



Linear Antennas

Radiation Group
Signals, Systems and Radiocommunications Department



Universidad Politécnica de Madrid
Andrés García Aguilar, Pablo padilla de la Torre
E-Mail: andreg@gr.ssr.upm.es, ppadilla00@gr.ssr.upm.es



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Linear Antennas



Outline

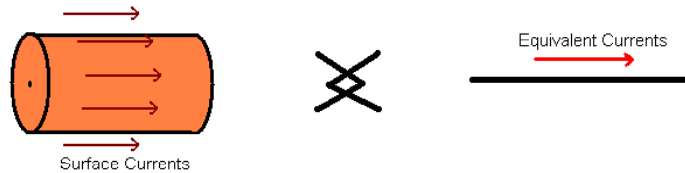
- 1.- Introduction
- 2.- The electric dipole, Baluns
- 3.- Monopole over ground plane
- 4.- Dipoles Parallel to the ground plane
- 5.- Yagi-Uda Antennas
- 6.- Other linear antennas



Linear Antennas



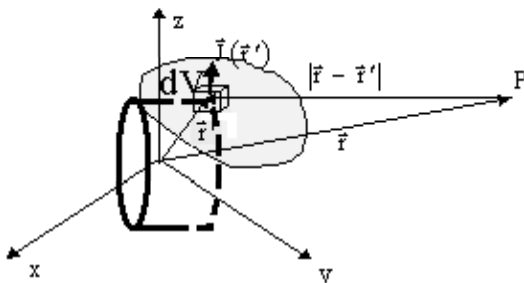
- Linear antennas are considered to be those antennas that imply the use of electrically thin conductors (wavelength \gg conductor diameter).
- Electric current flows over the conductor surface.
- In order to calculate radiated fields in these antennas, conductors are modeled as if they were current lines with no diameter. Its equivalent current is the surface current of the real model.



Linear Antennas



- Paying attention to the geometry, it is necessary to calculate the potential vector



Potential Vector:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_{L'} \frac{I(\vec{r}') e^{-jk_0|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} d\vec{l}'$$

Potential Vector ($\vec{r}' \ll \vec{r}$):

$$\vec{A} = \frac{\mu}{4\pi} \frac{e^{-jkr}}{r} \int_{L'} I(\vec{r}') e^{jk\hat{r}\cdot\vec{r}'} d\vec{l}'$$

- Potential vector allows us to obtain the electric field distribution:

$$\vec{H} = \frac{1}{\mu_0} \nabla \times \vec{A} \quad \vec{E} = \frac{1}{j\omega\epsilon_0} \nabla \times \vec{H}$$



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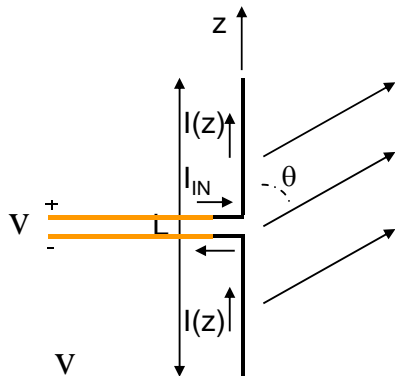
L/2



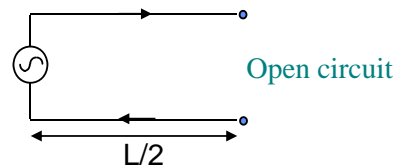
The electric dipole



- Mainly, an electric dipoles formed by two $\frac{L}{2}$ wires working together.



Although it is possible to calculate the analytical current distribution over the structure, it is preferable to assume some approximate results using *Transmission line theory*



In that conditions, we obtain the current distribution

$$I(z) = I_m \sin \left[k \left(\frac{L}{2} - |z| \right) \right] \quad |z| < \frac{L}{2}$$

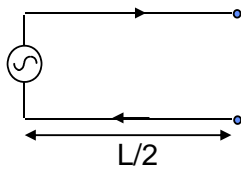


The electric dipole

$$I(z) = I_m \sin \left[k \left(\frac{L}{2} - |z| \right) \right] \quad |z| < \frac{L}{2}$$

Feeding current:

$$I_{IN} = I_m \sin \left[k \frac{L}{2} \right] \rightarrow I_m = \frac{I_{IN}}{\sin \left[k \frac{L}{2} \right]}$$

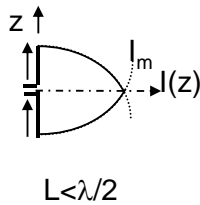


• The model yields a quite good agreement with the analytical result when considering

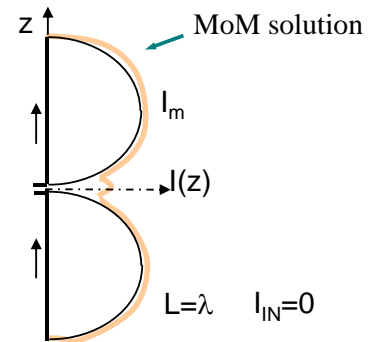
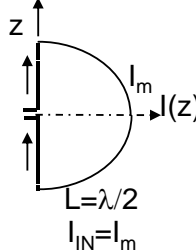
$$L = \frac{\lambda}{2}$$

Current distribution for different dipole lengths:

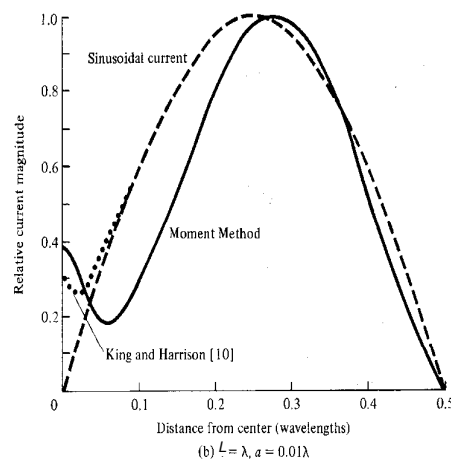
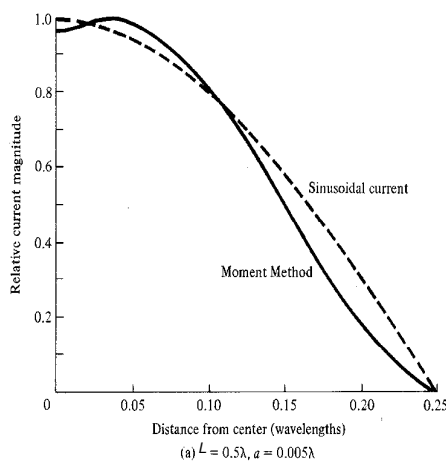
Short dipole:



Hertz dipole:



The electric dipole



Current distributions calculated using Moment Method

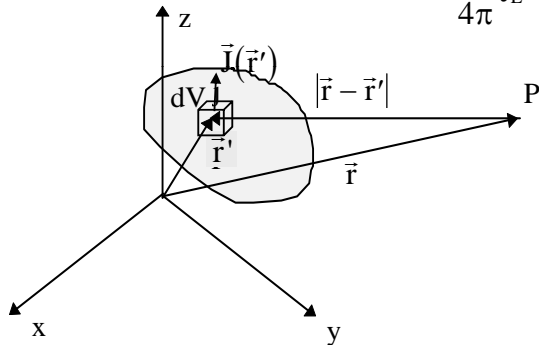


Radiated Field

Infinitesimal Dipole

Potential Vector:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_{L'} \frac{\vec{I}(\vec{r}') e^{-jk_0|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} d\vec{l}'$$



$$\vec{H} = \frac{1}{\mu_0} \nabla \times \vec{A}$$

$$\vec{E} = \frac{1}{j\omega\epsilon_0} \nabla \times \vec{H}$$

Electric and magnetic fields:

$$\vec{H} = \hat{\phi} \left[\frac{\partial}{\partial r} (rA_\theta) - \frac{\partial}{\partial \theta} A_r \right] = \hat{\phi} \frac{Idl \sin \theta}{4\pi r} \left(jk_0 + \frac{1}{r} \right) e^{-jk_0 r}$$

$$\vec{E} = \frac{j\eta Idl}{2\pi k_0} \left[\hat{r} \cos \theta \left(\frac{jk_0}{r^2} + \frac{1}{r^3} \right) + \hat{\theta} \frac{\sin \theta}{2} \left(-\frac{k_0^2}{r} + \frac{jk_0}{r^2} + \frac{1}{r^3} \right) \right] e^{-jk_0 r}$$



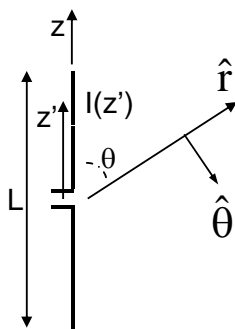
Radiated Field

Finite Dipole

Coordinates:

$$|\vec{r}-\vec{r}'| = r - z' \cos \theta + \frac{1}{r} \left(\frac{z'^2}{2} \sin^2 \theta \right) + \frac{1}{r^2} \left(\frac{z'^3}{2} \cos \theta \sin^2 \theta \right) + \dots$$

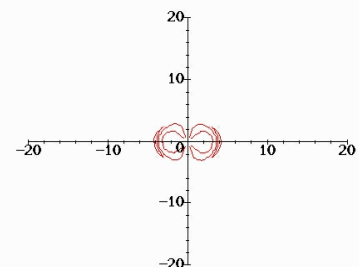
potential vector:



$$\vec{A} = \frac{\mu}{4\pi} \int \frac{\vec{I}(\vec{r}') e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} d\vec{l}' =$$

$$= \frac{\mu}{4\pi} \int_{-L/2}^{L/2} \frac{I_m \sin \left(k \left(\frac{L}{2} - |z'| \right) \right)}{r - z' \cos \theta + \frac{1}{r} \left(\frac{z'^2}{2} \sin^2 \theta \right) + \frac{1}{r^2} \left(\frac{z'^3}{2} \cos \theta \sin^2 \theta \right) + \dots} e^{jkr - z' \cos \theta} \hat{z} dz'$$

$$\text{Electric field: } |E| \propto K_1 \cdot \left(\frac{1}{k_2 r} + \frac{1}{k_3 r^2} + \dots \right)$$



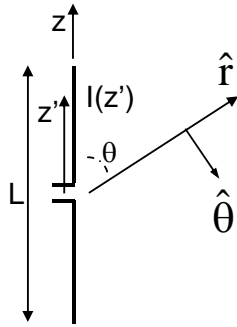


Radiated Field

Finite Dipole

Coordinates: $\hat{r} \cdot \vec{r}' = (\sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z}) \cdot (z' \hat{z}) = z' \cos \theta$

Far potential vector: $\vec{A} = \frac{\mu}{4\pi} \frac{e^{-jkr}}{r} \int_{L'} I(\vec{r}') e^{jk\hat{r} \cdot \vec{r}'} d\vec{r}' =$



$$= \frac{\mu}{4\pi} \frac{e^{-jkr}}{r} \int_{-L/2}^{L/2} I_m \sin \left(k \left(\frac{L}{2} - |z| \right) \right) e^{jkz' \cos \theta} \hat{z} dz' =$$

$$= \frac{\mu}{4\pi} \frac{e^{-jkr}}{r} \frac{2I_m}{k} \frac{\cos \left(\frac{kL}{2} \cos \theta \right) - \cos \left(\frac{kL}{2} \right)}{\sin^2 \theta} \left(\underbrace{\cos \theta \hat{r} - \sin \theta \hat{\theta}}_{\hat{z}} \right)$$

Could be considered as a 1st order approximation

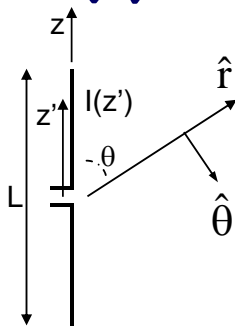
Far field: $\vec{E} = -j\omega(A_\theta \hat{\theta} + A_\phi \hat{\phi}) = j\eta \frac{e^{-jkr}}{2\pi r} I_m \frac{\cos \left(\frac{kL}{2} \cos \theta \right) - \cos \left(\frac{kL}{2} \right)}{\sin \theta} \hat{\theta}$

$$E_\phi = 0$$

Linear Polarization throughout θ



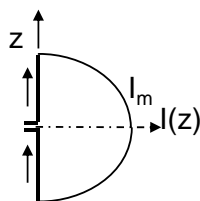
Radiated Power



$$\vec{E} = -j\omega(A_\theta \hat{\theta} + A_\phi \hat{\phi}) = j\eta \frac{e^{-jkr}}{2\pi r} I_m \frac{\cos \left(\frac{kL}{2} \cos \theta \right) - \cos \left(\frac{kL}{2} \right)}{\sin \theta} \hat{\theta}$$

$$\left\{ \begin{array}{l} P_{rad} = \int_{4\pi} U(\theta, \phi) d\Omega = \iint r^2 \frac{|E(\theta, \phi)|^2}{2\eta_0} \sin(\theta) d\theta d\phi \\ R_{rad} = \frac{2P_{rad}}{|I_0|^2} \end{array} \right.$$

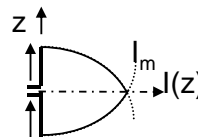
Hertz dipole:



$$P_{rad} = \frac{\pi}{3} \eta_0 |I_0|^2 \left(\frac{l}{\lambda} \right)^2$$

$$R_{rad} = 80\pi^2 \left(\frac{l}{\lambda} \right)^2$$

Short dipole:



$$P_{rad} = \frac{\pi}{12} \eta_0 |I_0|^2 \left(\frac{l}{\lambda} \right)^2 = \frac{P_{Hertz}}{4}$$

$$R_{rad} = 20\pi^2 \left(\frac{l}{\lambda} \right)^2$$

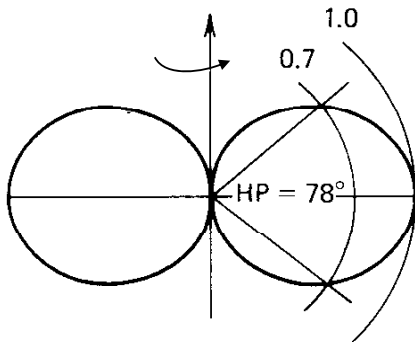


Radiation Parameters



Normalized Field diagrams:

$$\frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta}$$

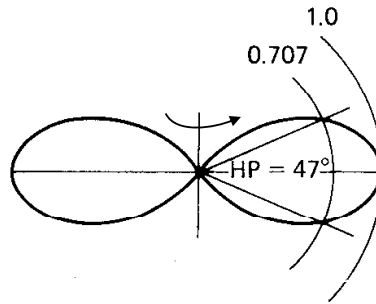


$L=0.5\lambda$

Directivity: $D_0=1,64 = 2,15 \text{ dBi}$

$R_{\text{radiation}} = 73 \Omega$

$$\frac{1 + \cos(\pi \cos \theta)}{2 \sin \theta}$$

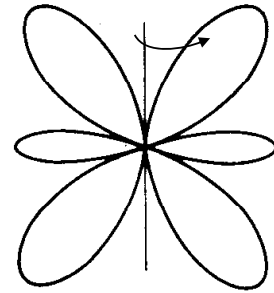


$L=\lambda$

$D_0=2,41$

$R_{\text{rad}}=\infty \Omega$

Multilobe Diagram



$L=1.5\lambda$

$D_0=2,17$

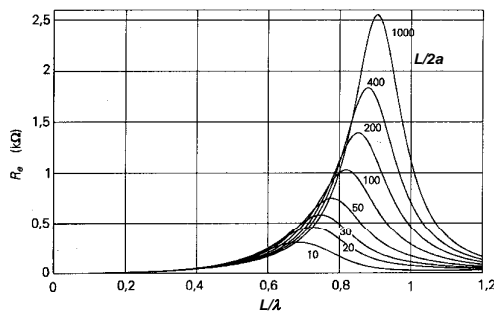
$R_{\text{rad}}=99,5 \Omega$



Input Impedance

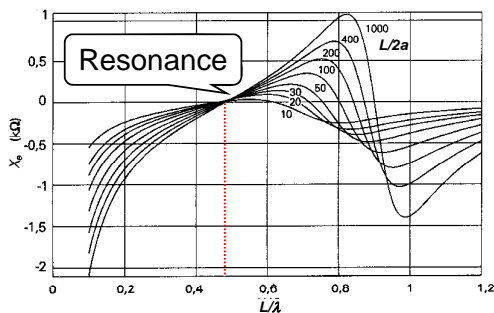


Input impedance: $(Z_{IN}=R_e+jX_e)$

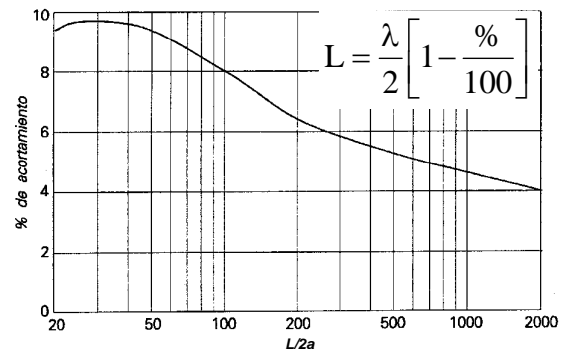


$$Z_{IN}(\lambda/2)=73+j42,5 \Omega \quad \text{when } a \rightarrow 0$$

a =dipole radius



Resonance Condition



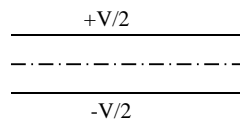


Dipole Feeding - Baluns

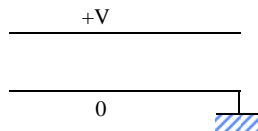


- Baluns are devices that transform balanced current lines into unbalanced ones. Its name comes from: “**balun**” = **balanced** to **unbalanced**.
- Baluns allow us to feed symmetric structures in a balanced way.
For instance: dipoles, fed with asymmetric transmission lines, coaxial wires... Used to transport energy from circuitry to the antenna.

Balanced lines:



Unbalanced lines:



Bifilar



Shielded Bifilar



Coplanar



Coaxial



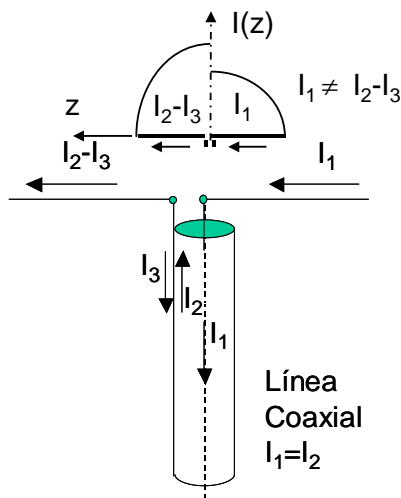
Microstrip



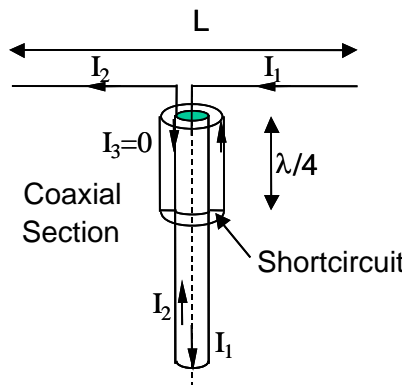
Stripline



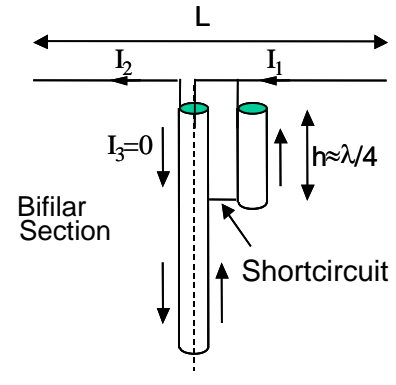
Baluns



Non symmetric feeding



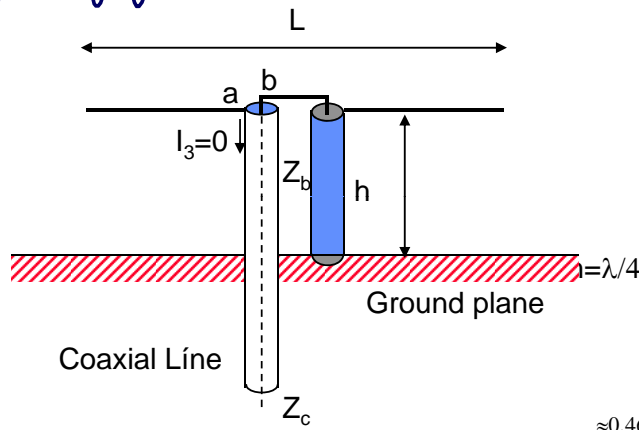
Bazooka or Sleeve Balun



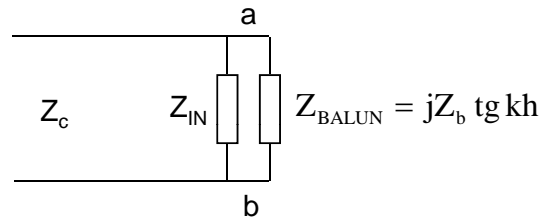
Broken Balun



Baluns

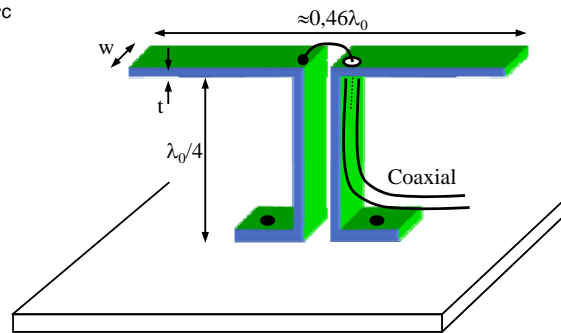


Equivalent Circuit



$$Z_{BALUN} = jZ_b \operatorname{tg} kh$$

$$\text{For } h = \lambda/4 \Rightarrow Z_{BALUN} = \infty$$



Linear Antennas



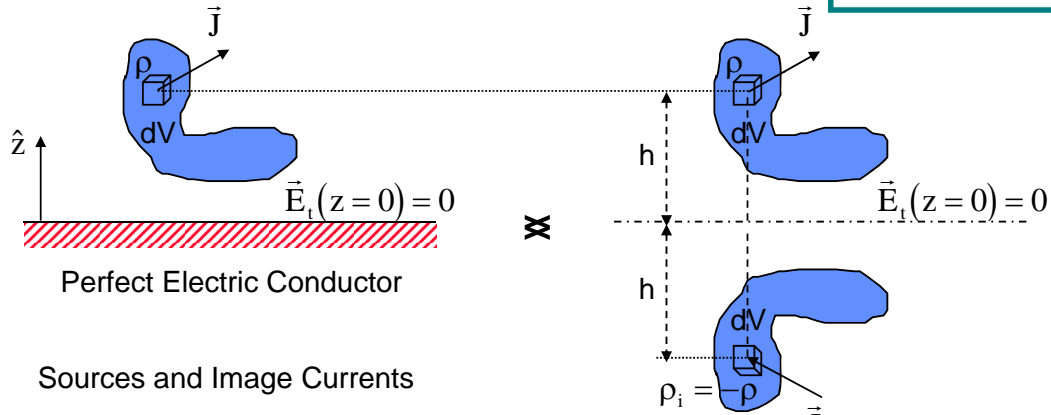
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Image Theorem



z ≥ 0 Valid Results



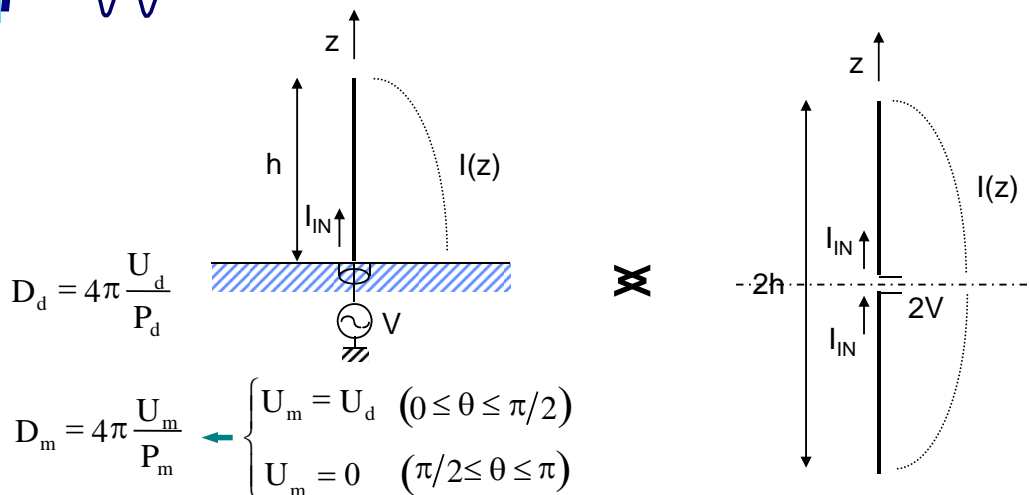
Perfect Electric Conductor

Sources and Image Currents

$$\begin{cases} \rho \\ \rho_i = -\rho \end{cases} \begin{cases} \vec{J} = J_x \hat{x} + J_y \hat{y} + J_z \hat{z} \\ \vec{J}_i = -J_x \hat{x} - J_y \hat{y} + J_z \hat{z} \end{cases}$$



Vertical Monopole over Ground Plane



$$D_d = 4\pi \frac{U_d}{P_d}$$

$$D_m = 4\pi \frac{U_m}{P_m} \left\{ \begin{array}{l} U_m = U_d \quad (0 \leq \theta \leq \pi/2) \\ U_m = 0 \quad (\pi/2 \leq \theta \leq \pi) \end{array} \right.$$

$$P_m = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} U_m(\theta, \phi) \sin \theta d\theta d\phi = \frac{1}{2} P_d$$

$$Z_{INdipolo} = \frac{2V}{I} = 2Z_{INMonopolo}$$

$$D_{monopolo} = 2D_{dipolo}$$

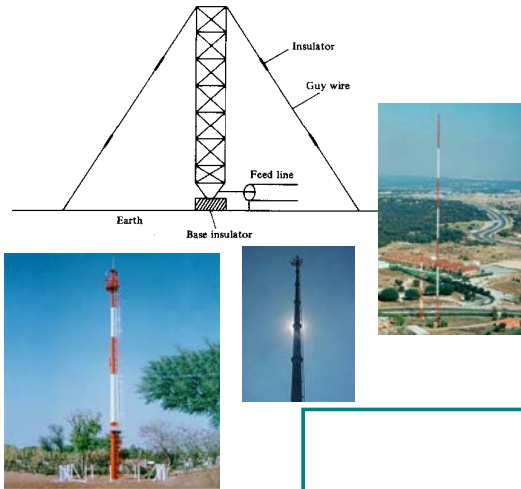
$$R_{rad\ monopolo} = \frac{1}{2} R_{rad\ dipolo}$$

$$Z_{INmonopolo} = \frac{1}{2} Z_{INdipolo}$$

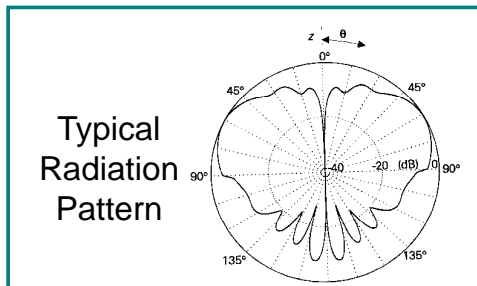
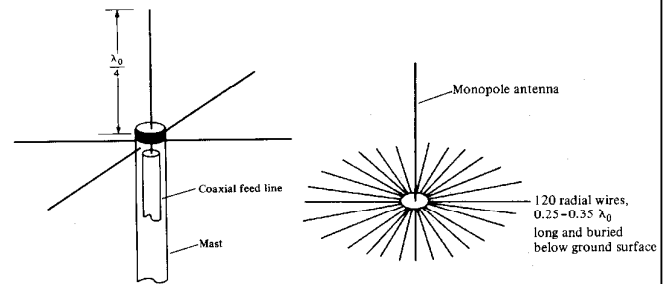


Vertical Monopoles

MW Monopole over ground.



Monopole over ground plane simulated with metallic wires

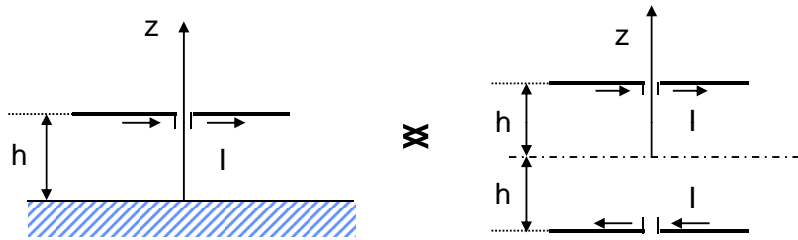


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Image Theorem: parallel dipoles over ground plane



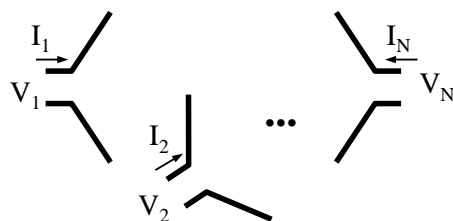
- When $h \ll \lambda$, the radiated field is quite small due to the destructive summation of direct and reflected field.
- When $h = \lambda/4$ the radiated field is enhanced due to constructive summation in z direction.
- Considering finite ground planes (higher than $\lambda \times \lambda$), the image theorem can be applied and undesired effects can be neglected.



Antenna mutual couplings



- When designing composed antenas, with several radiating elements, it is necessary to study mutual couplings between these elements.
 - From both, the radiation point of view (feeding currents) and the circuit point of view (impedance), the global antenna behaves as a **multiport linear network**.

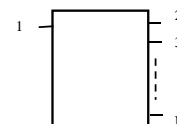


$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

Active impedance of *i* radiating element:

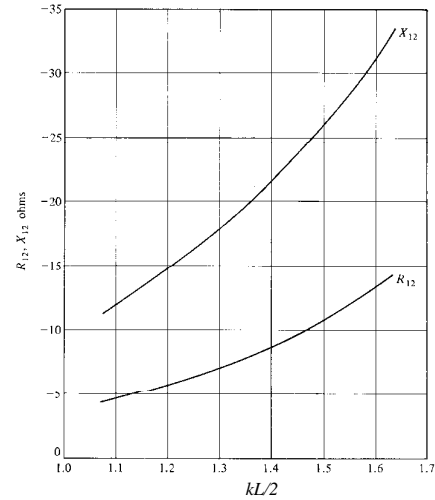
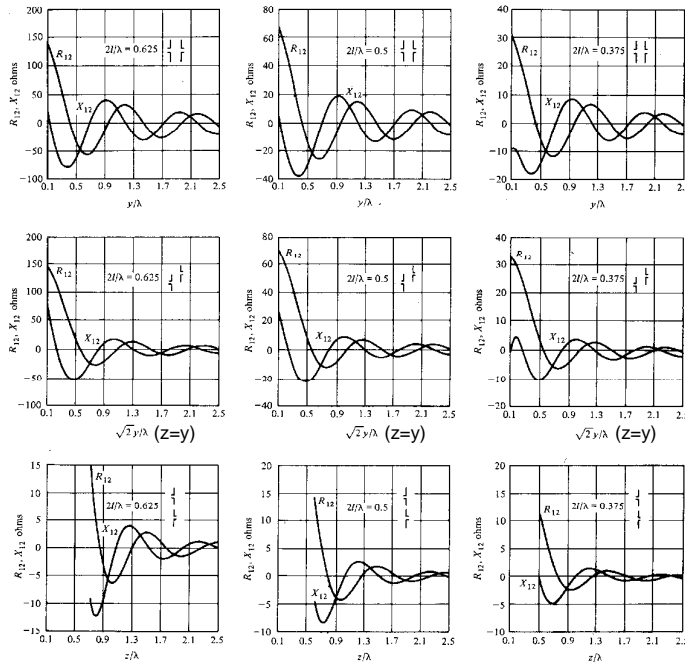
$$Z_i = \frac{V_i}{I_i} = \sum_{j=1}^N Z_{ij} \frac{I_j}{I_i} = Z_{ii} + \sum_{\substack{j=1 \\ i \neq j}}^N Z_{ij} \frac{I_j}{I_i}$$

Easily related to generalized multipole analysis





Mutual Impedances between Dipoles



Mutual impedance between two identical parallel dipoles, one in front of the other, at $\lambda/2$ distance

Fig. 7.24 The Mutual Impedance Between Two Identical Slender Center-Fed Cylindrical Dipoles versus Their Separation Along Various Paths; Rectangular Plots



Linear Antennas



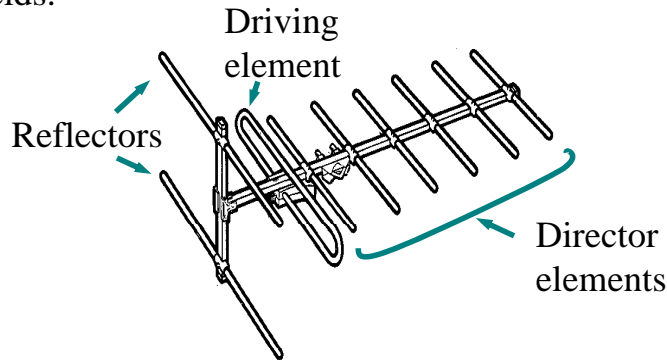
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Yagi Antennas



- These antennas are made of a lot of dipoles. Typically, only one of them is externally fed (active, driving element); the rest have induced currents due to mutual couplings (director elements).
- Passive dipoles behave whether reflecting (reflectors) or guiding (director elements) radiated fields.



A Yagi antenna shows a radiation Gain, approximated by:

$$G_{yagi} = G_{Dipole} + 10 \log(n + 1)$$

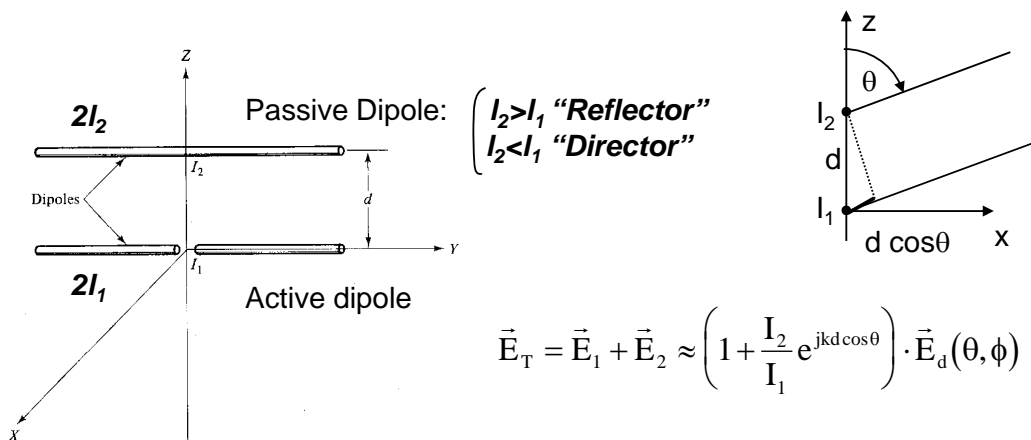
where n is the number of driving dipoles.



Yagi Antennas



- 2 elements Yagi-Uda antenna.



$$\vec{E}_T = \vec{E}_1 + \vec{E}_2 \approx \left(1 + \frac{I_2}{I_1} e^{jkd \cos \theta} \right) \cdot \vec{E}_d(\theta, \phi)$$

- Circuital equations:

$$\begin{cases} V_1 = Z_{11}I_1 + Z_{12}I_2 \\ 0 = Z_{21}I_1 + Z_{22}I_2 \end{cases}$$

$$\frac{I_2}{I_1} = - \frac{Z_{12}}{Z_{22}}$$

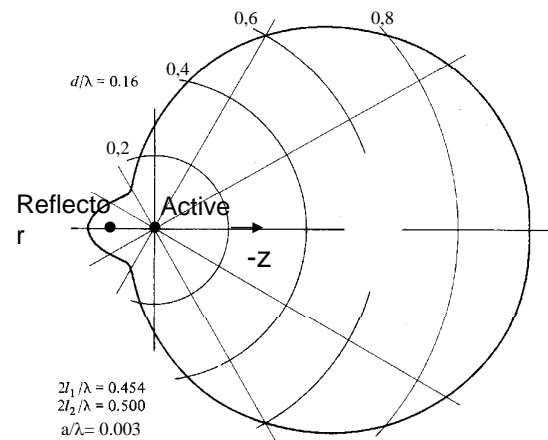
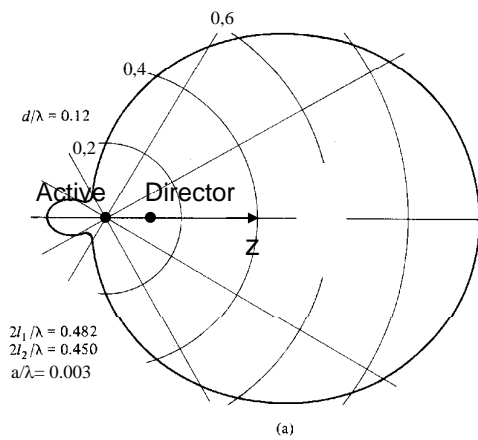
$$Z_{IN} = \frac{V_1}{I_1} = Z_{11} - \frac{Z_{12}^2}{Z_{22}}$$



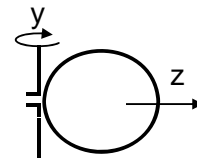
2 elements Yagi Antenna: Patterns



H Plane (XZ Plane): $E_d(\theta, \phi=0)=\text{constant}$; $F(\theta, \phi=0)=|1+I_2/I_1 \exp(jkdcos\theta)|$



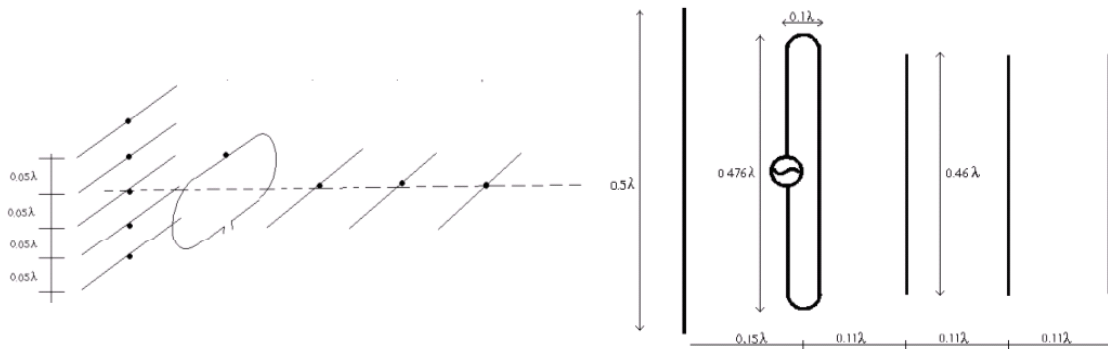
E Plane (YZ Plane): The radiation patten must include the dipole radiation pattern (like a donut).



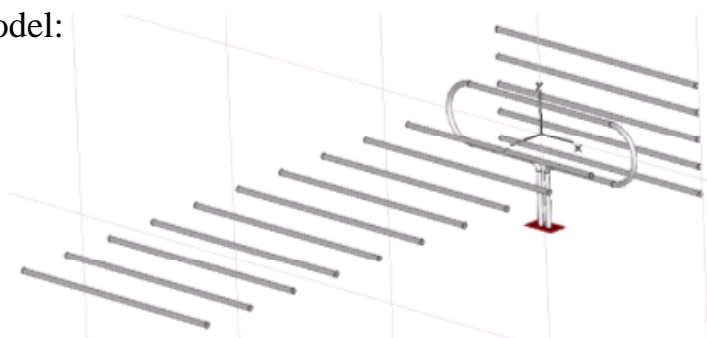
Yagi antennas.



Typical dimensions:



Simulation model:

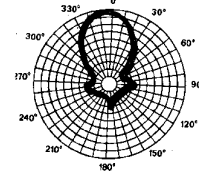
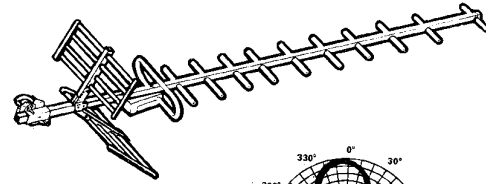




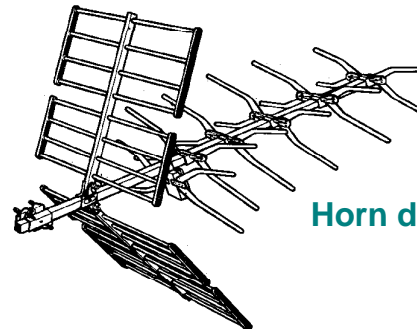
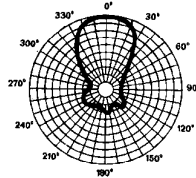
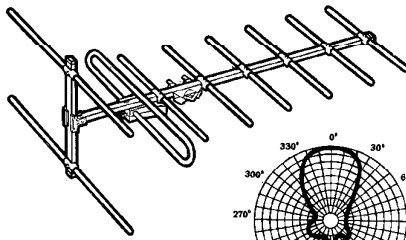
Yagi Examples

Usually, as active element, it is used a bended dipole in order to increase input impedance and band width.

Diedric reflector Yagi



Double reflector Yagi



Horn dipoles Yagi



Linear Antennas

- 1.- Introduction
- 2.- The electric dipole, Baluns
- 3.- Monopole over ground plane
- 4.- Dipoles Parallel to the ground plane
- 5.- Yagi-Uda Antennas
- 6.- Other linear antennas



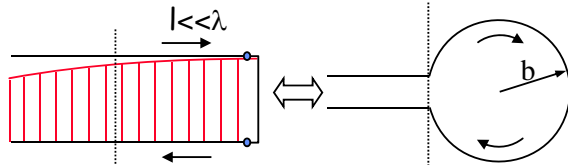
Loop Antennas



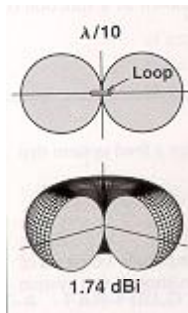
Approximate current distribution:

Short Loop (in terms of λ):

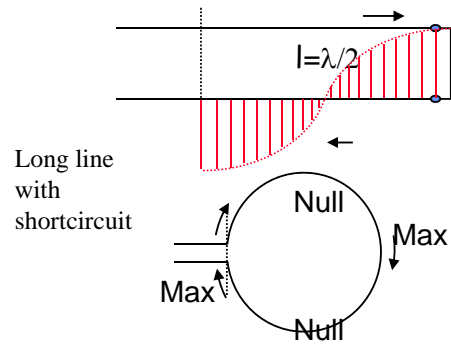
Short line with shortcircuit = **uniform current**



$$C = 2\pi b \approx 2l$$



Large Loop (in terms of λ):



• multilobe pattern, high efficiency.

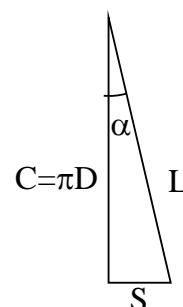
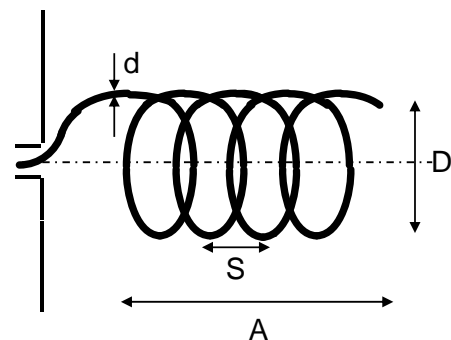


Helix



• The helix geometry depends on the next values:

- D= Helix diameter.
- C= Perimeter of the helix cylinder= πD
- S= length of one turn= $\pi D \tan \alpha$
- α = Inclination angle= $\text{atan}(S/C)$
- L= one turn length
- N= Number of turns.
- A= Total length= NS
- d= Wire diameter



• Helix antennas are usually used in its axial radiation mode, when C is similar to λ .



Helix

Axial Radiation Mode

- Electrically big helix, with dimensions $3/4 < C/\lambda < 4/3$ y $\alpha \approx 12^\circ - 15^\circ$:

- Progressive current wave through the helix:

$$I(l) = I_0 \exp(-jkl)$$

- Wide band: $f_{\text{sup}}/f_{\text{inf}} = 1,78$

- Approximately real impedance value:

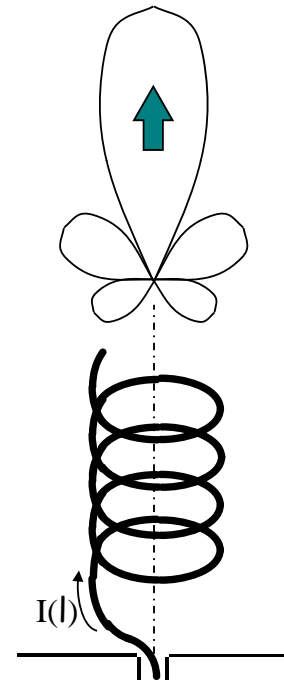
$$R_{\text{in}} \approx 140 \frac{C}{\lambda} \approx 140 \quad \Omega$$

- Circular polarization (left or right handed, the same as the wire turn)

- Quite directive radiation pattern. Secondary lobe level: -9 dB.

- Directivity:

$$D \approx 15 \left(\frac{C}{\lambda} \right)^2 \frac{NS}{\lambda} \approx 15 \frac{A}{\lambda}$$



Helix examples

