High Frequency Circuit Analysis

Basic Concepts, Smith Chart, Scattering Parameters.

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Outline

Introduction

High Frequency Analysis ; Graphic analysis

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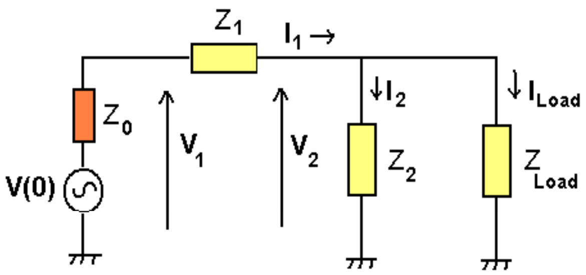
High Frequency Circuits Measurement Techniques

Introduction

A conventional low frequency circuit consists of:



$$Z_E = Z_1 + Z_2 // Z_{LOAD}$$

$$\frac{V_2}{Z_2 // Z_{load}} = \frac{V_0 - V_2}{Z_1 + Z_0} \rightarrow V_2 = \frac{Z_2 // Z_{Load} + Z_1 + Z_0}{(Z_2 // Z_{Load})(Z_1 + Z_0)} V_0$$

Low frequency condition:

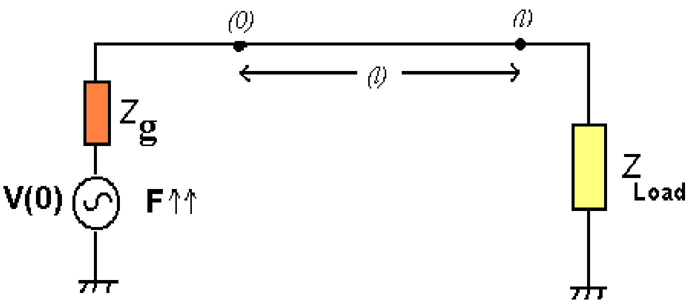
→ wires length << signal wavelength

High frequency condition

→ Wires length α signal wavelength

Introduction

A conventional High frequency circuit consists of:



The wires length is proportional to the signal wavelength

The wires current or potential phase depends on the distance (l)

$$\left.\begin{aligned} V(z) &= V^+ e^{-j\beta z} + V^- e^{j\beta z} \\ I(z) &= I^+ e^{-j\beta z} - I^- e^{j\beta z} \end{aligned}\right\} \rightarrow$$

The equivalent impedance depends on the position Z(d)

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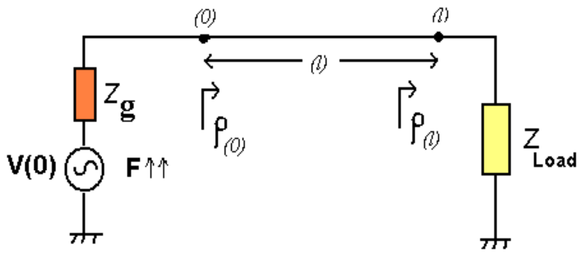
High Frequency Analysis



The reflection coefficient is defined:

$$\rho(d) = \frac{V_{ref}}{V_{inc}} = \frac{V^- \, e^{j\beta(-z)}}{V^+ e^{-j\beta(-z)}} = |\rho_l| e^{-2j\beta z}$$

There is a quite easy relation between Z and ρ.

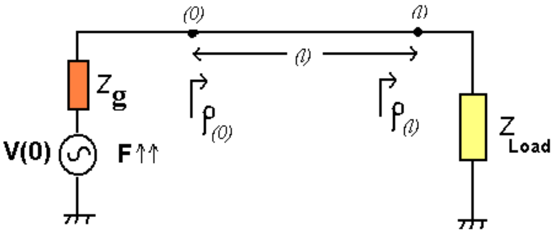
$$V(d) = V^+ e^{-j\beta z} + V^- e^{j\beta z} = V^+ e^{-j\beta z}\left[1+\rho(l) e^{2j\beta(z-l)}\right]$$

$$I(d) = \frac{V^+ e^{-j\beta z} - V^- e^{j\beta z}}{Z_o} = \frac{V^+}{Z_o} e^{-j\beta z}\left[1-\rho(l) e^{2j\beta(z-l)}\right]$$

$$\rightarrow Z(z) = Z_o \frac{1+\rho(z)}{1-\rho(z)} = Z_o \frac{1+\rho(l) e^{-2j\beta z}}{1-\rho(l) e^{-2j\beta z}} = Z_o \frac{Z_l \cos\beta l + jZ_o sen\beta l}{Z_0 \cos\beta l + jZ_l sen\beta l}$$

$$Z(z) = Z_o \frac{1+\rho(z)}{1-\rho(z)} \longleftrightarrow \rho(z) = \frac{Z(z) - Z_o}{Z(z) + Z_o}$$

High Frequency Analysis



VSWR:

$$VSWR = \frac{|V_{max}|}{|V_{min}|} = \frac{1+|\rho(l)|}{1-|\rho(l)|}$$

The transmitted Power:

$$P_T(z) = \frac{1}{2}\Re e\left[V(z)I(z)^*\right] = \frac{1}{2}\Re e\left[V^+e^{-j\beta z}\left[1+\rho(l)e^{2j\beta(z-l)}\right]\frac{V^+}{Z_o}e^{+j\beta z}\left[1-\rho(l)e^{-2j\beta(z-l)}\right]\right]$$

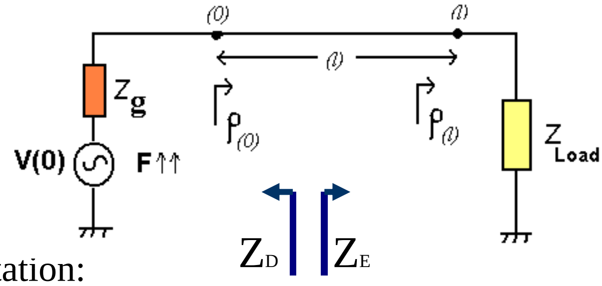
$$P_T(z) = \frac{1}{2Z_0}V^+\left[1-|\rho(l)|^2\right]$$

V^+ Easily solved from the relation:

$$Vg = -Z_g I(0) + V(0)$$

$$V(0) = Z_g\frac{V^+}{Z_o}\left[1-\rho(l)e^{2j\beta(-l)}\right] + V^+\left[1+\rho(l)e^{2j\beta(-l)}\right]$$

High Frequency Analysis

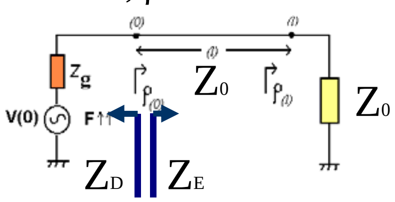


Power Adaptation:

When $Z_E = Z_D^*$, The power given to the load is the same as the power available at the generator.

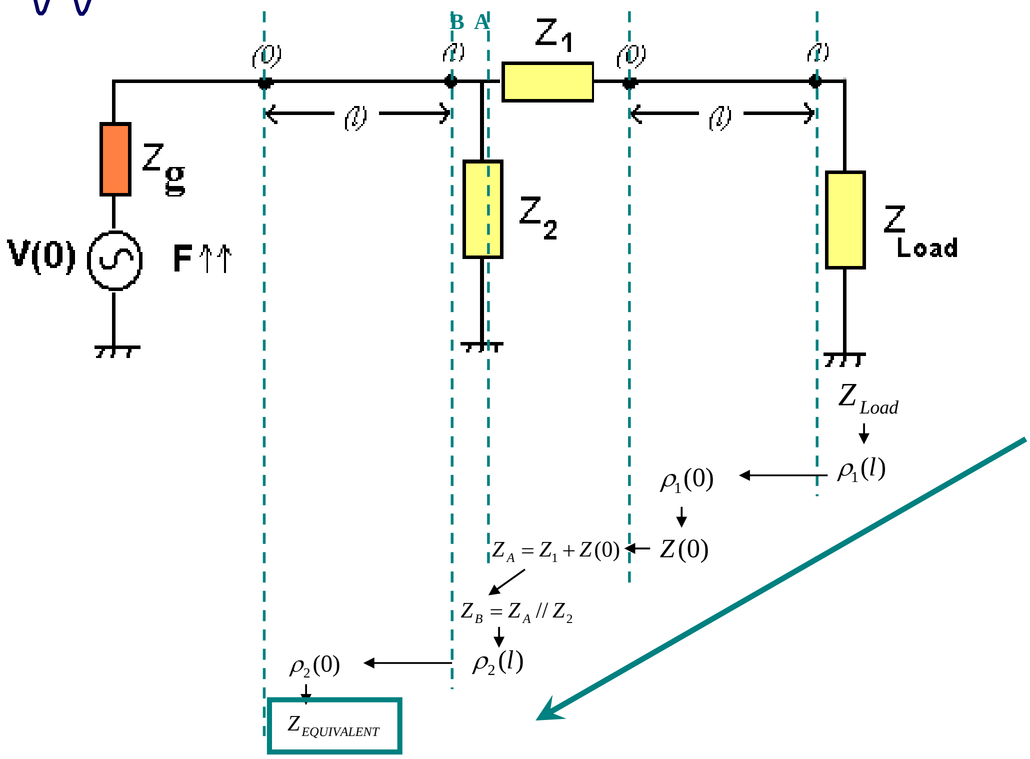
$$P_T(z) = \frac{1}{2Z_0}V^+\left[1-|\rho(l)|^2\right] = \frac{1}{8\,\mathrm{Re}(Zg)}|V(0)|^2$$

- However, ρ could be different from zero.



$\rho = 0$ → but $Z_E = Z_0 \neq Z_g$

High Frequency Analysis



High Frequency Analysis

➢A quite simple high frequency circuit could imply an embarrassing solution, with a lot of complex operations.

➢With this analysis method, there is no way to notice the circuit behavior without analyzing it.

Graphic Method of Solution:

➢In the 30's, Smith, engineer at Bell Laboratories developed an easy graphic method to solve high frequency circuits: The Smith Chart.

➢This method allows us to obtain a general idea of what the circuit behavior is without a complete analysis.

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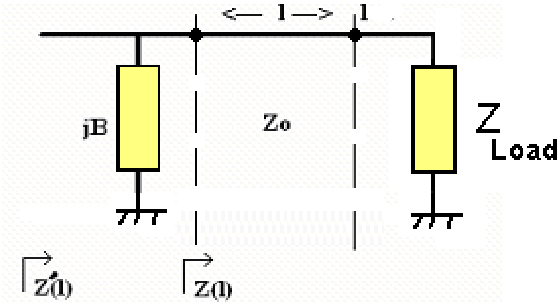
High Frequency Analysis: Smith Chart



Z and ρ, in complex form:

$$\bar{Z}(l) = \frac{Z(l)}{Z_o} = r + jx$$

Normalized impedance

$$\rho(l) = |\rho| e^{j\theta} = u + jv$$

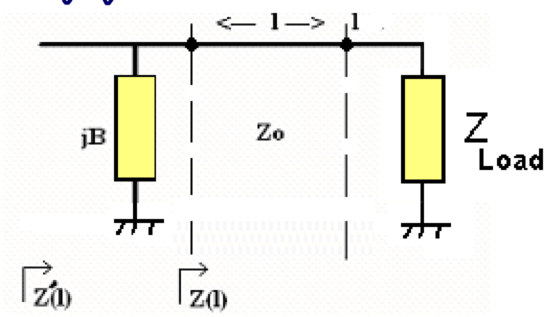
Going on with the relation between Z̄ and ρ :

$$r + jx = \frac{1 + (u + jv)}{1 - (u + jv)}$$

$$r = \frac{1 - (u^2 + v^2)}{(1-u)^2 + v^2} \rightarrow \left(u - \frac{r}{1+r}\right)^2 + v^2 = \frac{1}{(1+r)^2}$$

$$x = \frac{2v}{(1-u)^2 + v^2} \rightarrow (u-1)^2 + \left(v - \frac{1}{x}\right)^2 = \frac{1}{x^2}$$

High Frequency Analysis: Smith Chart



$$r = \frac{1 - (u^2 + v^2)}{(1-u)^2 + v^2} \rightarrow \left(u - \frac{r}{1+r}\right)^2 + v^2 = \frac{1}{(1+r)^2}$$

$$x = \frac{2v}{(1-u)^2 + v^2} \rightarrow (u-1)^2 + \left(v - \frac{1}{x}\right)^2 = \frac{1}{x^2}$$

Considering (u, v) the coordinates of our 2D coordinate system, the previous relation means:

For different values of ***r*** (real part of the normalized impedance) we will plot different circumferences, with features:

Center: $C\left(\frac{r}{1+r}, 0\right)$ and radius: $R\left(\frac{1}{1+r}\right)$

In the same way, for different values of ***x*** (imaginary part of the normalized impedance) we will plot different circumferences, with features:

Center: $C\left(1, \frac{1}{x}\right)$ and radius: $R\left(\frac{1}{x}\right)$

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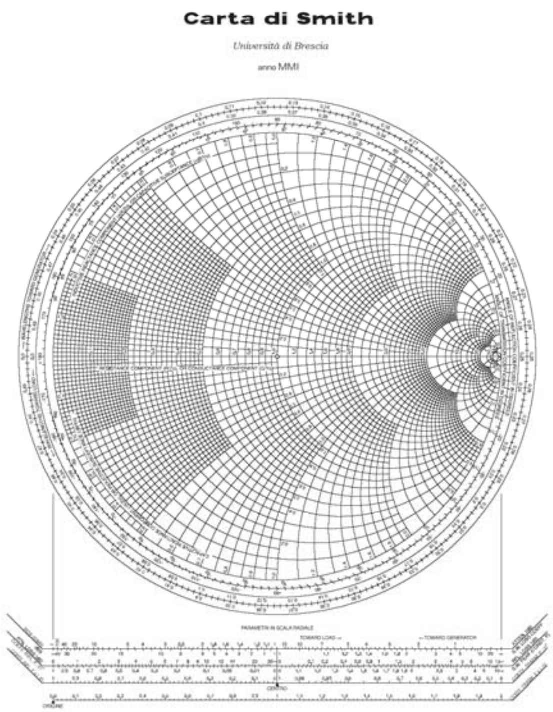
High Frequency Analysis: Smith Chart



$$r=\frac{1-(u^2+v^2)}{(1-u)^2+v^2}\rightarrow\left(u-\frac{r}{1+r}\right)^2+v^2=\frac{1}{(1+r)^2}$$

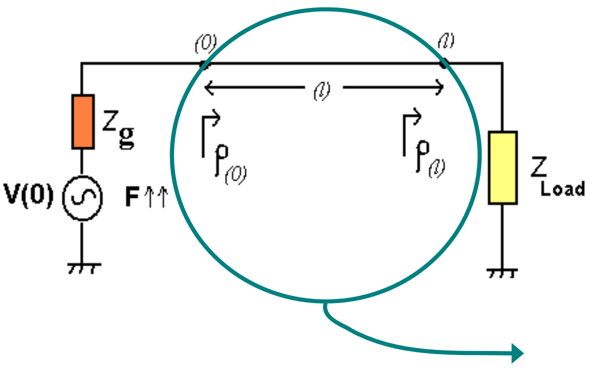
$$x=\frac{2v}{(1-u)^2+v^2}\rightarrow(u-1)^2+\left(v-\frac{1}{x}\right)^2=\frac{1}{x^2}$$

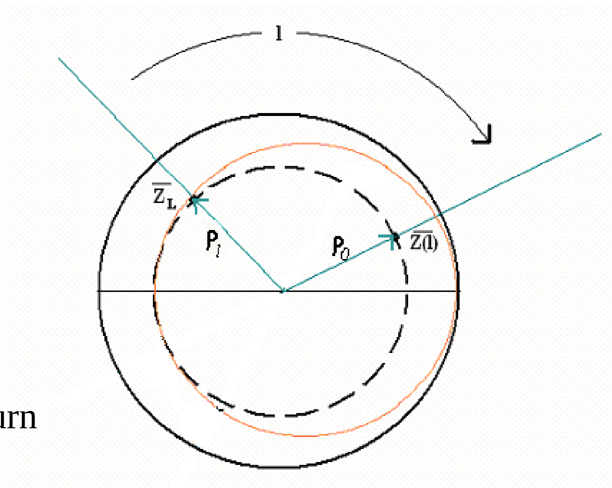
r: Center $C\left(\frac{r}{1+r},0\right)$ and radius $R\left(\frac{1}{1+r}\right)$

x: Center $C\left(1,\frac{1}{x}\right)$ and radius $R\left(\frac{1}{x}\right)$

High Frequency Analysis: Smith Chart

First case of study: movement over wires

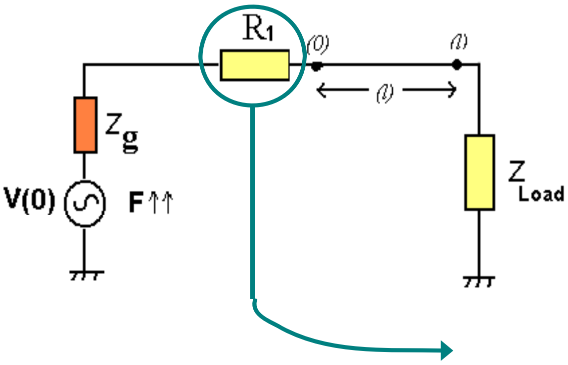


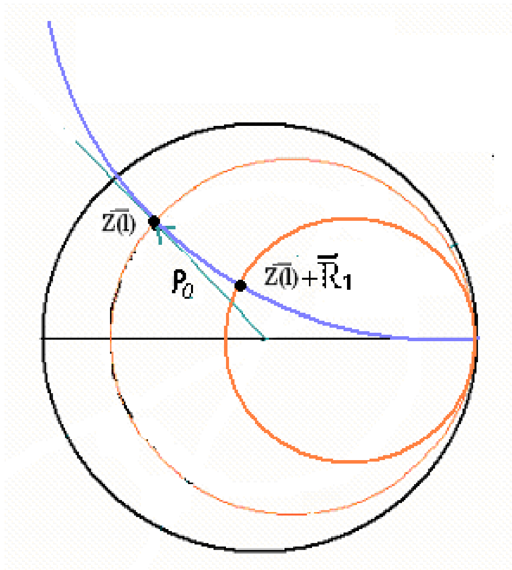


When moving towards generator, we turn towards right at the Smith Chart.

High Frequency Analysis: Smith Chart

Second case of study: adding series resistor.

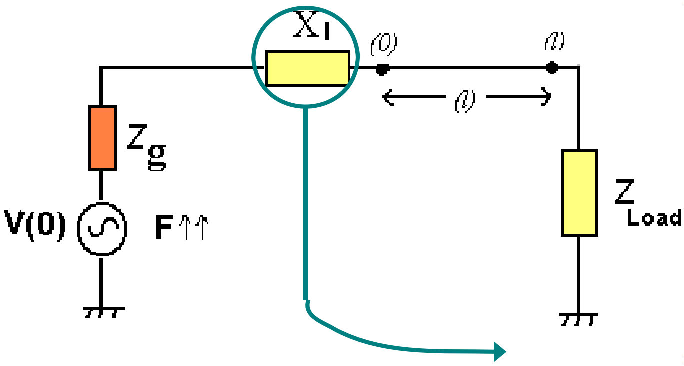


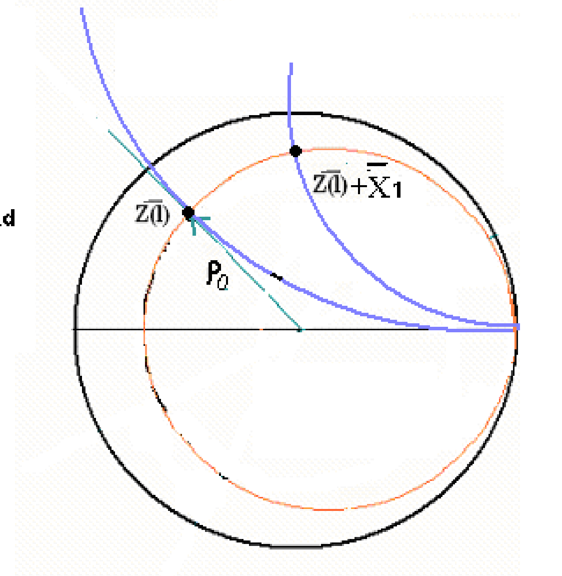


When adding resistor, we keep the imaginary part and change into another real part circumference at the Smith Chart.

High Frequency Analysis: Smith Chart

Third case of study: adding series impedance with no real part.

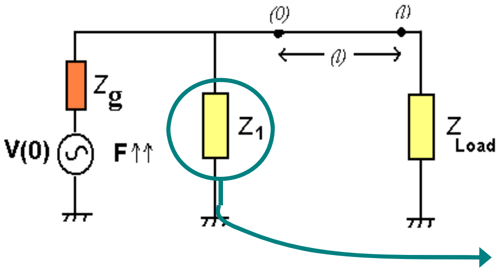


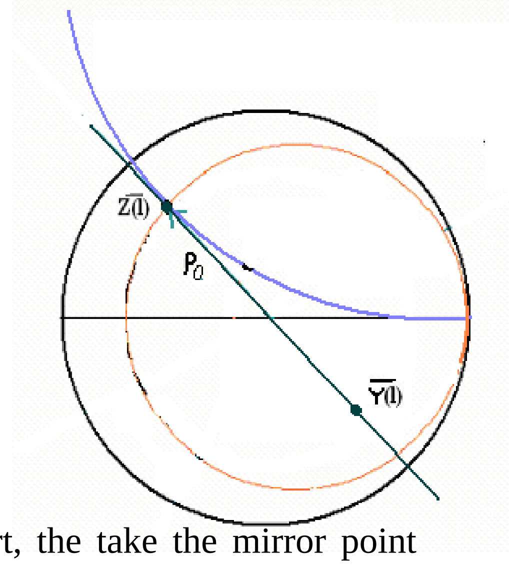


When adding impedance with no real part, we keep the real part and change into another imaginary part curve at the Smith Chart.

High Frequency Analysis: Smith Chart

Fourth case of study: adding parallel impedance.





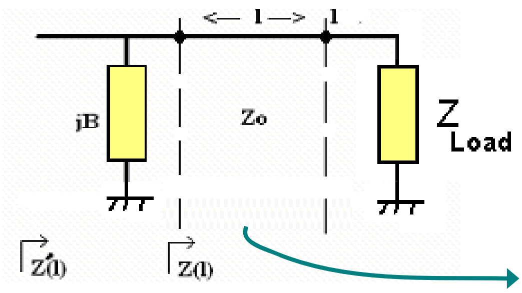
When adding parallel impedance, we transform impedance into admittance.

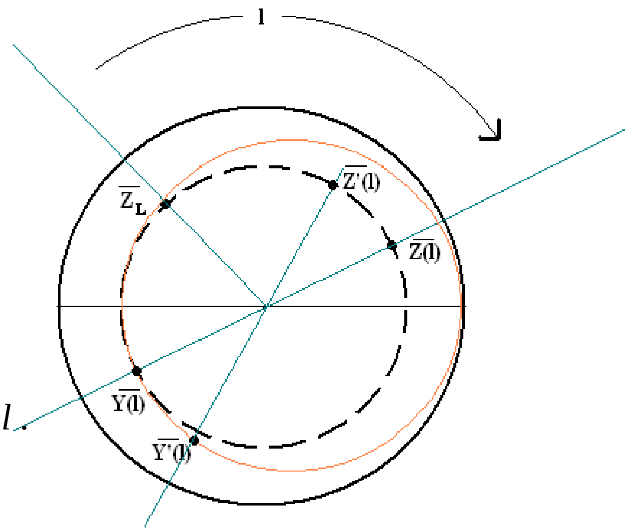
$$\overline{Z}(l) // \overline{Z}_1 = \overline{Y}(l) + \overline{Y}_1 \qquad \overline{Z}(l) = \frac{1}{\overline{Y}(l)}$$

To obtain the admittance at the Smith Chart, the take the mirror point related to the center.
We solve as in the previous cases, taking admittance or impedance when necessary.

High Frequency Analysis: Smith Chart

Circuit example:





1.-Normalize Z_{load}.
2.- plot Z_{load} at the S. chart.
3.- move towards the generator length *l*.
4.- change Z(*l*) into Y(*l*).
5.- add Y=jB; Y'(*l*)=Y(*l*)+Y .
6.- change Y'(*l*) into Z'(*l*).
7.- desnormalize Z'(*l*).

Introduction

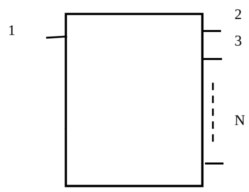
High Frequency Analysis; Graphic analysis

Scattering Parameters

High Frequency Circuits Measurement Techniques

Scattering Parameters

Low frequency circuits are usually characterized with their Z (impedance) parameters.



$$Z_i = \frac{V_i}{I_i} = \sum_{j=1}^{N} Z_{ij}\frac{I_j}{I_i} = Z_{ii} + \sum_{\substack{j=1 \\ i \neq j}}^{N} Z_{ij}\frac{I_j}{I_i}$$

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

There are other possibilities, in order to reduce the complexity of analysis in any particular case, as F parameters, T parameters…

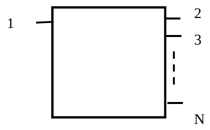
At *high frequency circuits* we define the S parameters. These parameters simplify measurements, as we analyze circuits in terms of transmission and reflection.

Scattering Parameters

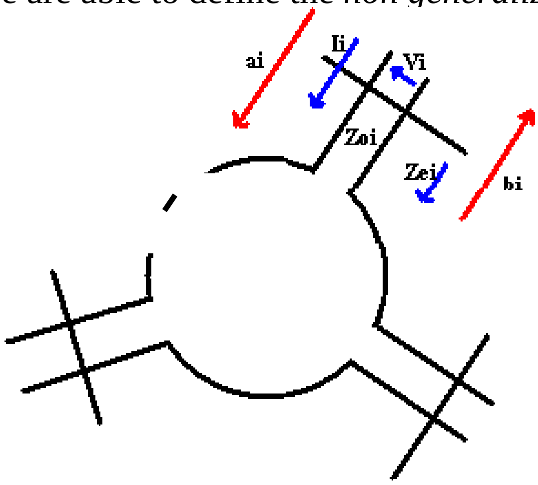
Considering the current and potential waves:

$$V_i(z) = V_i^+ e^{-j\beta z} + V_i^- e^{j\beta z}$$

$$I_i(z) = I_i^+ e^{-j\beta z} - I_i^- e^{j\beta z}$$



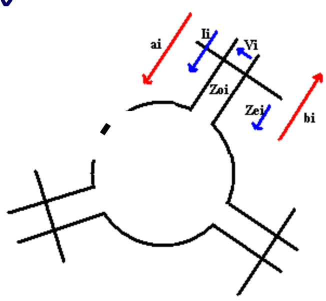
We are able to define the *non generalized power waves*.



$$a_i = \frac{V_i + Z_{oi} I_i}{\sqrt{8 Z_{oi}}}$$

$$b_i = \frac{V_i - Z_{oi} I_i}{\sqrt{8 Z_{oi}}}$$

Scattering Parameters



$$a_i = \frac{V_i + Z_{oi} I_i}{\sqrt{8 Z_{oi}}}$$

$$b_i = \frac{V_i - Z_{oi} I_i}{\sqrt{8 Z_{oi}}}$$

Taking into account, potential and current definitions:

$$V_i = V_i^+ + V_i^- \qquad I_i = \frac{1}{Z_{oi}}\left[V_i^+ - V_i^-\right]$$

We obtain results below:

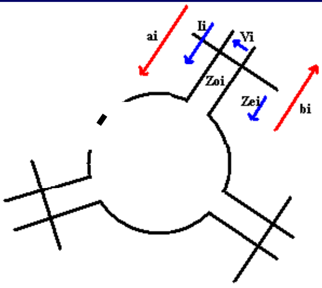
$$|a_i|^2 = \frac{|V_i^+|^2}{2Z_{oi}} = Pi^+ \qquad |b_i|^2 = \frac{|V_i^-|^2}{2Z_{oi}} = Pi^-$$

Incident and reflected power at *i* port.

$$\frac{b_i}{a_i} = \frac{V_i^-}{V_i^+} = \frac{Z_{ei} - Z_{oi}}{Z_{ei} + Z_{oi}} = \rho_{vi}$$

Reflection coefficient at *i* port.

Scattering Parameters



$$a_i = \frac{V_i + Z_{oi} I_i}{\sqrt{8 Z_{oi}}}$$

$$b_i = \frac{V_i - Z_{oi} I_i}{\sqrt{8 Z_{oi}}}$$

The scattering matrix exhibits the relation between a_i and b_i

$$B = S \cdot A \qquad \begin{pmatrix} b_1 \\ \dots \\ b_n \end{pmatrix} = \begin{pmatrix} S_{11} & \dots & S_{1n} \\ \dots & \dots & \dots \\ S_{n1} & \dots & S_{nn} \end{pmatrix} \begin{pmatrix} a_1 \\ \dots \\ a_n \end{pmatrix}$$

It is also possible to relate S parameters and Z parameters:

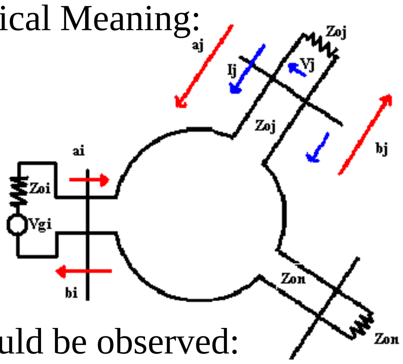
$$\left.\begin{matrix} B = S \cdot A \\ V = Z \cdot I \end{matrix}\right\} \rightarrow \begin{matrix} A = F[V + G \cdot I] = F[Z + G]I \\ B = F[V - G \cdot I] = F[Z - G]I \end{matrix}$$

Where: $F = diagonal\left(\frac{1}{\sqrt{8 Z_{oi}}}\right)$

$G = diagonal(Z_{oi})$

Scattering Parameters

Practical Meaning:



$$b_i = \sum_j S_{ij} a_j$$

It could be observed:

$$\forall j \neq i \mapsto V_j = -Z_{oj} I_j \mapsto a_j = \frac{V_j + Z_{oj} I_j}{\sqrt{8 Z_{oj}}} = 0$$

There is no input power when loading the j port with Z_{oj}.

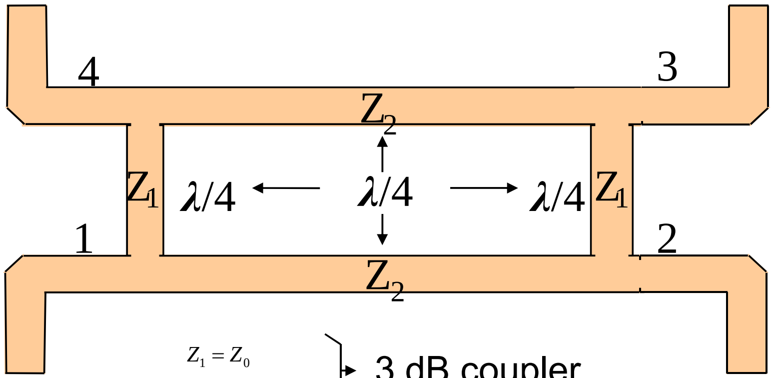
$$b_i = S_{ii} a_i \xrightarrow[j \neq i]{\forall a_j = 0} S_{ii} = \frac{b_i}{a_i} = \frac{V_i - Z_{oi} I_i}{V_i + Z_{oi} I_i} = \frac{Z_{ei} - Z_{oi}}{Z_{ei} + Z_{oi}} = \rho_{vi}$$

S_{ii} shows the reflection coefficient of the i port.

$$b_k = S_{ki} a_i \xrightarrow[j \neq i]{\forall a_j = 0} S_{ki} = \frac{b_k}{a_i} = \frac{V_k^- \sqrt{2 Z_{oi}}}{V_i^+ \sqrt{2 Z_{ok}}} = \frac{V_k^-}{V_i^+} \sqrt{\frac{Z_{oi}}{Z_{ok}}}$$

S_{ki} is proportional to power transmission from i to k port.

Scattering Parameters: example



$Z_1 = Z_0$

$Z_2 = Z_0/\sqrt{2}$

→ 3 dB coupler

$$S = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & -j & -1 & 0 \\ -j & 0 & 0 & -1 \\ -1 & 0 & 0 & -j \\ 0 & -1 & -j & 0 \end{bmatrix}$$

- Gate 1 : input
- Gate 2 : -90º output
- Gate 3 : -180º output
- Gate 4 : isolated port

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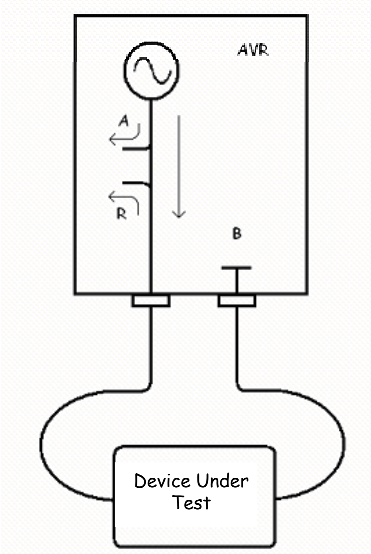
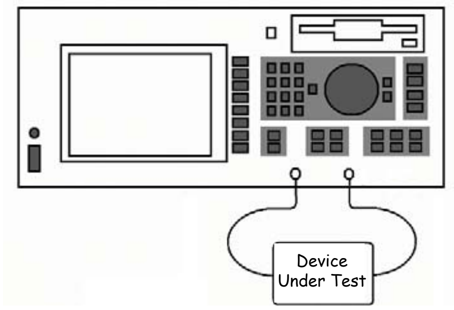
High Frequency Circuits Measurement Techniques

High Frequency Circuits Measurement Techniques.

High frequency circuits are measured with a *Network Analyzer:*



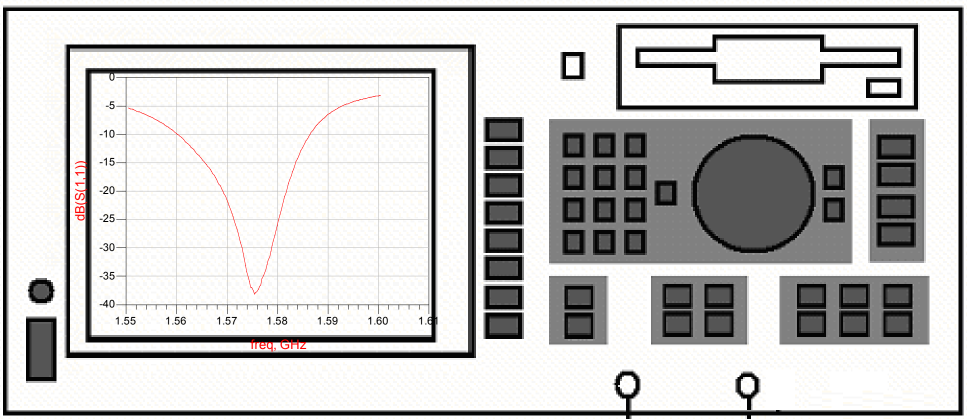


S parameters are measured and shown:

- ➢Transmission (S_{12})
- ➢Reflection (S_{11})
- ➢Smith Chart (S_{11})

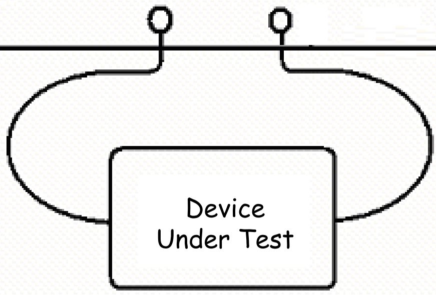
High Frequency Circuits Measurement Techniques.

The Network Analyzer presents the transmission and reflection for a selected frequency spam:



For the selected frequency spam:

- ➢Transmission (S_{12})
- ➢Reflection (S_{11})



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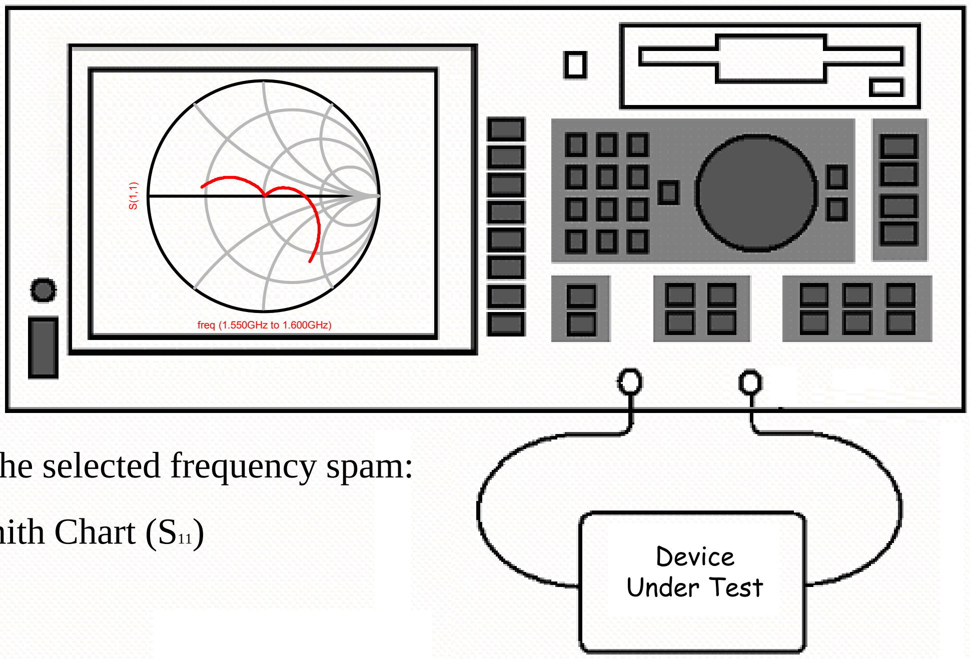

For the selected frequency spam:

➢Smith Chart (S_{11})