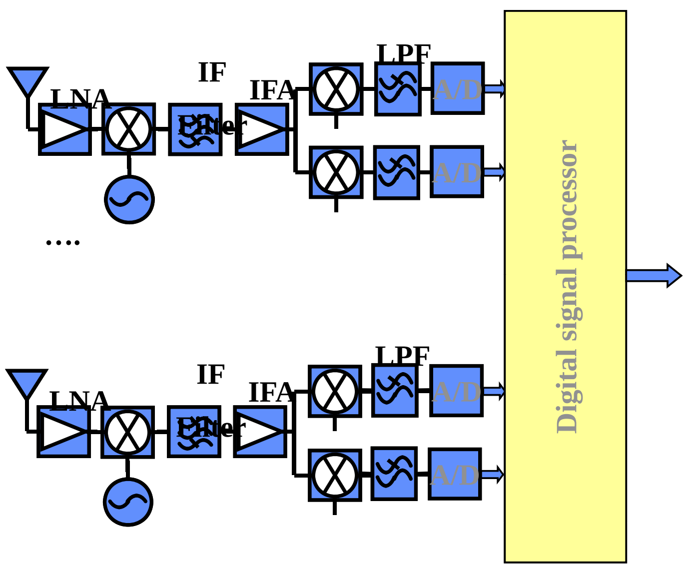




The active arrays are antennas with variable phase, that allow beam steering in a variable direction (very useful in Radar systems).

Arrays types: network (IV)
Adaptative arrays

- A digital processor allow:
  - Digital control of patterns
  - Patterns dependent of
    - » frequency
    - » time
    - » code
  - Simultaneous variables patterns



The adaptative arrays are kind of antennas that works with active feeding modifying instantaneously the radiation pattern depending of the signal that it receives (These antennas are very useful in communication systems)

Big arrays

Very Large Array (VLA).

Radiotelescope situated in Socorro, New Mexico.

Works in the band of 1 to 25GHz



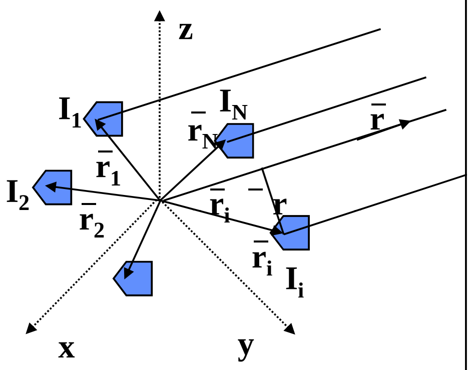
Array theory

Radiation pattern of an array (I)

- The Multiplication patterns principle, that characterize the arrays antennas, is based on the superposition principle derived of the Maxwell equations.

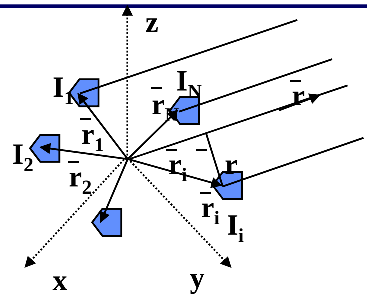
- **<u>Formulation condition</u>**:
  - Equal elements
  - Equal oriented elements

- An array describes with this principle is characterized by:
  - The position vectors of each elements:
  - The feeding currents of each elements: I_i
  - The radiation pattern of the radiating element :



Radiation pattern of an array (II)

- <u>Radiated field for one element:</u>



$$\vec{E}_i(r,\theta,\phi) = \vec{E}_e(r,\theta,\phi)\frac{I_i}{I_0}e^{jk_0\hat{r}\bar{r}_i}$$

Radiated field of an radiating element in the origin

Complex feeding coefficient

Relative phase for displacement out of the origin

$$\vec{E}_A(r,\theta,\phi) = \sum \vec{E}_i = \vec{E}_e(r,\theta,\phi)\sum A_i e^{jk_0\hat{r}\bar{r}_i} \Rightarrow F_A(\theta,\phi) = \sum A_i e^{jk_0\hat{r}\bar{r}_i}$$

$$\vec{E}_A(r,\theta,\phi) = \vec{E}_e(r,\theta,\phi)\cdot F_A(\theta,\phi)$$

The radiated field can be expressed as the product of the element field, situated in the origin, by the ARRAY FACTOR $F_A(\theta,\phi)$.

In function of:
- **Element positions**
- **Excitation A_i**
- **Frequency**

Multiplication patterns principle (I)

$$\left|\overline{E}_A(r,\theta,\phi)\right| = \left|\overline{E}_e(r,\theta,\phi)\right| \cdot \left|F_A(\theta,\phi)\right|$$

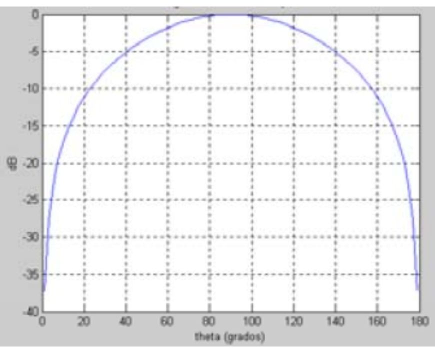
- The radiation pattern of an array is the product of the radiation pattern of the single radiating element and the array factor.
- The total radiated field polarization depends only on the used radiating element (F_A is a scalar value).
- The array factor allow to analyze how is the influence of the geometry and the excitation law on the radiation without considering what kind of radiating element we use.
- <u>In big arrays</u>, $F_A(\theta,\phi)$ varies more than $E_e(\theta,\phi)$ does, and we can approximate the total radiation pattern as the array factor.

Multiplication patterns principle (II)

- Example:

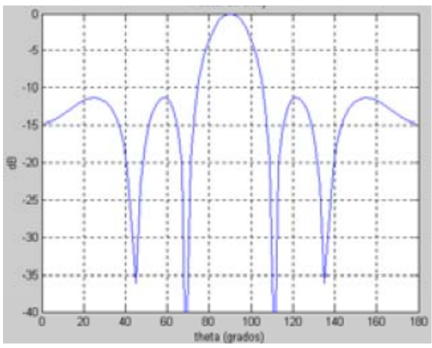
$$\left|\overline{E}_A(r,\theta,\phi)\right| = \left|\overline{E}_e(r,\theta,\phi)\right| \cdot \left|F_A(\theta,\phi)\right|$$

Element radiation pattern E_e



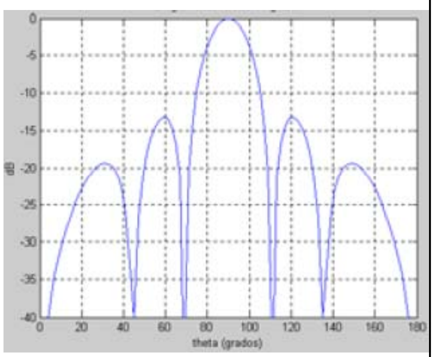
Theta (degree)

Array Factor F_A



Theta (degree)

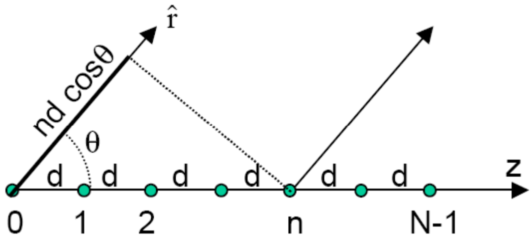
Array radiation pattern E_A



Theta (degree)

Equispace linear arrays (I)

- Array of N elements separated of a distance d and feed with A_n coefficients



$$F_A(\theta,\phi)=\sum_n A_n e^{jnk_0\hat{r}\vec{r}_n}=\sum_n A_n e^{jnk_0 d\cos\theta}=\sum_n A_n e^{jn\psi}$$

$$DFT^{-1}\{A_n\}!!$$

•As we can see in this expression, the array factor F_A is the DFT of the excitation law of the array.

•We can establish a parallelism between the studied concepts in signal processing with the arrays concepts.

•While in signal processing we pass from time domain to frequency spectrum, in arrays theory we pass from spatial domain (position and excitation law) to angular spectrum (radiation pattern).

•Some of the properties that we obtain studying the DFT are:

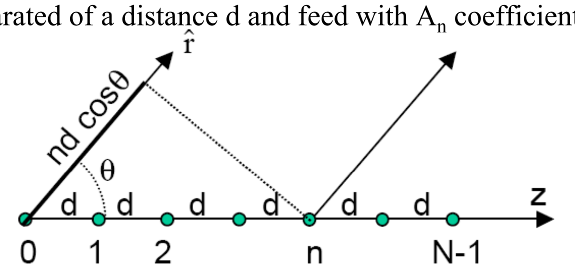
➢The array factor is a periodic function, of period 2π, in the ψ variable.

➢If the array is longer (electrically o in function of λ), the main lobe of it array factor will be narrower, the same that occur with the relation between narrow pulses and narrow frequency spectrum.

➢Excitation with laws that decrease from the centre to the edges have lower side lobe levels in the radiation pattern, although we have a wider main lobe in the radiation pattern.

Equispace linear arrays (II)

- Array of N elements separated of a distance d and feed with A_n coefficients



$$\left.\begin{array}{l} F_A(\theta,\phi)=\sum A_n e^{jk_0\hat{r}\vec{r}_n} \\ A_n=a_n e^{jn\alpha} \\ \vec{r}_n=nd\hat{z},\ \hat{r}\cdot\vec{r}_n=nd\cos\theta \end{array}\right\} \Rightarrow F_A(\theta,\phi)=\sum_{n=0}^{N-1} A_n e^{jnk_0 d\cos\theta}=\sum_{n=0}^{N-1} a_n e^{jn(k_0 d\cos\theta+\alpha)}=\sum_{n=0}^{N-1} a_n e^{jn\psi}$$

$$\psi=kd\cos\theta+\alpha$$

Radiating elements with progressive phase:

➢α = difference phase between 2 elements

Excitation laws most used:

➢Uniform in amplitude and phase, $A_n = 1\ \forall n$

➢Uniform in amplitude and the phase is progressive

➢Symmetry amplitude and decreasing from centre to edge and the phase is constant or progressive

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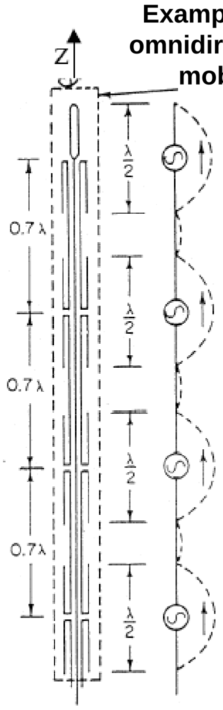
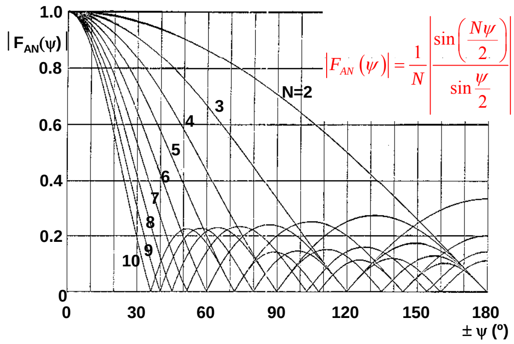
Linear arrays uniformly feed in amplitude and phase

Example of a broadside array: omnidirectional antenna used in mobile communications



$$A_n = 1 \Rightarrow F_A(\psi) = \sum_{n=0}^{N-1} e^{jn\psi} = \frac{e^{jN\psi}-1}{e^{j\psi}-1} = e^{j\frac{N-1}{2}\psi} \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}$$



Width between nulls of principal lobes:

$$\psi_{1N} = \pm 2\pi / N \Rightarrow k_0 d\cos\theta_{1N} = 2\pi/N \Rightarrow \cos\theta_{1N} = \frac{\lambda}{Nd}$$

$$\Rightarrow \sin\left(\frac{\pi}{2} - \theta_{1N}\right) = \frac{\lambda}{Nd}$$

$$\Delta\theta = 2\left(\frac{\pi}{2} - \theta_{1N}\right) = 2\arcsin\left(\frac{\lambda}{Nd}\right)$$

$$If\ Nd >> \lambda \Rightarrow \Delta\theta_{nulls} = \frac{2\lambda}{Nd} = \frac{2\pi}{L}\ [rad]$$

Width of the main lobe at -3dB:

$$\Delta\theta_{-3dB} = 0.886\frac{\lambda}{Nd}\ [rad]$$

Side lobe levels SLL:

$$F_{AN}\left(\frac{3\pi}{N}\right) = \frac{1/N}{\sin\left(\frac{3\pi}{2N}\right)}$$

$$SLL_{N,big} = \frac{2}{3\pi} \Rightarrow -13.46dB$$

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Linear arrays uniformly feed in amplitude and the phase is progressive (I)

- Uniform feeding in amplitude and the phase is progressive:

$$A_n = a_n e^{jn\alpha} = \frac{1}{N} e^{jn\alpha}$$

α = difference constant phase between 2 elements

$$F_A(\theta) = \frac{1}{N}\sum_{i=0}^{N-1} e^{ji(k_0 d\cos(\theta)+\alpha)} = \frac{1}{N}\sum_{i=0}^{N-1} e^{ji\psi} \quad \text{where } \psi = k_0 d\cos\theta + \alpha$$

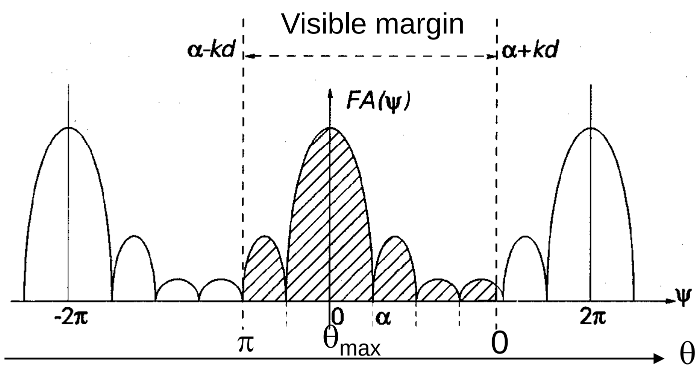
➢ The new variable α allow to adjust the steering direction of the main lobe of the array.

➢The progressive phase between elements α allow to compensate for a determined direction of the space, the phase difference associated to the propagation between the generated waves for the different elements, positioning the maximum radiation in this specified direction.

Linear arrays uniformly feed in amplitude and the phase is progressive (II)

$$|F_A(\theta)| = \frac{1}{N}\left|\frac{\sin\left(N\frac{\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}\right|$$

$$0 \le \theta \le \pi \Rightarrow -k_0 d + \alpha \le \psi \le +k_0 d + \alpha$$



Maximum in ψ=0, $\theta = \cos^{-1}\left(-\frac{\alpha}{k_0 d}\right)$

Periodic of period 2π in ψ

Nulls in ψ=2π/N

- If we observe this figure, we can deduce some arrays properties that can be generalized to equispace linear antennas:
  - If the excitation coefficient A_n (except the progresive phase component) are reals and positives, the maximum of the array factor is in ψ = 0, because it is in this direction where the contribution of all the arrays elements will sum in phase.
  - If the visible margin include ψ = 0, the maximum will be in the direction θ_{max}= arccos($-\alpha/k_0 d$). So varying the difference phase α, it allow to change the steering direction of the array main lobe. This method is actually used in electronical exploration radars, controlling digitally the phase shift between the elements.

Linear arrays uniformly feed in amplitude and the phase is progressive (III)

- The arrays are divided depending on it steering direction in these followings types:
  - **Broadside array:** has it radiation maximum in the perpendicular plane of the array (θ_{max} = π/2), where α = 0 and the visible margin is $-k_0 d < \psi < k_0 d$.
  - **Exploration array:** steer at a variable direction θ_{max} fixed by the difference constant phase α. The visible margin is the general one: $0 \le \theta \le \pi \Rightarrow -k_0 d + \alpha \le \psi \le +k_0 d + \alpha$

$$\psi = k_0 d \cos\theta_{max} + \alpha = 0 \Rightarrow \theta_{max} = \arccos\left(\frac{-\alpha}{k_0 d}\right)$$

  - **Endfire array:** has the radiation maximum in the array axis (θ_{max} = 0 or π).
    - For the case θ_{max} = 0, α = $-k_0 d$ and the visible margin is $-2k_0 d < \psi < 0$. in this case the radiaton pattern is pencil beam type with the same beamwidth in the two principal planes.
    - If the distance between elements d (and/or the phase shift α) are bigger than determined values (as d > λ , can appear "**grating lobes**" or diffraction lobes (emerge similar of the main lobe). So this why d = 0.6 – 0.8λ for broadside arrays and d = 0.4 – 0.45λ for enfire arrays.
    - When the elements are fed with the same amplitude $|A_n|$ = 1, the array factor F_A continue to be a periodic sinc and the diagram in ψ (slide(22)) is still valid.

Directivity with linear arrays uniformly feed in amplitude and the phase is progressive

- Uniform feeding ($a_n = 1, \forall n$):

$$D = \frac{N^2}{N + \sum_{m=0}^{N-1} 2(N-m) \frac{\sin(mk_0 d)}{mk_0 d} \cos m\alpha}$$

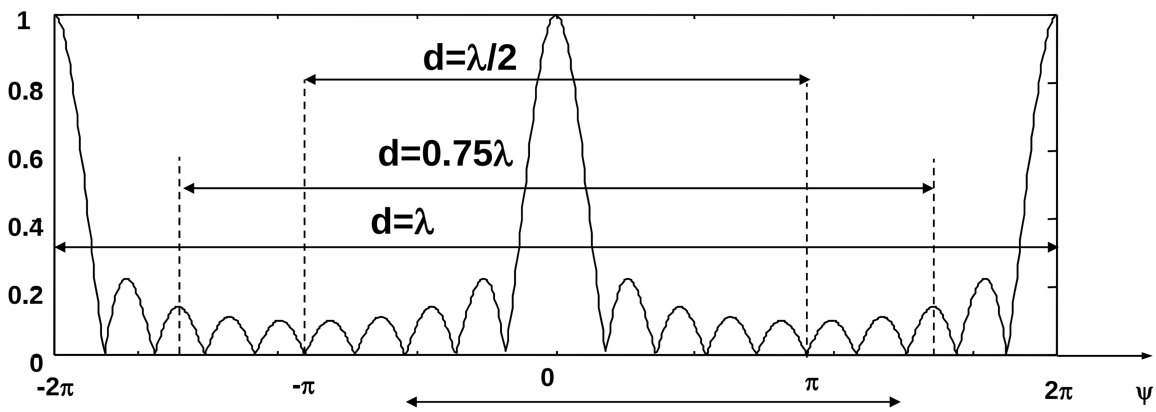
- Interest case:
  - Multiple separation of $\lambda/2$: $d = k\frac{\lambda}{2}$ $\quad D = N$
  - Broadside array (α=0): $d \approx \frac{\lambda}{2},\ d < \lambda$ $\quad D = 2N\frac{d}{\lambda} = 2\frac{L}{\lambda}$
  - Endfire array: $d \leq \left(1 - \frac{1}{2N}\right)\frac{\lambda}{2}$ $\quad D = 4N\frac{d}{\lambda} = 4\frac{L}{\lambda}$

L=Nd

Broadside array

Uniform feeding in amplitude and phase: The visible margin is

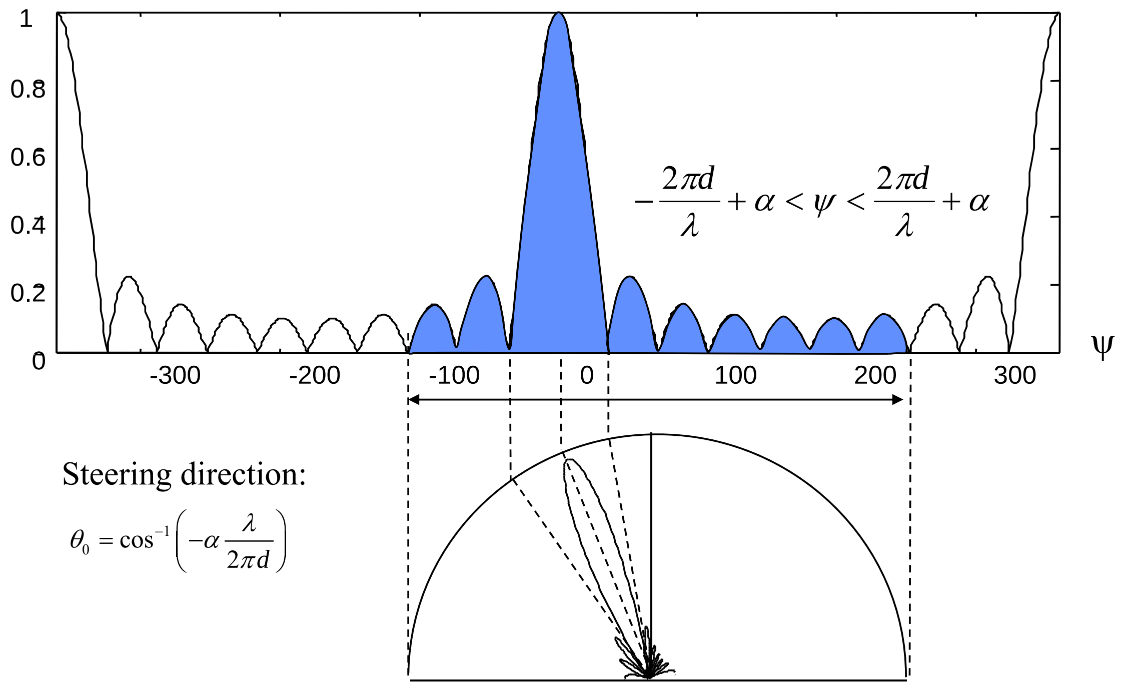
Maximum: $\psi = 0 \Rightarrow \theta_{max} = \pi/2$



Conclusion: For the same element numbers, when we increase the separation distance, the directivity of the antenna increase.

Exploration array

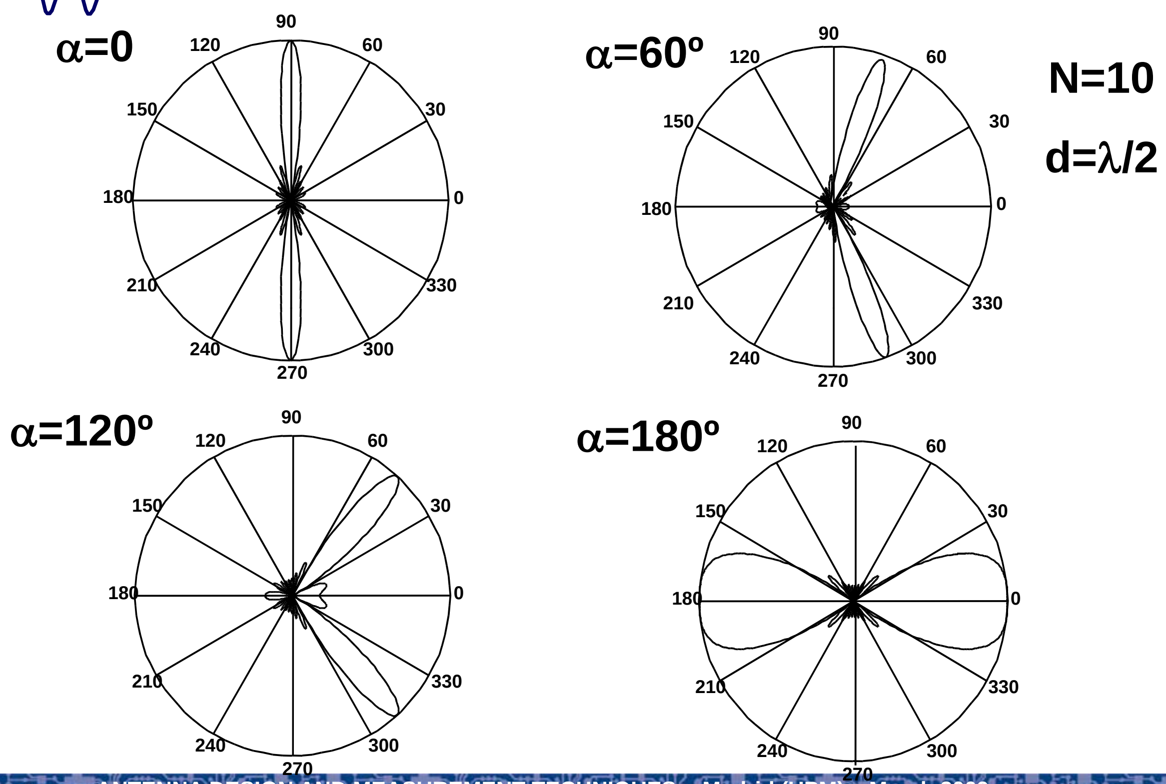
Exploration array: steer at a variable direction θ_{max} fixed by the difference constant phase α.



Steering direction:

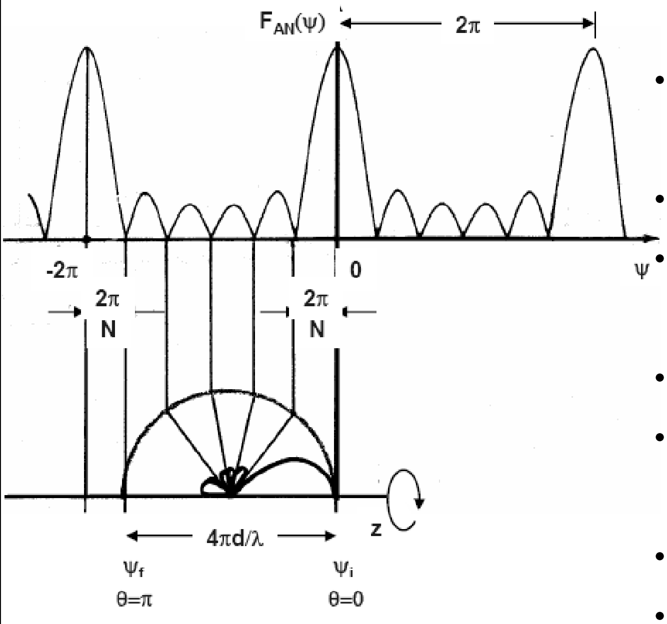
$$\theta_0=\cos^{-1}\left(-\alpha\frac{\lambda}{2\pi d}\right)$$

Effect of varying the difference phase elements α



Endfire array

The endfire array is characterized to achieve a pencil beam type main lobe



Depending on the array axis:

- Main maximum: $\theta=0$ or ($\theta=\pi$)
- Visible margin: $\dfrac{-4\pi d}{\lambda} < \psi < 0$
- Required progressive phase:

$$\alpha = -k_0 d = -2\pi \frac{d}{\lambda}$$

- Width between the nulls of the main lobe:
- If $Nd >> \lambda \Rightarrow BW_{nulls} = 2\sqrt{\dfrac{2\lambda}{Nd}}$ *rad*
- Width of the main lobe at -3dB:
- If $Nd >> \lambda \Rightarrow BW_{-3dB} = 2\sqrt{\dfrac{0.88\lambda}{Nd}}$ *rad*

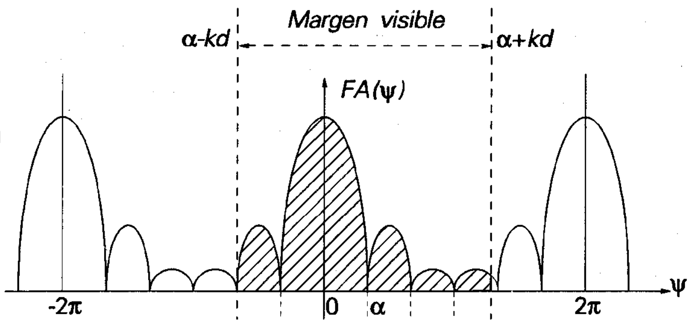
Resume: Equispace linear array uniformly feed in amplitude and the phase is progresive

- $F_A(\psi)$ is always a periodic function with period $\psi = 2\pi$: the valid margin of the radiation pattern is the margin with possible values of θ: between 0 y π

$$\psi \equiv kd\cos\theta + \alpha$$

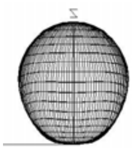
$$F_A(\theta,\phi) = \sum A_i e^{ji\psi}$$

Graphic representation



Broadside

- **Phase:** $\alpha = 0$ $\quad \psi = kd\cos\theta$ **Uniform phase**
  - **Visible margin:** $-kd < \psi < kd$
- **Maximum:** $\psi = 0 \Rightarrow \theta_{max} = \pi/2$



Exploration

$$\psi = kd\cos\theta + \alpha$$

Progressive phase

$$-kd + \alpha < \psi < kd + \alpha$$

$$\psi = 0 \Rightarrow \theta_{max} = \mathrm{acos}\left(\frac{-\alpha}{kd}\right)$$

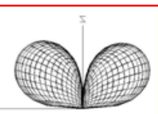
Endfire

$$\theta = 0, \psi(\theta = 0) = kd + \alpha = 0$$

$$\alpha = -kd = -2\pi d/\lambda$$

$$-4\pi d/\lambda < \psi < 0$$

$$\theta_{max} = 0 \quad (o' \ \theta_{max} = \pi)$$

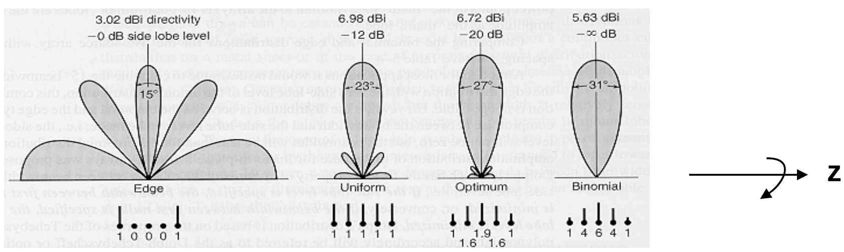


Linear arrays with symmetry amplitude, decreasing from centre to edge and the phase is constant or progressive

- **With a phase variation α**, we can control the steering direction.
- So **with an amplitude variation**, we can control the side lobe levels (SLL).
- **With symmetry amplitude, decreasing from centre to edge**, it achieve to reduce the side lobe lels (SLL) and wider the main lobe and therefore reduce the array directivity.
- The side lobe levels (SLL) reduction achieve with symmetry amplitude, decreasing from centre to edge is equivalent to the problems of signal theory when we use no rectangular windows like (Hanning, Hamming, Triangular,…).
- As in signal theory, the side lobe levels (SLL) reduction have resolution loss that is equivalent to wider beamwidth.

Control of side lobe levels (SLL)

- With symmetry amplitude, decreasing from centre to edge, we achieve to reduce the side lobe levels and wider the main lobe so the directivity D_0 is reduced.
- Some examples for a broadside array of 5 isotropic elements separated of λ/2.
  - ➢As we can observe the maximum directivity is given by the uniform excitation
  - ➢The minimum side lobe levels (SLL) is given by the binomial feeding, with a strong reduce directivity
  - ➢If a progressive phase shift α is introduced, the side lobe levels (SLL) are maintained when the beam explore.



- We observe the potential of the design that is in the arrays theory: we can control the side lobe levels (SLL), control the steering direction,…

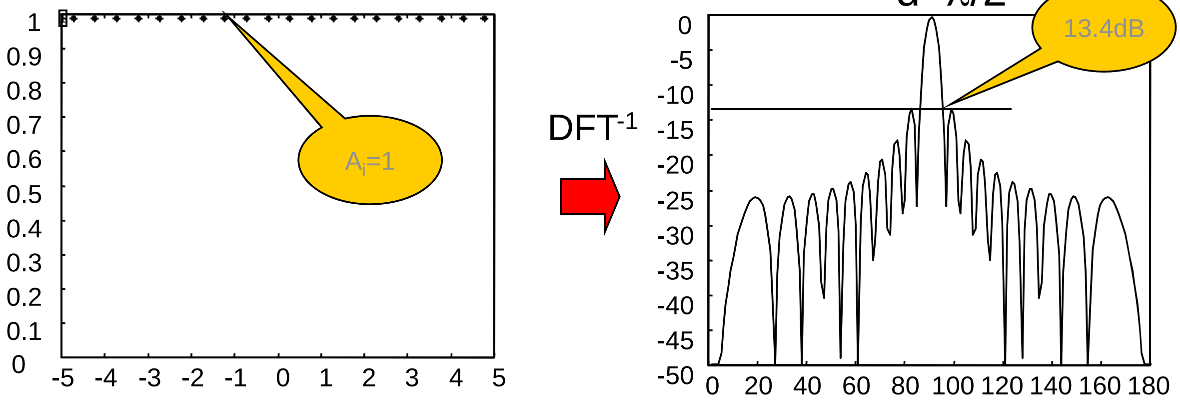
Effect of the feeding elements (I)

- Uniform feeding: when $A_n = 1 \cdot e^{jn\alpha}$ for i=0 to N-1

  - ➢Control of steering direction
  - ➢The directivity is maximum ⇒ D = N for d = λ/2
  - ➢The side lobe levels (SLL) is -13.4dB for N high
  - ➢BW_{-3dB} = 0.88λ/Nd sinθ

N=20

d=λ/2



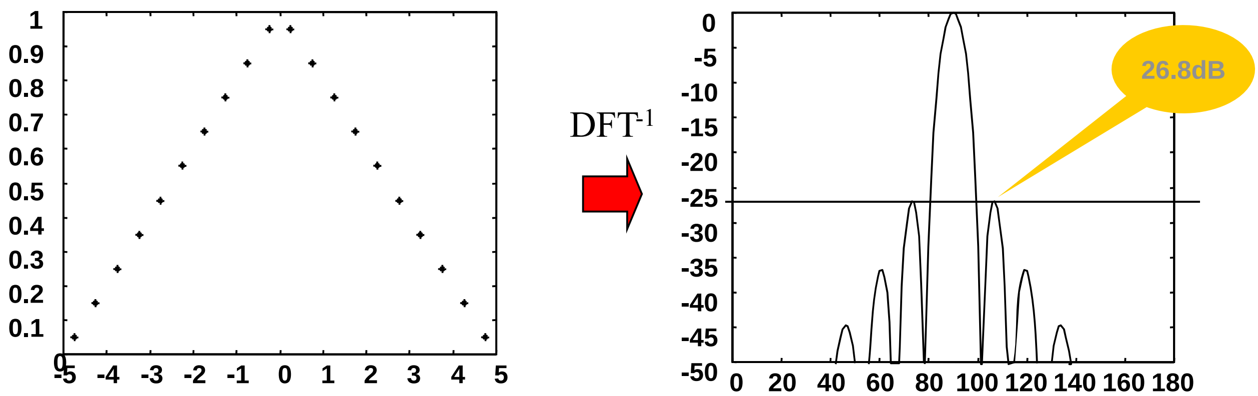
Effect of the feeding elements (II)

- Triangular feeding: when A_n=[1-abs(-(n-1)/2+i)/(n/2))] ; for n=0 to i-1

  - ➢The side lobe levels (SLL) fall until -26.8dB
  - ➢The directivity fall to ¾ of the maximum ⇒ D = 3N/4 for d = λ/2
  - ➢BW-3dB = 1.75λ/Nd sinθ

N=20

d=λ/2



Effect of the feeding elements (III)

- Cosines feeding on pedestal:
  for i=0 to n-1

➢Control the reduced side lobe levels (SLL)

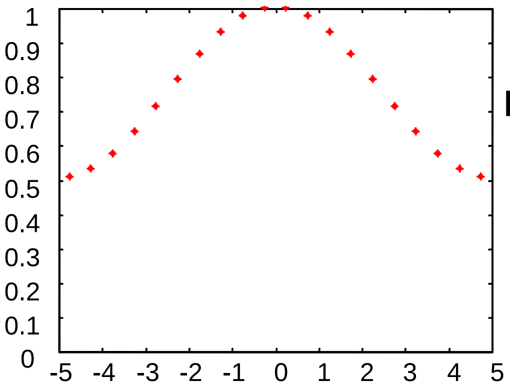
➢The directivity is reduced

➢The beamwidth increase

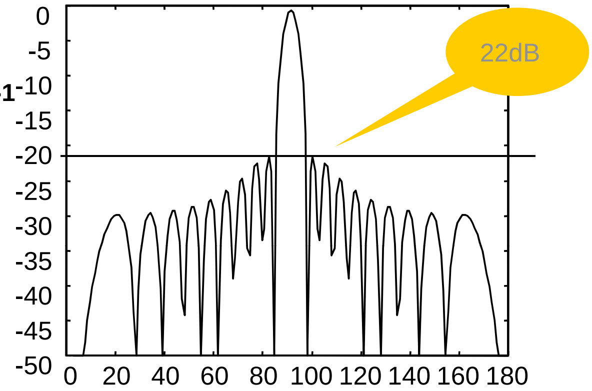
N=20

d=λ/2

H=0.5



DFT-1



Effect of the feeding elements

- Binomial feeding: when
  for i=0 to N-1

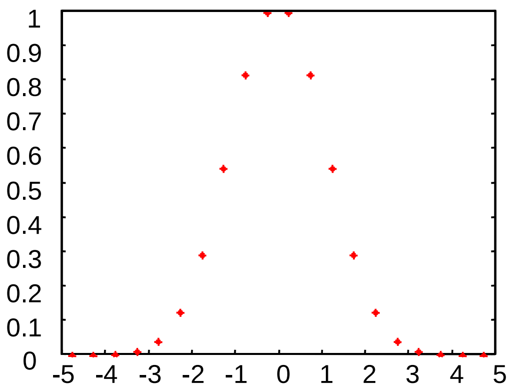
➢The side lobe levels (SLL) disappear

➢The directivity is reduced

➢The main beamwidth increase

N=20

d=λ/2



DFT-1