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Inferring consensus weights from pairwise comparison matrices without suitable properties

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Abstract Pairwise comparison is a popular method for establishing the relative importance of n objects. Its main purpose is to get a set of weights (priority vector) associated with the objects. When the information gathered from the decision maker does not verify some rational properties, it is not easy to search the priority vector. Goal programming is a flexible tool for addressing this type of problem. In this paper, we focus on a group decision-making scenario. Thus, we analyze different methodologies for getting a collective priority vector. The first method is to aggregate general pairwise comparison matrices (i.e., matrices without suitable properties) and then get the priority vector from the consensus matrix. The second method proposes to get the collective priority vector by formulating an optimization problem without determining the consensus pairwise comparison matrix beforehand.

Keywords Pairwise comparisons · Group decisions · Preference aggregation · Goal programming

Group decision making involves aggregating individual preferences into a single collective preference. The existence of a satisfactory aggregation procedure depends on what type of data there is about these individual preferences. The data types and scales are classified according to how decision makers show their preferences towards the different alternatives (attributes) involved (see e.g., Herrera et al. 2001).

A particular case for representing individual preferences with respect to a finite set of alternative is defined by means of pairwise comparisons. The pairwise comparison (pc) method is a popular method for establishing the relative importance of n objects. The

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information gathered from the decision maker (DM) is recorded in a matrix structure, $\mathbf{M} = (m_{ij})_{i,j=1,\dots,n}$, named as *pc matrix*.

The problem of inferring the relative importance of n objects in a collective pc scenario is the main subject of this paper. This is a fertile problem in the context of the Analytic Hierarchy Process (AHP) introduced by Saaty (1980). Ramanathan and Ganesh (1994) give a good introduction to the group aggregation procedure in this specific pairwise comparison scenario. However, some rational attitudes are expected from DMs in this type of work.

Our purpose here is to deal with a more general scenario where DMs may provide a pc matrix containing data type without any specific properties.

In the single case, the main purpose of the pc method is to get a set of weights, $w_1, w_2, \dots, w_n$ associated with the objects. Thus, the challenge is how to infer these weights when DM opinions are expressed through a pc matrix that fails to satisfy some rational properties. In this case, a theoretical framework based on approximations is needed. This is the origin of "distance minimization" methods.

"Distance minimization" methods can be divided into two classes:

(a) A rational pc matrix approximates the original pc matrix. The weights are taken from this rational matrix. This is the approach adopted by Chu (1998), Koczkodaj and Orlowski (1999) or Dopazo and González-Pachón (2003).
(b) The ratio of weights $\frac{w_i}{w_j}$ is closed "as much as possible" to the value m_{ij}. This is equivalent to the problem of minimizing the distance between matrix $\mathbf{M}$ and the rank one matrix $(\frac{w_i}{w_j})_{i,j}$. This is the perspective adopted by a lot of the work recorded in Choo and Wedley (2004).

We will adopt the second point of view, where Goal Programming (GP) has been proposed as a flexible tool (see Jones and Mardle 2004 or González-Pachón and Romero 2004) for other applications. The above theoretical framework may be extended to a collective decision-making scenario by considering three problems of inference based on three possible consensus problems: (i) inferring consensus weights from individual weights taken from an individual pc matrix, (ii) inferring consensus weights as associated with a consensus pc matrix or (iii) inferring consensus weights directly, without getting a consensus pc matrix. In all cases, the inference of weights from pc matrices with no suitable properties is the cornerstone of the analysis undertaken.

Problem (i) was already addressed by Linares and Romero (2002). Hence, this paper is designed to convincingly tackle problems (ii) and (iii). Consequently, the organisation of the paper is as follows. In Sect. 2, the problem of searching weights from a general pc matrix is solved by reducing a distance function optimisation problem to a GP formulation. In Sect. 3, the problem of searching a consensus pc matrix is analysed to yield the consensus weights from this matrix. In Sect. 4, the GP model proposed in Sect. 2 is adapted to yield consensus weights without beforehand determining a consensus pc matrix. Finally, some conclusions are discussed in the last section.

1 Inferring weights from a pc matrix without suitable properties

Let us consider a decision-making problem with n objects. Suppose we have a DM who expresses his or her preference using Saaty's scale (see Saaty 1980) as an $n \times n$ pc matrix $\mathbf{M} = (m_{ij})_{i,j=1,\dots,n}$.

Where **M** verifies suitable properties, as reciprocity and consistency, there exists a set of positive numbers, $w_1, w_2, \ldots, w_n$, such that

$$\frac{w_i}{w_j} = m_{ij}, \quad \forall i, j. \tag{1}$$

These positive numbers define the set of priority weights associated with the n objects.

The challenge is to search a set of priority weights that synthesize preference information contained in non-reciprocal and/or inconsistent pc matrices. In this case, we adopt an approach based on approximations rather than identities; i.e., the equality (1) is transformed as follows:

$$\frac{w_i}{w_j} \approx m_{ij}, \quad \forall i, j \Rightarrow m_{ij} w_j - w_i \approx 0. \tag{2}$$

Using this approximation shown in (2), the problem of getting weights from unsuitable pc matrices can be formulated as the following optimization problem for a generic metric p:

$$\begin{aligned} &\text{Min} \left[\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} |m_{ij} w_j - w_i|^p \right]^{1/p} \\ &\text{s.t.} \\ &\sum_{i=1}^{n} w_i = 1, \\ &w_i > 0, \quad \forall i. \end{aligned} \tag{3}$$

Because the objective function is not a true distance function between matrix **M** and the rank one matrix $(\frac{w_i}{w_j})_{i,j}$, the optimal solution cannot be scaled arbitrarily. So, the constraint $\sum_{i=1}^{n} w_i = 1$ needs to be added. We find that, for $p = 2$, this weights estimation method is the Preference Weighted Least Square method described in Choo and Wedley (2004).

For $p = \infty$, the objective function becomes the following expression:

$$\begin{aligned} &\text{Min} \ \underset{ij}{\text{Max}} \ |m_{ij} w_j - w_i| \\ &\text{s.t.} \\ &\sum_{i=1}^{n} w_i = 1, \\ &w_i > 0, \quad \forall i. \end{aligned} \tag{4}$$

It is far from easy to optimise models (3) and (4). However, it is straightforward to show how, taking into account the following change of variables (Charnes and Cooper 1977):

$$n_{ij} = \frac{1}{2}[|m_{ij} w_j - w_i| + (m_{ij} w_j - w_i)], \tag{5}$$

$$p_{ij} = \frac{1}{2}[|m_{ij} w_j - w_i| - (m_{ij} w_j - w_i)]. \tag{6}$$

The above optimization problems can be reduced to GP formulations. In fact, based on Romero (2001), the following Extended GP model represents a unified approach for the different p-metrics ($p \in [1, \infty]$), encompassing model (3)

$$\begin{aligned}
&\text{Min}\,(1-\lambda)D + \lambda \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} (n_{ij} + p_{ij}) \\
&\text{s.t.} \\
&m_{ij} w_j - w_i + n_{ij} - p_{ij} = 0, \quad i, j = 1, \ldots, n, \ i \neq j \\
&n_{ij} + p_{ij} \leq D, \quad i, j = 1, \ldots, n, \ i \neq j \\
&n_{ij} \geq 0, \qquad p_{ij} \geq 0, \quad i, j = 1, \ldots, n, \ i \neq j, \\
&\lambda \in [0, 1] \quad \text{(control parameter).}
\end{aligned} \tag{7}$$

We find that $\lambda = 1$ leads to the solution for metric $p = 1$, whereas $\lambda = 0$ leads to the solution for metric $p = \infty$. For values of control parameter λ belonging to the open interval $(0, 1)$, solutions for intermediate values of metric p, if they exist, are yielded.

It should be noted that solutions for metric $p = 1$ (i.e., $\lambda = 1$) imply a weights structure (priority vector) that minimizes the aggregated deviations associated with the optimum solution of system of (2). On the other hand, the solution for metric $p = \infty$ (i.e., $\lambda = 0$) implies a priority vector that minimizes the maximum deviation between the $n(n-1)$ deviations associated with system (2). For other values of the control parameter λ, intermediate solutions, if they exist, can be calculated. In short, parameter λ trade-offs amount of consensus obtained from the point of view of the majority with amount of consensus obtained from the point of view of the minority (González-Pachón and Romero 2004).

Example The above ideas will be illustrated with the help of the following example. We have the following pairwise comparison matrix over four objects (a_1, a_2, a_3, a_4). Saaty's scale has been used for valuations, yielding the following general pc matrix, which is non-reciprocal and inconsistent:

$$\mathbf{M} \equiv \begin{pmatrix} 1 & 5 & 1/9 & 3 \\ 1/4 & 1 & 3 & 1/3 \\ 2 & 1/2 & 1 & 1/2 \\ 1/3 & 3 & 2 & 1 \end{pmatrix}.$$

To infer the importance weights from pc matrix $\mathbf{M}$, we resort to model (7), getting the following Extended GP modeling

Achievement function:

$$\text{Min}(1-\lambda)D + \lambda \left[\sum_{i=1}^{4} \sum_{\substack{j=1 \\ j \neq i}}^{4} (n_{ij} + p_{ij}) \right]$$

Goals and constraints:

$$\begin{aligned}
&w_1 - 5w_2 + n_{12} - p_{12} = 0, \quad 4w_2 - w_1 + n_{21} - p_{21} = 0, \quad w_3 - 2w_1 + n_{31} - p_{31} = 0, \\
&3w_4 - w_1 + n_{41} - p_{41} = 0, \quad 9w_1 - w_3 + n_{13} - p_{13} = 0, \quad w_2 - 3w_3 + n_{23} - p_{23} = 0, \\
&2w_3 - w_2 + n_{32} - p_{32} = 0, \quad w_4 - 3w_2 + n_{42} - p_{42} = 0, \quad w_1 - 3w_4 + n_{14} - p_{14} = 0, \\
&3w_2 - w_4 + n_{24} - p_{24} = 0, \quad 2w_3 - w_4 + n_{34} - p_{34} = 0, \quad w_4 - 2w_3 + n_{43} - p_{43} = 0,
\end{aligned}$$

Table 1 Associated priority weights and rankings to different values of λ

Control parameter (λ)	Associated weights	Associated ranking
$\lambda = 1$	$\mathbf{w}^1 \equiv (0.120, 0.160, 0.240, 0.480)$	$a_4 > a_3 > a_2 > a_1$
$\lambda = 0.20$	$\mathbf{w}^{0.20} \equiv (0.150, 0.120, 0.360, 0.370)$	$a_4 > a_3 > a_1 > a_2$
$\lambda = 0$	$\mathbf{w}^0 \equiv (0.132, 0.190, 0.350, 0.330)$	$a_3 > a_4 > a_2 > a_1$

$$n_{ij} + p_{ij} - D \leq 0, \quad i, j = 1, \dots, 4,\ i \neq j,$$
$$w_1 + w_2 + w_3 + w_4 = 1.$$

Alternative vectors of priority weights can be obtained using different values of control parameter λ. For illustrative purposes, the two opposite solutions plus an intermediate one are shown in Table 1.

2 Inferring consensus weights from the consensus pc matrix

There are two basic ways to aggregate individual preferences depending on whether the group wants to act as a unit or as separate individuals; see Forman and Peniwati (1998). In this section, we take the first approach where individual identities are lost in a consensus pc matrix. So, the group becomes a new individual with its own pc matrix. This point of view defines the scenario for the second problem of inference described in the introduction that will be developed in this section.

The literature propose two widely used methods for aggregating pc matrices (Bolloju 2001): the geometric mean method (see e.g. Aczel and Saaty 1983; Barzilai and Golany 1994) and the weighted arithmetic mean method (see e.g. Dyer and Forman 1992). These two methods are basically used because they are computationally friendly and also satisfy the reciprocal property for every pc matrix.

In this paper, we deal with a pc matrix with no suitable properties. So, we propose a distance-metric-based approach for searching the consensus pc matrix where constraints for rational properties are not needed. In this way, let us consider now that the n objects (alternatives) are evaluated by a group of k DMs, who express their preferences through k pc matrices $\mathbf{M}^1, \dots, \mathbf{M}^k$ using Saaty's scale. These k matrices do not enjoy suitable properties (i.e., reciprocity and consistency). In this context, we are concerned with the computation of a consensus pairwise comparison matrix $\mathbf{M}^C$, i.e. we want to search an $n \times n$ pairwise comparison matrix that differs from $\mathbf{M}^1, \dots, \mathbf{M}^k$, "*as little as possible*".

A specific meaning for the assertion "*as little as possible*" is needed. For this purpose, we again resort to a general optimization problem based upon a generic metric p, such as:

$$\begin{aligned}&\text{Min}\left[\sum_{t=1}^{k}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}|m_{ij}^t - m_{ij}^C|^p\right]^{1/p}\\ &\text{s.t.}\\ &0.111 \leq m_{ij}^C \leq 9, \quad i, j \in \{1, \dots, n\},\end{aligned} \tag{8}$$

where m_{ij}^t are the elements of matrix $\mathbf{M}^t$ $(t = 1, \dots, k)$ and m_{ij}^C are the elements of the consensus matrix $\mathbf{M}^C$ to be searched. The constraint set must establish some scale conditions. In our case, we have resorted to the Saaty's scale conditions.

This optimization problem is very complex from a computational point of view. However, by resorting again to the change of variables (5) and (6), it is straightforward to reduce the above problem to the following Extended GP formulation:

$$\begin{aligned}
&\text{Min}\,(1-\lambda)D + \lambda\left[\sum_{t=1}^{k}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(n_{ij}^t + p_{ij}^t\right)\right]\\
&\text{s.t.}\\
&m_{ij}^C - m_{ij}^t + n_{ij}^t - p_{ij}^t = 0, \quad i, j \in \{1, \dots, n\},\ t \in \{1, \dots, k\},\\
&\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}(n_{ij}^t + p_{ij}^t) - D \le 0, \quad k \in \{1, \dots, k\},\\
&0.111 \le m_{ij}^C \le 9, \quad i, j \in \{1, \dots, n\},\\
&\mathbf{n} \ge \mathbf{0}, \qquad \mathbf{p} \ge \mathbf{0},\\
&\lambda \in [0, 1] \quad \text{(control parameter).}
\end{aligned} \tag{9}$$

A consensus pc matrix $\mathbf{M}^C \equiv (m_{ij}^C)_{i,j}$ without suitable properties does not pose a problem for deriving priority weights; see Sect. 2. Hence, normative conditions are not imposed on m_{ij}^C elements. In short, the above mathematical program is focused exclusively on getting consensus matrix values.

In this scenario, two distance-metric-based approaches are stated, one to get the consensus pc matrix and the other to get the priority vector from the above collective pc matrix. Employing the terminology of the respective Extended GP problems, the above facts are equivalent to stating two different control parameter values: λ_1 for the first approach and λ_2 for the second.

It is also interesting to recall that for metric $p = 1$ (i.e., $\lambda_1 = 1$) the respective consensus matrix represents a solution for which the sum of individual disagreements is minimized. For metric $p = \infty$ (i.e., $\lambda_1 = 0$), the resulting consensus matrix implies the minimization of the disagreement of the most displaced DM. For intermediate values of control parameter λ_1, compromises between these two solutions, if they exist, can be reached (González-Pachón and Romero 1999; Linares and Romero 2002).

Example Let us illustrate how the theory presented in this section works with the help of a simple example. Thus, let us assume that we have four DMs $\{d_1, d_2, d_3, d_4\}$ with the following pc matrices over four objects $\{a_1, a_2, a_3, a_4\}$, again using Saaty's scale:

$$\mathbf{M}^1 \equiv \begin{pmatrix} 1 & 1/5 & 5 & 3\\ 3 & 1 & 1/7 & 1/3\\ 1/5 & 7 & 1 & 1/3\\ 1/3 & 3 & 3 & 1 \end{pmatrix}, \qquad \mathbf{M}^2 \equiv \begin{pmatrix} 1 & 3 & 1/3 & 1/3\\ 1/3 & 1 & 1 & 5\\ 3 & 1 & 1 & 7\\ 5 & 1/5 & 1/5 & 1 \end{pmatrix},$$

$$\mathbf{M}^3 \equiv \begin{pmatrix} 1 & 1 & 1/2 & 7\\ 1 & 1 & 1/4 & 5\\ 2 & 4 & 1 & 8\\ 1/5 & 1/5 & 1/8 & 1 \end{pmatrix}, \qquad \mathbf{M}^4 \equiv \begin{pmatrix} 1 & 7 & 7 & 3\\ 5 & 1 & 1 & 1/5\\ 1/7 & 1 & 1 & 1/5\\ 1/3 & 5 & 5 & 1 \end{pmatrix}.$$

The resulting consensus pc matrices for metrics $p = 1$ (i.e., $\lambda_1 = 1$) and $p = \infty$ (i.e., $\lambda_1 = 0$), with the respective associated priority weights (appear in Table 2).

Table 2 Associated priority weights and rankings to different values of λ_1 and λ_2

Control parameter (λ_1)	Consensus matrix	Control parameter (λ_2)	Associated weights/ Associated ranking
$\lambda_1 = 1$	$\begin{pmatrix} 1 & 2.11 & 4.43 & 3 \\ 1 & 1 & 1 & 0.33 \\ 2 & 1 & 1 & 7 \\ 1/3 & 3 & 0.2 & 1 \end{pmatrix}$	$\lambda_2 = 1$	$\mathbf{w}^1 \equiv (0.492, 0.233, 0.111, 0.164)/$ $a_1 > a_2 > a_4 > a_3$
		$\lambda_2 = 0$	$\mathbf{w}^0 \equiv (0.414, 0.239, 0.229, 0.118)/$ $a_1 > a_2 > a_3 > a_4$
$\lambda_1 = 0$	$\begin{pmatrix} 1 & 3 & 2.20 & 3 \\ 1 & 1 & 1 & 0.5 \\ 2 & 1.88 & 1 & 7 \\ 1/3 & 5 & 3.1 & 1 \end{pmatrix}$	$\lambda_2 = 1$	$\mathbf{w}^1 \equiv (0.472, 0.157, 0.214, 0.157)/$ $a_1 > a_3 > a_2 \sim a_4$
		$\lambda_2 = 0$	$\mathbf{w}^0 \equiv (0.457, 0.158, 0.255, 0.170)/$ $a_1 > a_3 > a_2 > a_4$

3 Inferring consensus weights without consensus between pc matrices

Let us now suppose that the group of DMs wants to act as separate individuals. So, they are not interested in getting a consensus pc matrix to define themselves as a unit. There are two ways to derive a system of common priority weights:

(a) Individual priorities are computed and combined in a consensus priority vector. Traditionally, in an AHP scenario, this combination is calculated by using an arithmetic mean (Ramanathan and Ganesh 1994). Although, recently, a distance-metric approach modeled by GP has been proposed; see Linares and Romero (2002).
(b) The consensus priority vector is calculated directly by extending the methodology described in Sect. 2 for a single matrix to a group decision-making scenario.

In this section, we focus on the second approach; i.e., to get consensus weights without previously searching for a consensus pc matrix. This proposal is equivalent to formulating the following incompatible system of equations:

$$m_{ij}^t w_j - w_i = 0, \quad i, j = 1, \ldots, n, \ t = 1, \ldots, k,$$

where m_{ij}^t represents the estimation of quotients $\frac{w_i}{w_j}$ given by DM tth.

There are two different sources or practical explanations for the incompatibility of this new system of equations:

(i) Estimations given by different DMs.
(ii) Incompatibility between estimations and rational DMs.

Table 3 Associated priority weights and rankings to different values of λ

Control parameter (λ)	Associated weights	Associated ranking
$\lambda = 1$	$\mathbf{w}^1 \equiv (0.536, 0.179, 0.179, 0.106)$	$a_1 > a_2 \sim a_3 > a_4$
$\lambda = 0.5$	$\mathbf{w}^{0.5} \equiv (0.486, 0.162, 0.216, 0.136)$	$a_1 > a_3 > a_2 > a_4$
$\lambda = 0$	$\mathbf{w}^0 \equiv (0.453, 0.184, 0.212, 0.151)$	$a_1 > a_3 > a_2 > a_4$

One way of dealing with these incompatibilities is a straightforward adaptation of model (3) as follows:

$$\begin{aligned} &\text{Min} \left[\sum_{t=1}^{k} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} |m_{ij}^t w_j - w_i|^p \right]^{1/p} \\ &\text{s.t.} \\ &\sum_{i=1}^{n} w_i = 1, \\ &w_i > 0, \quad \forall i. \end{aligned} \tag{10}$$

Again the computational complexity of model (10) can be avoided by formulating the following Extended GP model, equivalent to the above model:

$$\begin{aligned} &\text{Min}\,(1-\lambda)D + \lambda \left[\sum_{t=1}^{k} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} (n_{ij}^t + p_{ij}^t) \right] \\ &\text{s.t.} \\ &m_{ij}^t w_j - w_i + n_{ij}^t - p_{ij}^t = 0, \quad i, j \in \{1, \dots, n\},\ t \in \{1, \dots, k\}, \\ &\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} (n_{ij}^t + p_{ij}^t) - D \leq 0, \quad t \in \{1, \dots, k\}, \\ &\mathbf{n} \geq \mathbf{0}, \quad \mathbf{p} \geq \mathbf{0}, \\ &\lambda \in [0, 1] \quad \text{(control parameter)}. \end{aligned} \tag{11}$$

It should be noted that there is only one control parameter, λ, in this formulation. In this sense, this formulation is less complex than for the model discussed in Sect. 3.

Again the priority vectors calculated from model (11) have different preferential interpretations according to the value of the metric/control parameter used. Thus, for metric $p = 1$ (i.e., $\lambda = 1$), the aggregated disagreement is minimized. For $p = \infty$ (i.e., $\lambda = 0$), the disagreement of the most displaced DM is minimized. And, finally, for intermediate values λ, compromises between these two opposite consensuses can be reached. By applying model (11) to the four pc matrices of the example given in Sect. 3, the opposite consensus weights plus an intermediate one were calculated (see Table 3).

4 Concluding remarks

When individuals show their preferences by pc matrices, a group decision-making problem (i.e., the aggregation problem) can be formulated according to the following scenarios:

(a) To search individual priority vectors and, then, to aggregate them.

This has been how a group decision-making problem has been traditionally stated in the context of AHP. This paper is not, however, exclusively confined to the AHP methodology. We deal with a general scenario where pc matrices do not necessarily verify any suitable property. Therefore, this point of view is not considered in this paper.

(b) To search a consensus pc matrix and then derive a common priority vector.

This problem arises when the group wants to act as a unit. In this way, DMs aggregate their preferences into a consensus pc matrix. In this paper, the Extended GP formulation has been shown to be a flexible tool for computing the common pc matrix and for deriving the common priority vector from it. Because we use two Extended GP formulations, the solutions to this problem are determined by two control parameter values.

(c) To derive a common priority vector directly from individual pc matrices.

This is the case where the group wants to act as separate individuals. In this case, they do not want to build a common pc matrix, but get a common priority vector directly from individual pc matrices. Again an Extended Goal Programming formulation is a flexible tool for computing the consensus priority vector. However, only one control parameter is needed in this case.

It is noteworthy that, in all the cases studied, Extended GP appears to be a friendly tool for solving all the underlying optimization problems. Moreover, this type of approach provides a clear preferential interpretation for all the resulting solutions.

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References

Aczel, J., & Saaty, T. L. (1983). Procedures for synthesizing ratio judgements. *Journal of Mathematical Psychology*, *27*, 93–102.

Barzilai, J., & Golany, B. (1994). AHP rank reversal, normalization and aggregation rules. *INFOR*, *32*, 57–64.

Bolloju, N. (2001). Aggregation of analytic hierarchy process models based on similarities in decision makers' preferences. *European Journal of Operational Research*, *128*, 499–508.

Charnes, A., & Cooper, W. W. (1977). Goal programming and multiple objective optimization—part 1. *European Journal of Operational Research*, *1*, 39–54.

Choo, E. U., & Wedley, W. C. (2004). A common framework for deriving preference values from pairwise comparison matrices. *Computers & Operations Research*, *31*, 893–908.

Chu, M. T. (1998). On the optimal consistent approximation to pairwise comparison matrices. *Linear Algebra and its Applications*, *272*, 155–168.

Dyer, R. F., & Forman, E. H. (1992). Group decision support with the analytic hierarchy process. *Decision Support Systems*, *8*, 99–124.

Dopazo, E., & González-Pachón, J. (2003). Consistency-driven approximation of a pairwise comparison matrix. *Kybernetika*, *39*, 561–568.

Forman, E., & Peniwati, K. (1998). Aggregating individual judgments and priorities with the analytic hierarchy process. *European Journal of Operational Research*, *108*, 165–169.

González-Pachón, J., & Romero, C. (1999). Distance-based consensus methods: a goal programming approach. *OMEGA-International Journal of Management Science*, *27*, 341–347.

González-Pachón, J., & Romero, C. (2004). A method for dealing with inconsistencies in pairwise comparisons. *European Journal of Operational Research*, *158*, 351–361.

Herrera, F., Herrera-Viedma, E., & Chiclana, F. (2001). Multiperson decision-making based on multiplicative preference relations. *European Journal of Operational Research*, *129*, 372–385.

Jones, D. F., & Mardle, S. J. (2004). A distance-metric methodology for the derivation of weights from a pairwise comparison matrix. *Journal of the Operational Research Society*, *55*, 869–875.

Koczkodaj, W. W., & Orlowski, M. (1999). Computing a consistent approximation to a generalized pairwise comparisons matrix. *Computers and Mathematics with Applications*, *37*, 79–85.

Linares, P., & Romero, C. (2002). Aggregation of preferences in an environmental economics context: a goal programming approach. *OMEGA-International Journal of Management Science*, *30*, 89–95.

Ramanathan, R., & Ganesh, L. S. (1994). Group preference aggregation methods employed in AHP: an evaluation and an intrinsic process for deriving members' weightages. *European Journal of Operational Research*, *79*, 249–265.

Romero, C. (2001). Extended lexicographic goal programming: a unifying approach. *OMEGA-International Journal of Management Science*, *29*, 63–71.

Saaty, T. L. (1980). *The Analytic Hierarchy Process*. McGraw–Hill: New York.